Heterogeneous Facility Location Games

Eleftherios Anastasiadis
London Southbank University
London, Uk
lefterans@gmail.com

Argyrios Deligkas
Technion
Haifa, Israel
argyris@technion.ac.il

ABSTRACT

We study heterogeneous k-facility location games on a real line segment. In this model there are k facilities to be placed on a line segment where each facility serves a different purpose. Thus, the preferences of the agents over the facilities can vary arbitrarily. Our goal is to design strategy proof mechanisms that locate the facilities in a way to maximize the minimum utility among the agents. For $k = 1$, if the agents' locations are known, we prove that the mechanism that locates the facility on an optimal location is strategy proof. For $k \geq 2$, we prove that there is no optimal strategy proof mechanism, deterministic or randomized, even when $k = 2$ and there are only two agents with known locations. We derive inapproximability bounds for deterministic and randomized strategy proof mechanisms. Finally, we provide strategy proof mechanisms that achieve constant approximation. All of our mechanisms are simple and communication efficient. As a byproduct we show that some of our mechanisms can be used to achieve constant factor approximations for other objectives as the social welfare and the happiness.

KEYWORDS

facility location; mechanism design; communication complexity;

ACM Reference Format:
Eleftherios Anastasiadis and Argyrios Deligkas. 2018. Heterogeneous Facility Location Games. In *Proc. of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2018), Stockholm, Sweden, July 10–15, 2018,* IFAAMAS, 9 pages.

1 INTRODUCTION

Facility location games lie in the intersection of AI, game theory, and social choice theory and have been studied extensively over the past years. The basic version of the problem on a real line was firstly studied by Procaccia and Tennenholtz [25]. In their setting, a central planner has to locate a facility based on the *reported* locations of selfish agents. The goal of the planner is to locate the facility in a way that the sum of the utilities of the agents is maximized. [1] However, the agents can *misreport* their locations in order to manipulate the planner and increase their utility. One main objective of the planner is to design procedures to locate the facility, called *mechanisms*, that incentivize the agents to report their true locations, i.e., the mechanisms are *strategy proof.*

[1]Note that in [25] the objective was to minimize the social cost, which w.l.o.g. is the same as maximizing the social utility.

Proc. of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2018), M. Dastani, G. Sukthankar, E. André, S. Koenig (eds.), July 10–15, 2018, Stockholm, Sweden.

When monetary payments are not allowed, that is the planner cannot pay the agents or demand payments from them, it is not always possible to design mechanisms that implement an optimal solution and remain strategy proof. Thus, the goal is to design mechanisms that *approximately* maximize an objective function under the constraint that they are strategy proof. The term *approximate mechanism design without money*, introduced by Procaccia and Tennenholtz, is usually deployed for problems like the one described above. Procaccia and Tennenholtz studied *homogeneous* facility location games, where one or two *identical* facilities had to be placed on a real line and every agent wanted to be as close as possible to any of them. In this setting, the agents were reporting to the planner a point on the line and the objectives studied were the maximization of the *social welfare* and the *minimum utility* over the agents.

In many real life scenarios though, both facilities and agents' preferences are *heterogeneous*; every facility serves a different need and every agent has potentially different needs from the others. Consider for example the case where the government is planning to build a school and a factory on a street. Citizens' preferences for these facilities might significantly differentiate. Those who work at the factory and also have children that go to school wish both facilities to be built close to their homes. Citizens without children might want the school to be build far because of the noise. Finally, those who do not work at the factory prefer its location to be far from their home to avoid the emitted pollution.

The example above shows that an agent might want to be *close* to a facility, be *away* from a facility, or be *indifferent* about its presence. Feigenbaum and Sethuraman [8] studied 1-facility heterogeneous games where each agent reported his preferred location on the line, while it was known to the planner whether he wanted to be close to or away from the facility. Zou and Li [32] extended the model of [8] for heterogeneous 2-facility games and studied the social utility objective for several different scenarios of the information the planner knows. We note that none of the papers above studied the case where some agents were indifferent for some facilities. Serafino and Ventre [26] studied heterogeneous 2-facility games on discrete networks. In their setting, each agent is located on a node of a graph and either is indifferent or wants to be close to each facility and the planner knows the location of every agent but not their preferences for the facilities.

