	Boolean Games	iBGs
C-MEMBERSHIP	coNP	PSPACE
$\exists$-STABILITY	Σ_2^p	2EXPTIME
$\forall$-STABILITY	Π_3^p	in 3EXPTIME
$\exists$-STABLE-MAXIMAL	D_2^p	2EXPTIME
$\forall$-STABLE-MAXIMAL	in Π_4^p	in 3EXPTIME

Table 1: Overview of main complexity results.

reasoning about local (Nash) equilibria may be harder than reasoning about the "global" concept of Nash equilibrium. A summary of our main complexity results is in Table 1.

Additional complexity results. An inspection of many of the proofs underlying our results shows that minor variations can be applied so that the same complexities can be obtained for (some of) the problems we studied with respect to other solution concepts and models of games. For instance, with respect to the former, we know that the same complexities hold for the maximality problems if we consider subgame perfect Nash equilibrium or dominant strategies. On the other hand, with respect to the latter, also the same complexities hold for the maximality problems if we consider SRML games [8].

Local equilibria. To the best of our knowledge the concept of *local equilibria* has not been previously studied in the way we have presented it in this paper. However, there are games where the concept of locality has been investigated in other forms. For instance, in graphical games [5]—where games are given as graphs with each node representing a player and edges representing players' choices—a player may be required to best respond only to the behaviour of its neighbours. This is clearly a different notion of locality, but it also embodies the idea that a player's behaviour may be analysed only with respect to a subset of the overall set of players in the game. The concept of local (Nash) equilibrium has also been studied in the context of economic games [1], continuous games [17], and social networks [21]. These are all different notions of locality, both between them and with respect to ours. However, these other notions of locality share one common feature: they try to characterise the fact that if the space of strategies is reduced then a global optimum may not be achieved, only a local one (as constrained by the reduced space of strategies under consideration), which may be easier to compute or model more faithfully the fact that players in a game may have only bounded rationality.

Strategy Logic and Formal Verification. Our results also relate to recent work on rational synthesis [6, 11] and rational verification [7, 8, 20]. In these papers the concept of locality is not present, neither the concept of cost functions in the way we use them here. However, the solutions in such papers make use of Strategy Logic (SL), which at one point we also use here. Although SL can easily provide some upper bounds, as shown in this paper, this does not immediately mean that they are optimal (cf., $\forall$-STABILITY). In addition, SL does not yield optimal upper bounds for BGs and other related settings. For these we would need another logic.

Future work. From a practical point of view, it should be possible to have an implementation of local equilibria for BGs or for games with imperfect recall using MCMAS [12], since MCMAS supports SL with memoryless strategies [4]. From a theoretical point of view, ways to lower the overall complexity of the problems we considered should be investigated. Two promising directions are to consider games with simpler types of goals, or games with simpler strategy models.

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