## People are Processors: Coalitional Auctions for Complex Projects

# (Extended Abstract)

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## ABSTRACT

To successfully complete a complex project, be it a construction of an airport or of a backbone IT system, agents (companies or individuals) must form a team (a coalition) having required competences and resources. A team can be formed either by the client based on individual agents' offers (centralized formation); or by the agents themselves (decentralized formation) bidding for a project as a consortium-in that case many feasible teams compete for the employment contract. In these models, we investigate rational strategies of the agents (what salary should they ask? with whom should they team up?) under different organizations of the market. We propose concepts allowing to characterize the stability of the winning teams. We show that there may be no (rigorously) strongly winning coalition, but the weakly winning and the auction-winning coalitions are guaranteed to exist. In a general setting, with an oracle that decides whether a coalition is feasible, we show how to find the winning coalitions with a polynomial number of calls to the oracle. We also determine the complexity of the problem in a special case in which a project is a set of independent tasks. Each task must be processed by a single agent, but processing speeds differ between agents and tasks.

### **Categories and Subject Descriptors**

I.2.11 [Artificial Intelligence]: Distributed Artificial Intelligence— Multiagent systems

## Keywords

game theory; co-opetition; cooperative game theory; coalition formation; equilibria; skill games; scheduling.

#### 1. INTRODUCTION & CONTRIBUTION

We present a new class of coalition games that models cooperation and competition between agents for the employment in a complex project. We consider two organizations of the market: (i) The winning coalition is selected by a central mechanism; the agents are strategic about the salaries they ask. (ii) The coalition formation is decentralized—the already-formed coalitions bid for the project, and the agents are strategic about asking salaries and choosing teams.

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	solution	Exist	Checking	Finding
Decentr.	RSW	no	$O(n^2 \cdot fcfc)$	$O(n^5 \log(nv) fc fc)$ (*-)
	SW	no	open	
	WW	yes	$O(n^{\circ} \log(nv) fc fc)$ (*-)	
	AW	yes	O(ffc)	$O(v \cdot ffc)$
Centr.	WN	N/A	O(fcfc)	
	SNE	yes (*)	O(fcfc)	$O(n^3 \log(nv) fcfc))$ (*-)
		no (+)	v	

Table 1: Our results for the general model. The column "Exist" indicates whether a coalition/equilibrium always exists. The column "Checking" shows the complexity of checking whether a given coalition satisfies the solution concept. The column "Finding" shows the complexity of finding a coalition/equilibrium (*ffc* and *fcfc* are the complexities of the problems FFC and FCFC). Asterisks (\*) denote results only for the project salary model; crosses (+) — only for the hourly salary model; dashes (-) — only if the salaries of the agents are rational numbers. Solution concepts: (R)SW = (rigorously) strongly winning, WW = weakly winning, AW = auction winning, WN = winning coalition (provided we have asking salaries), SNE = Strong Nash Equilibrium.

The contributions of this paper are as follows: (i) We identify and formalize a new class of coalition games. These games describe the agents gathering into groups and competing with other teams for the employment in a complex project. In a general setting we consider an oracle that decides which teams of agents (further referred to as coalitions) have sufficient skills to complete the project on time. In this way our games resemble cooperative skill games [1] and coalitional resource games [3] (these games, however, consider the problems in the grand coalition and interaction between its members; our approach is to expose multiple coalitions' competition). Thus, we do not apply the typical cooperative game theory concepts but instead model cooperation and competition of the agents as a non-cooperative game.

Next we explore two organizations of the market. In the *central-ized setting*, where the agents communicate only with the client, (ii) we prove that a Strong Nash Equilibrium (SNE) always exists unless there is no feasible coalition. We show how to find a SNE, and for the client—how to select the most profitable, but still winning coalition, with a polynomial number of calls to the oracle. In the *decentralized setting* (iii) we propose two concepts of a winning coalition. We prove that a strongly winning coalition may not exist, but a weakly winning coalition is guaranteed to exist (provided there exists a feasible one). We show how to find weakly/strongly

winning coalitions. (iv) We propose two mechanisms that the client can apply to find the winning coalition. We introduce the concept of an auction-winning coalition and show how to find one.

In this version we only give an outline of the results (Table 1). For the full version we refer the reader to [2].

#### 2. MODEL & SOLUTION CONCEPTS

We consider a model in which a client submits a single complex project to be executed. The project has a deadline d. The client has a certain valuation v of the project, that is the maximal price that she is able to pay for completing the project. The client has no additional utility from completing the project before the deadline: if she had, it could be expressed by changing the project description and submitting a project with shorter deadline.

A coalition C is a triple  $\langle N_C, \phi_C, c_C \rangle$  consisting of the set of participating agents  $N_C \subseteq N$ , a salary function  $\phi_C : N_C \to \mathbb{N}$  assigning salaries to member agents, and the total cost of the coalition  $c_C \in \mathbb{N}$ . Every agent *i* has her *minimal salary*,  $\phi_i^{\min} > 0$ , for which she is willing to work.