In this paper we extend the aforementioned models and study heterogeneous k-facility location games (simply k-facility games) on a given real line segment. Our main focus is to maximize the minimum utility among all the agents, termed Egalitarian. As byproduct we derive results for the social welfare, termed Utilitarian, and the recently proposed minimum *happiness* objective, termed Happiness. Happiness, which is reminiscent of the proportionality notion in resource allocation problems, is a fairness criterion for facility location problems introduced in [22]. The happiness of an agent is the ratio between the utility he gets under the locations of the facilities over the maximum utility the agent could get under any location. To the best of our knowledge, there is no prior work on this model. We note that while our model is a natural extension of the aforementioned models almost none of those results apply in our case.

1.1 Our contributions

We study several questions regarding heterogeneous k-facility games on a given line segment. For $k = 1$, we assume that the locations of the agents are known to the planner. We prove [2] that, for a large family of maxmin objectives, the mechanism that places the facility on an optimal location for the reported preferences of the agents is strategy proof. The only constraint we impose on the objective functions is that they are increasing with the minimum utility of the agents. We note that this result holds for a broad family of objective functions of the agents, including Egalitarian and Happiness. Our result complements the result of [8] where it was proven that there is no deterministic strategy proof mechanism with bounded approximation for Egalitarian for 1-facility games with known preferences but unknown locations.

Next, we focus on the Egalitarian objective. We prove that there is no optimal deterministic strategy proof or strategy proof in expectation mechanism for k-facility games even for instances with $k = 2$, two agents, and known locations for the agents. We complement these results by deriving inapproximability bounds for deterministic and randomized strategy proof mechanisms. The techniques we use are fundamentally different from [26], since in our model the facilities can be located anywhere on the segment without any constraint, making the analysis more complex.

Then, we focus on 2-facility games and we propose strategy proof mechanisms that achieve constant approximation ratio for the Egalitarian objective. All of our mechanisms are *simple* (i.e. if it requires minimal information from the agents (bit-wise)) and require *limited communication*. To the best of our knowledge, this is the first paper to study the communication complexity on facility location problems and how communication affects approximation. We propose two deterministic and two randomized mechanisms. The first deterministic mechanism, called `Fixed`, requires zero communication between the planner and the agents. On any instance, `Fixed` locates the facilities symmetrically away from the middle of the segment without requiring any information from the agents. Although this mechanism might seem naive, it achieves constant approximation. Furthermore, we prove that `Fixed` is *optimal* when no communication is allowed. No communication means that the agents do not transmit any bits to the planner before the locations for the facilities are decided, or equivalently that the facilities have to be located without getting any information from the agents. The second mechanism, termed `Fixed`$^+$, utilizes the intuition gained from `Fixed` and chooses between five different location-combinations for the facilities and locates the facilities in one of them by using the information it got from the agents. Furthermore, every agent has to communicate only 5 bits of information to the agent. Both of our randomized mechanisms are universally strategy proof. Our first randomized mechanism, termed `Random`, locates with half probability both facilities on the beginning of the segment and with half probability both facilities on the end of the segment. `Random` seems naive, but it achieves $\frac{1}{2}$-approximation and requires zero communication. The second randomised mechanism, `Random`$^+$, combines the ideas of `Random` and `Fixed`$^+$ and improves upon `Random` by requiring again only 5 bits of information per agent.

For the special case where agents' locations are known to the mechanism and all the agents are indifferent or want to be close to the facilities, then we show how we can utilize the optimal mechanism for the 1-facility game and get a $\frac{3}{4}$-approximate strategy proof mechanism for Egalitarian when $k = 2$.

