Strategy-proof Mechanism Design for Facility Location Games: Revisited *

(Extended Abstract)

Lili Mei
College of Computer Science, Zhejiang University
meilili@zju.edu.cn

Minming Li
Department of Computer Science, City University of Hong Kong
minming.li@cityu.edu.hk

Deshi Ye
College of Computer Science, Zhejiang University
yedeshi@zju.edu.cn

Guochuan Zhang
College of Computer Science, Zhejiang University
zgc@zju.edu.cn

ABSTRACT

In facility location games, one aims at designing a mechanism to decide the facility location based on the addresses reported by all agents. In the standard facility location game, each agent wants to minimize the distance from the facility, while in the obnoxious facility game, each agent prefers to be as far away from the facility as possible. In this paper we revisit the two games on a line network by finely defining more reasonable agent cost (utility) functions in terms of their satisfaction degree with respect to the facility location. Namely, a happiness factor within $[0, 1]$ is introduced to measure the difference between the best facility location for an agent and the one given by the mechanism. Agents aim at a largest possible happiness factor while the social satisfaction is to maximize the total factors. For the standard facility location game, we observe that the median mechanism [4] is of $3/2$-approximation. We then devise a $5/4$-approximation group strategy-proof mechanism. For the obnoxious facility game, we show the majority mechanism [1] is best possible with approximation ratio of two.

Categories and Subject Descriptors

F.7.2 [**Theory of computation**]: Theory and algorithms for application domains—*Algorithmic game theory and mechanism design*

General Terms

Algorithms, Economics, Theory

*This work was partly supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China [Project No. CityU 117913] and the Nature Science Foundation of Zhejiang Province [Project NO. LQ15A010001].

Appears in: *Proceedings of the 15th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2016), J. Thangarajah, K. Tuyls, C. Marsella, S. Jonker (eds.), May 9–13, 2016, Singapore.*

Keywords

Algorithmic Mechanism Design, Facility Location Game, Happiness Factor, Social Satisfaction

1. INTRODUCTION

Facility location is one of the fundamental optimization problems, that assigns one or several facilities to a given network so that all clients in the network are served and the total cost is minimized. It is assumed that the full information of clients is known as public knowledge. In a game setting, however, the location of each client (we call an agent in the terminology of game theory) is private, that is not known before an algorithm is designed. The system manager (mechanism designer) is required to publicize an algorithm (mechanism) first. Then the agents report their locations (addresses) in the network, based on which the mechanism decides the facility locations. In the standard facility location game, each agent wants to minimize the distance from the facility, while in the obnoxious facility game, each agent prefers to be as far away from the facility as possible. It is of great interests if all agents are willing to tell the truth. The mechanism designer thus aims at strategy-proof mechanisms (where no agents have incentives to lie) maximizing the total utility (or minimizing the total cost) of the agents.

In the literature, the research of the facility location game and the obnoxious facility game has rich history. However, to the best of our knowledge, in both games, the cost or utility of each agent is simply the distance from the agent to the facility location. We observe that for agents at different positions, the best utilities or costs they can achieve from any mechanism will be different. To reflect the relative happiness of each agent compared to the best she can achieve, in this paper, we propose the happiness factor of each agent, which considers the agent's degree of satisfaction for the facility location.

Related results. In the facility location game, each agent wants to minimize the distance from the facility. Moulin [3] and Schummer and Vohra [5] characterized a class of generalized median voter schemes for the single-peak preference on the line and other networks, respectively. The study of approximation mechanism design for facility location games was first introduced by Procaccia and Tennenholtz [4]. They proposed the best possible strategy-proof mechanisms for the facility location game for minimizing the sum of all the agents' costs (minSum) and the maximum cost (minMax).

For the obnoxious facility game, each agent wants to stay far away from the facility. Cheng et al. [1] first proposed and studied the obnoxious facility game for maximizing the sum of all the agents' utilities (maxSum). Ibara and Nagamochi [2] characterized all the strategy-proof mechanisms.

2. PROBLEM STATEMENT

We study the setting where the underlying network is a closed interval $I = [0, 1]$. There are n agents in the interval, denoted as $N = \{1, 2, \ldots, n\}$. Each *agent* i has a location $x_i \in I$, which is private information. Let $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in I^n$ denote the agents' location profile. In our setting, a mechanism f is a function $f : I^n \to I$ where the input is the reported location profile and the output is the facility location. As mentioned before, in this paper, each agent wants to maximize her satisfaction degree, which is named happiness factor in our setting.

Let $y \in I$ denote the facility location. Since our setting is an interval, let $d(y, x_i) = |y - x_i|$ be the distance between *agent* i and the facility. Let $d^i_{max} = \max\{d(0, x_i), d(1, x_i)\}$ denote the longer distance from agent i to two endpoints. For the facility location game, the happiness factor function $h(y, x_i)$ for agent i is $h(y, x_i) = 1 - \frac{d(y,x_i)}{d^i_{max}}$. And for the obnoxious facility game, the happiness factor function is $h(y, x_i) = \frac{d(y,x_i)}{d^i_{max}}$. In our setting, the social satisfaction is the sum of all the agents' happiness factors, *i.e.*, $SH(y, \mathbf{x})$ is $\sum_{i \in N} h(y, x_i)$. In this paper, we aim to design mechanisms to elicit the location profile and maximize the social satisfaction as much as possible.

