Optimal Crowdfunding Design

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Crowdfunding Campaign

- n potential consumers
- Pre-buy price τ for a product
- Threshold N

⇒ When the number of consumers who commit to pre-buy exceeds the threshold, the crowdfunding succeeds and the seller gets the pre-buy payments.

⇒ Raising funding for developing and producing new products

Mechanism Design Problem

Standard Scheme: One Price-Threshold Pair

- Each consumer i has private value $v_i \in \{v_L, v_H\}$
  - with probability p, $v_i = v_H$, with probability $1-p$, $v_i = v_L$
- Each consumer chooses action:
  - $a_i = 1$: contributes to the product
- Each consumer's utility:

$$u_i(a_i, a_{-i}) = \begin{cases} v_i - \tau, & \text{if } a_i = 1, \sum_{j=1}^{n} a_j \geq N \\ 0, & \text{otherwise} \end{cases}$$

- Bayesian Nash Equilibrium (BNE):

$$E_{v_{-i}\sim p}\left[u_i\left(a_i(v_i), a_{-i}(v_{-i})\right)\right] \geq E_{v_{-i}\sim p}\left[u_i\left(a_i'(v_i), a_{-i}(v_{-i})\right)\right], \quad \forall i, v_i, a_i'$$

- Non-trivial equilibrium:

$$E_{v_i\sim p}\left[\sum_{i=1}^{n} a_i(v_i)\right] \geq N$$

- Seller: maximize expected profit by choosing optimal pair (N, τ)

Results:

- Unique symmetric BNE
- Maximized expected profit:

$$R^* = \max\{R(1, v_H), R(1, v_L)\}$$

where

$$R(1, v_H) = (A - \gamma B)(1-p)^n + \sum_{k=1}^{n} (A - \gamma \max\{B - k(v_H - \tau_0), 0\} + \max\{k(v_H - \tau_0) - B, 0\}) \binom{k}{n} (1-p)^{n-k} p^k$$

$$R(1, v_L) = A - \gamma \max\{B - n(v_L - \tau_0), 0\} + \max\{n(v_L - \tau_0) - B, 0\}$$

- A: future profit ; γ: interest rate for outside option ; B: initial cost to start production ; τ_0: marginal cost for each product

Variation 1: Bulk Discount

Two Price-Threshold Pair

- Additional threshold N_2 $(1 \leq N \leq N_2)$ for the discounted price τ_2 $(\tau_2 \leq \tau)$
- Each consumer's utility:

$$u_i(a_i, a_{-i}) = \begin{cases} v_i - \tau, & \text{if } a_i = 1, N \leq \sum_{j=1}^{n} a_j < N_2 \\ v_i - \tau_2, & \text{if } a_i = 1, \sum_{j=1}^{n} a_j \geq N_2 \\ 0, & \text{otherwise} \end{cases}$$

- Seller: maximize expected profit by choosing optimal pair (N, τ, N_2, τ_2)

Results:

- Unique symmetric BNE
  - Depending on the relationship among v_L, v_H, τ, τ_2
- Maximized expected profit:

$$R^* = \max\{R(1, v_H, 1, v_H), R(1, v_L, 1, v_L)\}$$

where

$$R(1, v_H, 1, v_H) = R(1, v_H),\ R(1, v_L, 1, v_L) = R(1, v_L)$$

- Setting an additional pair of threshold and price does NOT help increase the seller's expected profit!

In reality:

- No exact number of consumers, but a rough estimation
- Relatively large τ and small N to guarantee a successful crowdfunding and some amount of money
- Discounted price (τ_2) with a larger threshold (N_2) for possibly large amount of money

Variation 2: Product Differentiation

Two Threshold

- Additional threshold N_1 $(1 \leq N_1 \leq N)$ for simplified version
- Each consumer i's value $v_{i1} \in \{v_l, v_h\}$ for simplified version
  - $v_l < v_L < v_h < v_H$ also follows binary distribution p
- Each consumer's utility:

$$u_i(a_i, a_{-i}) = \begin{cases} v_{i1} - \tau, & \text{if } a_i = 1, N_1 \leq \sum_{j=1}^{n} a_j < N \\ v_i - \tau, & \text{if } a_i = 1, \sum_{j=1}^{n} a_j \geq N \\ 0, & \text{otherwise} \end{cases}$$

- Seller: maximize expected profit by choosing optimal pair (N_1, N, τ)

Results:

- Unique symmetric BNE
  - Depending on the relationship among v_L, v_H, τ
- Lemma: for any integer $\hat{N}$ satisfying $N \leq \hat{N} \leq n+1$,

$$\arg\max_{N_1 \leq N \leq \hat{N}} R(N_1, N, \tau) = N_1 \text{ or } \hat{N}$$

- To maximize the profit, the seller should ONLY provide the standard version, or ONLY provide the simplified version!
- Additional threshold is NOT the cause for possible increase in profit!

In reality:

- Simplified version requires less initial and marginal cost.
- Also considered to guarantee a successful crowdfunding and some amount of money