As a byproduct, we show that `Fixed` and `Random` achieve the same approximation guarantee for Happiness and Utilitarian. Thus, we establish lower bounds that were not known before and complement the results of [32].

1.2 Further related work

There is a long line of work on homogeneous facility location games [1, 7, 13, 14, 20, 21, 31]. Different objectives and different utility functions have been studied as well. In [9] the objective was the sum of L_p norms of agent's utilities, while in [10] it was the sum of least squares. [12] introduced double-peaked utility functions. The obnoxious facility game on the line, where every agent wants to be away from the facilities, was introduced in [5] and later the model was extended for trees and cycles in [6]. In [29] the objective of least squares for obnoxious agents was studied. [4] recently introduced the maximum envy as an objective for facility location games. In that paper as well as in [15], the authors studied the approximation of mechanisms according to additive errors. [27] studied false-name proof mechanisms for the location of two identical facilities. [28] gave a characterization of strategy proof and group strategy proof mechanisms in metric networks for 1-facility games with private locations of the agents.

Simple mechanisms received a lot of attention lately; see [16] for example and the references therein for simple auctions. Informally, a simple mechanism is easy to implement and allows the agents to "easily" deduce the strategy proofness of the mechanism. One way to capture simplicity is to use *verifiably truthful* mechanisms [3], where agents can check whether a mechanism is strategy proof by using some, possibly exponential, algorithm. [18] formalized simple mechanisms by introducing *obviously* strategy proof mechanisms. [11] analysed this type of mechanisms for homogeneous 1-facility games.

After a long history in theoretical computer science [17], communication complexity problems studied on auction settings [2] and for more general mechanism design problems [23, 30]. To the best of our knowledge though, no one studied the communication complexity of facility location games.

2 MODEL

In a *k-facility game*, there is a set $N = \{1, \ldots, n\}$ of agents located on the line segment $[0, \ell]$ and a set of k distinct facilities

[2]Due to space constraints some of our proofs are omitted from the main body of the paper.

$F = \{1, \ldots, k\}$ that need to be located on the segment. Each agent i is associated with a location $x_i \in [0, \ell]$ and a vector $t_i \in \{-1, 0, 1\}^k$ that represents his preferences for the facilities.

If agent i wants to be *far* from the facility j, then $t_{ij} = -1$, if he is *indifferent*, then $t_{ij} = 0$, and if he wants to be *close* to j, then $t_{ij} = 1$. We will use $\mathrm{y} = (\mathrm{y}_1, \ldots, \mathrm{y}_k)$ to denote the locations of the facilities and $s = (s_1, \ldots, s_n)$ to denote the profile of the agents, i.e. their declared tuples $s_i = (x_i, t_i), \forall i \in N$. A vector $s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$ is the vector of tuples excluding s_i thus we can denote a profile as (s_i, s_{-i}). A *mechanism* M is an algorithm that takes as input a profile s and outputs the locations of the facilities, i.e. $\mathrm{y} = M(s)$. A mechanism is *deterministic* if it chooses y deterministically and *randomized* if y is chosen according to a probability distribution.

Let $\mathrm{OPT}(s)$ and $M(s)$ denote the optimal value and the value of mechanism M for an objective function under the profile s. A mechanism M achieves approximation ratio $\alpha \leq 1$, or it is α-approximate, if for any type profile s, it holds that $M(s) \geq \alpha \cdot \mathrm{OPT}(s)$.

The utility agent i gets from facility j, denoted as u_{ij}, depends on the distance $|x_i - \mathrm{y}_j|$ and on agent's preference for that facility. Following the literature [25, 26], the cost that agent $i \in N$ incurs from facility $j \in F$ is defined as:

$$c_{ij}(x_i, t_{ij}, \mathrm{y}_j) = \begin{cases} \ell - |x_i - \mathrm{y}_j|, & \text{if } t_{ij} = -1 \\ 0, & \text{if } t_{ij} = 0 \\ |x_i - \mathrm{y}_j|, & \text{if } t_{ij} = 1. \end{cases}$$

