How to Amend a Constitution: Model, Axioms, and Supermajority Rules

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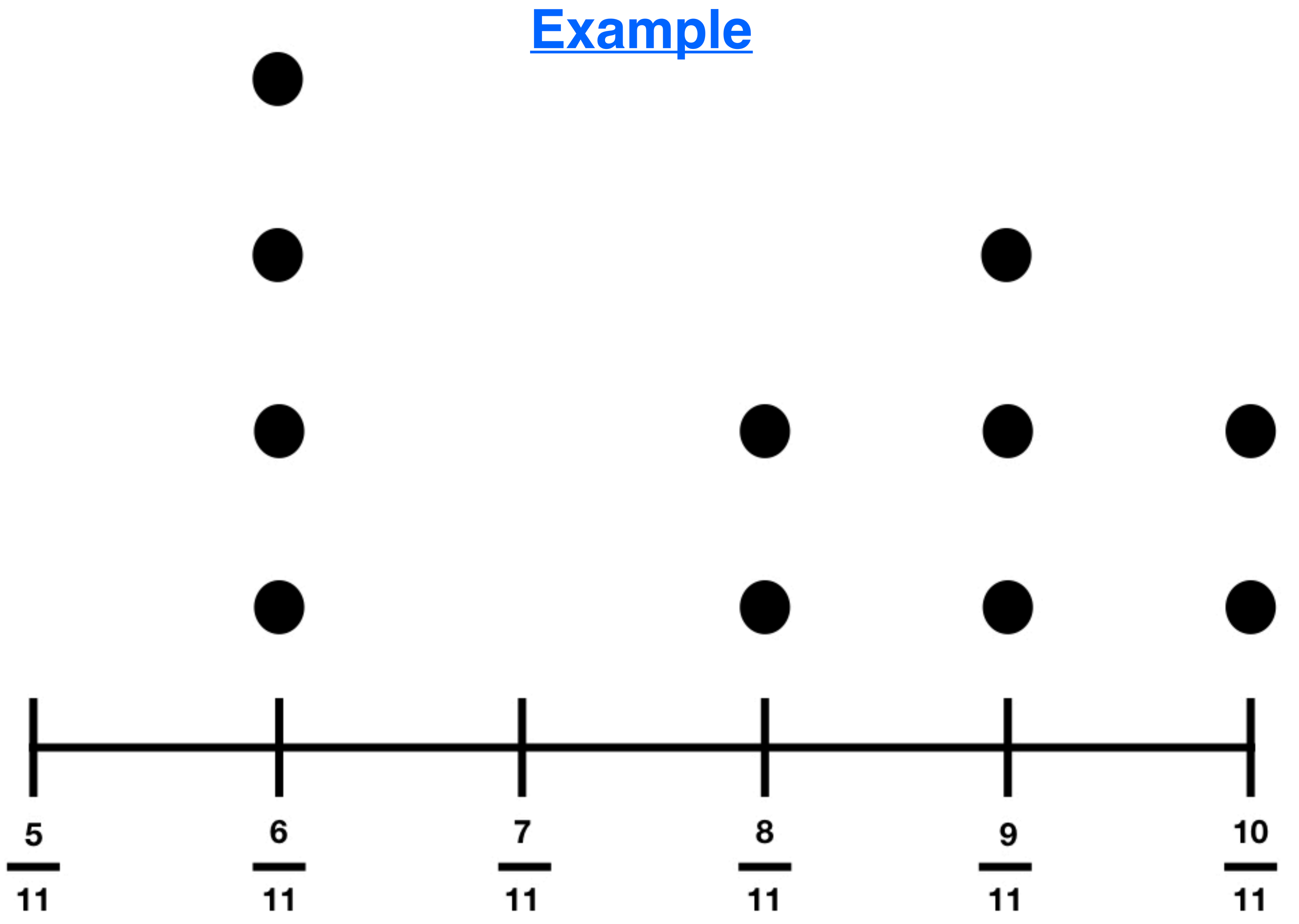
Introduction

We consider a simplified model of constitutions which consist of a set of possible decision rules, and an amendment procedure for choosing a decision rule, given that some decision rule is currently in use as the status quo. If the status quo is too easy to change, the constitution can be chaotic. For example, if the amendment procedure uses majority rule, then 3/5 of the voters could vote to lower the decision rule from 2/3 to 3/5, so that they can have their way. Conversely, if the status quo is too difficult to change, this can lead to instability in the form of revolt. We want to develop stable constitutions whose amendments will be acceptable to the agents.

