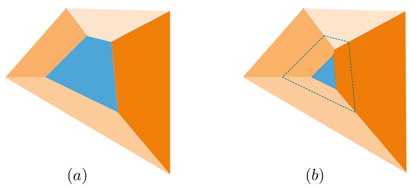

Figure 1: From (a) to (b), company in blue colour suffers from decrease of market area and neighbors due to price increment.

We now turn to the 2D case. The biggest challenge in analyzing the 2D market is that companies' neighbors may change when prices vary (Figure 1), while in the 1D market, company i's neighbors remain unchaged. Due to the change in neighbors, each company's utility function will be piecewise continuous (Figure 2), i.e., it changes every time a neighbor comes or goes. Moreover, since companies' locations are arbitrary, the shape of a company's market area may be irregular, which makes the analysis more difficult, even proving existence of equilibrium is non trivial.

THEOREM 2. *In the 2D market with $q = 0$, Nash equilibrium always exists, and when the market is at Nash equilibrium, we have:*

$$P_i = \frac{1}{\sum_{j \in \mathcal{NR}(i)} \frac{l_{ij}}{2d_{ij}}} S_i, \quad \forall i \in \mathcal{N}. \tag{3}$$

where l_{ij} is the length of border line between i, j and d_{ij} is the distance between i, j.
Similarly to the 1D case, the factor $\gamma = \sum_{j \in \mathcal{NR}(i)} \frac{l_{ij}}{2d_{ij}}$ represents the competition intensity. For a company i, farther distance to competitors (bigger d_{ij}) can reduce the competition intensity, while longer contiguous border (bigger l_{ij}) increases it.

4. MARKET EQUILIBRIUM WHEN $Q = 1$

In this section, we discuss the situation when $q = 1$, i.e., when the market area has a linear relationship with the brand name. We show that the interesting "wipe out" phenomenon appears when $q > 0$.

The "wipe out" phenomenon substantially increases the difficulty in analyzing the problem, because in this case a company's market area can suddenly shrinks to zero after some threshold price. In this case, its neighbors' utility functions are not continuous. This is exactly the same problem as in the classic Hotelling model, where "undercut" destroys the continuity of the utility function, and therefore leads to the non-existence of equilibrium.

THEOREM 3. *Nash equilibrium always exists in the 1D market with $q = 1$.*
Note that proving the existence of equilibrium under the possibility of "wipe out" is highly non-trivial, since any "sudden death" company may lead to chain reaction of all companies' pricing strategies. In fact, we can show that for any company i, the necessary and sufficient condition of surviving in the market is $\beta < \frac{2d_i d_{i-1}}{d_i + d_{i-1}}$. In another world, companies can survive better with farther distances to neighbors (greater d_i, d_{i-1}) or in a market with less brand effect (smaller β). Moreover, we can also show results similar to Theorem 2, but due to the "wipe-out" phenomenon, companies' strategies will be more conservative, and equilibrium prices will be lower than those under $q = 0$.

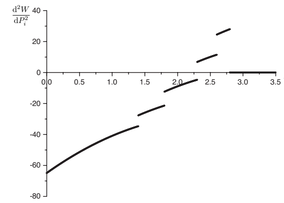

Figure 2: An example of second order derivative of the utility function. Each piece is a fragment from an increasing downward parabola.

5. CONCLUSION

In this paper, we study equilibrium properties based on a variant Hotelling model, considering brand name effect in the market by including a market area term into customers' aggregate price. We prove the existence of Nash equilibrium in single-feature and dual-feature market, and also derive explicit characterizations of equilibrium prices and market areas. Our results reconcile the common belief that company's pricing power is proportional to its market area over competition intensity, and offer insight into pricing under brand name effect and market positioning.

Specifically, our results offer the following insight: (i) When there is no brand effect or equivalent brand effect, i.e., β or q is zero, the equilibrium price of a company is proportional to its market area (market power) over the competition intensity with its neighbors (boundary over distance). (ii) When brand name has a positive effect in attracting customers, it is important to lower the price and seize more market area. (iii) New companies should try to avoid markets where the brand factor is large, and to avoid positioning at market points where competition is intense, because they can be "wiped out".

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REFERENCES

[1] C. d'Aspremont, J. J. Gabszewicz, and J.-F. Thisse. On hotelling's" stability in competition". *Econometrica: Journal of the Econometric Society*, pages 1145–1150, 1979.

[2] F. A. Fetter. The economic law of market areas. *The Quarterly Journal of Economics*, pages 520–529, 1924.

[3] H. Hotelling. Stability in competition. *The Economic Journal*, 39(153):pp. 41–57, 1929.

[4] C. D. Hyson and W. P. Hyson. The economic law of market areas. *The Quarterly Journal of Economics*, pages 319–327, 1950.

[5] E. Veendorp and A. Majeed. Differentiation in a two-dimensional market. *Regional Science and Urban Economics*, 25(1):75–83, 1995.

[6] J. M. Villas-Boas. Consumer learning, brand loyalty, and competition. *Marketing Science*, 23(1):134–145, 2004.

[7] B. Wernerfelt. Brand loyalty and market equilibrium. *Marketing Science*, 10(3):229–245, 1991.