If a mechanism f elicits the true location profile, we say the mechanism is strategy-proof, *i.e.*, no agent can improve her happiness factor by misreporting. Formally, given any location profile $\mathbf{x}$, we have $h(f(\mathbf{x}), x_i) \geq h(f(x'_i, \mathbf{x}_{-i}), x_i)$ for all $x'_i \in I$, where $\mathbf{x}_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$ is the location profile without agent i. Moreover, a mechanism f is group strategy-proof if no any coalition of agents can improve their happiness factors by misreporting their locations simultaneously, *i.e.*, given any location profile $\mathbf{x}$, for any non-empty subset $S \subseteq N$, there exists $i \in S$ such that $h(f(\mathbf{x}), x_i) \geq h(f(\mathbf{x}'_S, \mathbf{x}_{-S}), x_i)$ for any $\mathbf{x}'_S \in I^{|S|}$, where $\mathbf{x}_{-S}$ is the location profile without agents in S.

Let y^* denote the optimal facility location. A mechanism f is ρ-approximation if $SH(y^*, \mathbf{x}) \leq \rho \cdot SH(f(\mathbf{x}), \mathbf{x})$ for any location profile $\mathbf{x}$.

3. MAIN RESULTS

3.1 The facility location game

We first study the median mechanism in [4], which returns the leftmost median agent's location as the facility location. In their setting, the mechanism gets an optimal solution. However, in our setting, it cannot guarantee optimality any longer.

THEOREM 3.1. *The median mechanism is a group strategy-proof mechanism with approximation ratio* $3/2$ *for maximizing the social satisfaction.*

Then we consider a better mechanism below.

MECHANISM 1. *Given a location profile* $\mathbf{x} \in I^n$*, without loss of generality, we assume that* $x_1 \leq x_2 \leq \cdots \leq x_n$*. Mechanism* f *outputs*

$$f(\mathbf{x}) = \begin{cases} 1/5 & \text{if } med(\mathbf{x}) \in [0, \frac{1}{5}] \\ 4/5 & \text{if } med(\mathbf{x}) \in [\frac{4}{5}, 1] \\ med(\mathbf{x}) & \text{otherwise} \end{cases},$$

where $med(\mathbf{x})$ *is the output of the median mechanism.*

THEOREM 3.2. *Mechanism 1 is a group strategy-proof mechanism with approximation ratio* $5/4$ *for maximizing the social satisfaction.*

We then consider lower bounds for the facility location game. We obtain the following theorem.

THEOREM 3.3. *Let* $N = \{1, 2, \cdots, n\}$*, where* $n \geq 2$*. Any strategy-proof mechanism cannot have an approximation ratio less than* $8 - 4\sqrt{3}$ *for maximizing the social satisfaction.*

We finally consider a mechanism when there are only two agents. We establish the following group strategy-proof mechanism and the approximation ratio is $8 - 4\sqrt{3}$.

MECHANISM 2. *Given a location profile* $\mathbf{x} \in I^2$*, if two agents is in different sides of* $1/2$*, mechanism* f *outputs* $1/2$*; otherwise outputs the one which is closer to* $1/2$*.*

3.2 The obnoxious facility game

In [1], a majority mechanism is presented. If the number of agents in $[0, 1/2]$ is larger than (or equal to) that in $(1/2, 1]$, the mechanism outputs 1, otherwise outputs 0.

THEOREM 3.4. *The majority mechanism is a group strategy-proof mechanism with approximation ratio* 2 *for maximizing the social satisfaction.*

Now we turn to consider lower bounds and we get the following theorem which implies that the majority mechanism is the best.

THEOREM 3.5. *Let* $N = \{1, 2, \cdots, n\}$*, where* $n \geq 2$*. Any strategy-proof mechanisms cannot have an approximation ratio less than* $2 - \epsilon$ *for maximizing the social satisfaction, where* $\epsilon > 0$ *is an arbitrary small number.*

REFERENCES

[1] Cheng, Y. and Yu, W. and Zhang, G.: Mechanisms for obnoxious facility game on a path. The 5th Annual International Conference on Combinatorial Optimization and Applications (COCOA), pp. 262–271, 2011.

[2] Ibara, K. and Nagamochi, H.: Characterizing mechanisms in obnoxious facility game. The 6th Annual International Conference on Combinatorial Optimization and Applications (COCOA), pp. 301–311, 2012.

[3] Moulin, H.: On strategy-proofness and single peakedness. Public Choice 35(4), pp 437-455, 1980.

[4] Procaccia, A.D. and Tennenholtz, M.: Approximate mechanism design without money. The 10th ACM conference on Electronic Commerce (EC), pp. 177–186, 2009.

[5] Schummer, J. and Vohra, R.V.: Strategy-proof location on a network. Journal of Economic Theory 104(2), pp 405-428, 2002.