Model

- A set of n agents V are currently using a δ-supermajority rule R^δ for decisions. Their constitution specifies an amendment procedure M for voting on whether to select a new decision rule.
- A δ-supermajority rule R^δ elects a proposal p over the status quo r if and only if it is preferred by strictly greater than δn agents. We denote the set of δ-supermajority rules for n agents by $\mathcal{A} = \frac{k}{n} : \lfloor \frac{n}{2} \le k < n, k \in \mathbb{N} \rfloor$.
- Every agent $v \in V$ has an ideal δ, or bliss point (also denoted v), and their preferences over $\mathcal{A}$ are single-peaked.
- An amendment procedure is a function $\mathcal{A} \times \mathcal{A}^n \to \mathcal{A}$ that updates the decision rule.
- We consider amendment rules that can be expressed as choosing a δ to use to choose between each possible set of decision rules $\{r, p\}$ for each amendment. We can write an amendment in two steps as $M(r,p) = \delta$, $R^\delta(r,p) = w \in \{r,p\}$ but we abbreviate this as $M^\delta(r,p) = w$.

Example



$V = \{6/11, 6/11, 6/11, 6/11, 8/11, 8/11, 9/11, 9/11, 9/11, 10/11, 10/11\}$.
The set of self-stable rules is $\{7/11, 8/11, 9/11, 10/11\}$.
The rules $7/11$ and $8/11$ are necessarily other-stable, while $9/11$ and $10/11$ may or may not be.

Results

Theorem. Constitution 1 ultimately elects a self-stable rule regardless of the initial status quo.

Theorem. Constitution 2 ultimately elects a self-stable and other-stable rule regardless of the initial status quo.

Theorem. Constitution 1 is complaint-free if $r \le m$, where m is the median of V.

Theorem. All decision rules $\delta \in [h, m]$ are self-stable and other-stable

Theorem. Constitution 3 is strategy-proof

Theorem. If voter preferences are 1-Euclidean, Constitution 3 has distortion with a tight upper bound of $\frac{h}{1-h}$.

Amendment Types

Definition (Evolution)**.** Amendment $M(r,p) = w$ is an evolution if $M^r(r,p) = w$

Definition (Revolution)**.** Amendment $M(r,p) = w$ is a revolution if $M^p(r,p) = w$

Proposed Axioms

Definition (Self-Stability)**.** A decision rule $r \in \mathcal{A}$ is self-stable with respect to a given profile if $M^r(r,p) = r$ for all $p \in \mathcal{A}$.

Definition (Other-Stability)**.** A decision rule $r \in \mathcal{A}$ is other-stable with respect to a given profile if $M^p(r,p) = r$ for all $p \in \mathcal{A}$.

Definition (Complaint-Freeness)**.** An amendment vote $M(r,p) = w$ is complaint-free if for all voters $v \in V$ either $M^v(r,p) = w$, or $w \succ_v \{r,p\} \backslash w$.

Constitutions

Constitution 1 Incremental Evolutionary Constitution

$r \in \mathcal{A}$ initialized or exists as status quo
for $p = r - \frac{1}{n}; p \in \mathcal{A}; p{-}= \frac{1}{n}$ **do**
  $r \leftarrow R^r(r,p)$
end for
for $p = r + \frac{1}{n}; p \in \mathcal{A}; p{+}= \frac{1}{n}$ **do**
  $r \leftarrow R^r(r,p)$
end for
elect r

Constitution 2 Incremental Revolutionary Constitution

$r \in \mathcal{A}$ initialized or exists as status quo
for $p = r - \frac{1}{n}; p \in \mathcal{A}; p{-}= \frac{1}{n}$ **do**
  $r \leftarrow R^p(r,p)$
end for
for $p = r + \frac{1}{n}; p \in \mathcal{A}; p{+}= \frac{1}{n}$ **do**
  $r \leftarrow R^r(r,p)$
end for
elect r

Constitution 3 The h-Rule

Elect $h = \arg\max_{\delta \in \mathcal{A}} |\{v \in V : v \ge \delta\}| \ge \delta n$ regardless of the status quo

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