We consider two models of agents' compensation. Let  $\phi_{\mathcal{C}}^{tot}(i)$  denote the total amount of money agent *i* gets in coalition  $\mathcal{C}$  (naturally,  $c_{\mathcal{C}} = \sum_{i \in N_{\mathcal{C}}} \phi_{\mathcal{C}}^{tot}(i)$ ). In the project salary model  $\phi_{\mathcal{C}}^{tot}(i)$  is equal to the salary of the agent  $\phi_{\mathcal{C}}(i)$  (and thus does not depend on the amount of work assigned to that agent). In the hourly salary model  $\phi_{\mathcal{C}}^{tot}(i)$  is equal to the product of the salary  $\phi_{\mathcal{C}}(i)$  and the time  $t_i$  that *i* spends on processing her part of the project (thus, we implicitly assume that there exists a schedule from which the coalition members can extract  $t_i$ ).

The coalition C is *feasible* if there exist a schedule such that: (i) the project can be finished before the deadline (if the coalition does not have some required competences, we model this as a coalition that never finishes a project); (ii) the project budget is not exceeded  $(c_C \leq v)$ ; (iii) the cost  $c_C$  of the coalition C is consistent with the salaries  $\phi_C$ . Specifically, in the project salary model  $c_C = \sum_{i \in N_C} \phi_C(i)$ . In the hourly salary model there must exist a schedule in which each member *i* of the coalition C spends  $t_i$  time units on the project and  $c_C = \sum_{i \in N_C} t_i \phi_C(i)$ . Moreover, the salaries are higher than the minimal salaries,  $\phi_C(i) \geq \phi_i^{\min}$ .

We assume that there is an oracle that decides whether a given coalition is feasible (the project will be finished before the deadline). More precisely, we assume that there is an oracle solving the FFC problem, defined below:

PROBLEM 1. (FFC: FIND FEASIBLE COALITION). An instance of FFC consists of a project (with a deadline d and a budget v) and the set of the agents N with (known) minimal required salaries  $\phi_i^{\min}$ . The question is to find any feasible coalition or to claim there is no such.

In our results we also use the subproblem of finding a *cheapest* coalition (denoted as FCFC).

We consider two models of forming coalitions. First, we consider the *centralized formation*. Agents submit their bids—(asking) salaries  $\phi_i$ —directly to the client. The client chooses the members of the team that is awarded the project (we will call the winning team the coalition to use the same vocabulary as in the second part of the paper). Naturally, the client chooses the members so that the project is completed before the deadline for the smallest price. The members of the winning coalition are payed according to their asking salaries  $\phi_i$ .

Second, we consider the *decentralized formation* of the coalition. Agents communicate and are able to form coalitions by binding agreements. A coalition sends a bid—the total cost  $c_c$ —to the client; the bid represents the compensation the coalition expects to

get for completing the whole project. The cheapest coalition  $C^*$  wins the project and is payed  $c_{C^*}$ ; then  $c_{C^*}$  is allotted to the members of  $C^*$  according to the salary function  $\phi_{C^*}$ .

In the centralized model we consider the complexity of finding a winning coalition (provided we know the asking salaries of the agents). Further we consider the existence and the problem of finding a Strong Nash Equilibrium. In the decentralized model we consider the following notions of stability.

DEFINITION 1. The vector of actions  $\pi$  is a Rigorously Strong Nash Equilibrium (RSNE) if and only if there is no subset of the agents  $N_C$  such that the agents from  $N_C$  can make a collaborative action C (a set of actions played by agents) after which the payoff of each agent i from  $N_C$  would be at least equal to her payoff under  $\pi$  and the payoff of at least one agent  $i \in N$  would change.

Informally, a coalition is (rigorously) strongly winning if it constitutes a (rigorous) Strong Nash Equilibrium, i.e., the members will not deviate to other coalitions.

DEFINITION 2. The feasible coalition C is rigorously strongly winning if and only if there is an RSNE in which the agents from  $N_C$  get positive payoffs  $\phi_C$ . The feasible coalition C is strongly winning if and only if there is an SNE in which the agents from  $N_C$ get positive payoffs  $\phi_C$ .

The following theorem characterizes winning coalitions.

DEFINITION 3. A feasible coalition C is explicitly endangered by a coalition C' if (i) C' is feasible, (ii)  $N_C \cap N_{C'} = \emptyset$  and (iii) C'is cheaper than C.

A feasible coalition C is implicitly endangered by a coalition C' if (i) C' is feasible, (ii)  $N_C \cap N_{C'} \neq \emptyset$  and each agent from  $N_C \cap N_{C'}$ gets in C' at least as good salary as in C, and (iii) either  $N_C \neq N_{C'}$ or  $\phi_C \neq \phi_{C'}$ .

THEOREM 1. The coalition C is rigorously strongly winning if and only if C is not explicitly nor implicitly endangered by any coalition.

We also need weaker notions as a strongly winning coalition does not always exist.

DEFINITION 4. A feasible coalition C is weakly winning if it is not explicitly endangered by any coalition and for each feasible coalition C' such that C is implicitly endangered by C', there exists a feasible coalition C'' such that C' is explicitly or implicitly endangered by C''.

DEFINITION 5. A coalition C is auction-winning if and only if there is no feasible coalition C' such that  $b_{C'} < b_C$  and for each agent  $i \in N_C \cap N_{C'}$ , i gets better salary in C',  $\phi_{C'}(i) \ge \phi_C(i)$ .

Table 1 shows a summary of our results. See [2] for the proofs.

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