However, in our model, where all the three types of agents are available, there are cases in which the cost of the optimal solution is 0. Thus, the approximation ratio of a mechanism is undefined. In order to circumvent such cases we define the utility of an agent as his satisfaction over a produced outcome. For normalization purposes we assume that the maximum value of the satisfaction of an agent i is ℓ. For that reason we define his utility for a facility j as the difference between ℓ and cost he suffers. Formally,

$$u_{ij}(x_i, t_{ij}, \mathrm{y}_j) = \begin{cases} |x_i - \mathrm{y}_j|, & \text{if } t_{ij} = -1 \\ \ell, & \text{if } t_{ij} = 0 \\ \ell - |x_i - \mathrm{y}_j|, & \text{if } t_{ij} = 1. \end{cases} \quad (1)$$

The total (expected) utility agent i gets under y is defined as the sum of the utilities he gets for each of the facilities, i.e. $u_i(x_i, t_i, \mathrm{y}) = \sum_{j \in [k]} u_{ij}(x_i, t_{ij}, \mathrm{y}_j)$. We consider three different objective functions: Egalitarian, defined as $\max_{\mathrm{y}} \min_i u_i(x_i, t_i, \mathrm{y})$, Utilitarian defined as $\max_{\mathrm{y}} \sum_i u_i(x_i, t_i, \mathrm{y})$, and Happiness defined as $\max_{\mathrm{y}} \min_i \frac{u_i(x_i, t_i, \mathrm{y})}{u_i^*(x_i, t_i)}$ where $u_i^*(x_i, t_i) = \max_{\mathrm{y}} u_i(x_i, t_i, \mathrm{y})$.

A mechanism is called strategy proof if no agent can benefit by misreporting his preferences. Formally, a mechanism M is strategy proof if for any true profile (s_i, s_{-i}) it returns locations y and any misreported profile (s_i', s_{-i}) it returns y', it holds that $u_i(x_i, t_i, \mathrm{y}) \geq u_i(x_i, t_i, \mathrm{y}')$. A randomized mechanism is universally strategy proof if it is a probability distribution over deterministic strategy proof mechanisms and strategy proof in expectation if no agent can increase his *expected* utility by misreporting his type. Furthermore a mechanism is called *false-name proof* if no agent can benefit by using multiple and different identities in the game.

3 1-FACILITY GAMES

As a warm up we first study the case where the locations of the agents are publicly known and only one facility has to be placed on the segment. Although this scenario seems similar to the classical single peaked setting studied by Moulin [24], a closer look shows that it is not the same. The utility functions are defined in such a way that single peaked utility functions are a special case of ours.

Theorem 3.1. *When the locations of the agents are known, the mechanism that locates the facility on the left most location that maximizes* $\min_i u_i(x_i, t_i, \mathrm{y})$ *is strategy proof.*

Theorem 3.1 complements the result of [8]. There it was proven that there is no deterministic strategy proof mechanism with bounded approximation for Egalitarian for 1-facility games with known preferences but unknown locations.

4 INAPPROXIMABILITY RESULTS

For the rest of the paper, unless specified otherwise, we study Egalitarian. In this section we provide inapproximability results for strategy proof mechanisms for 2-facility games. We prove that, the extension of the optimal mechanism for two facilities, i.e. placing the facilities on the locations that maximize the objective under the declared preferences of the agents, is not strategy proof even in settings with two agents. Furthermore, we provide inapproximability results for strategy proof mechanisms.

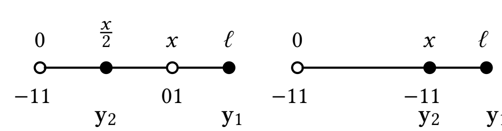
We prove that there is no 0.851-approximate deterministic strategy proof or strategy proof in expectation mechanism.

Theorem 4.1. *There is no α-approximate deterministic strategy proof mechanism for the 2-facility game with $\alpha \geq 0.851$.*

Proof. Let us consider the instances I and I' depicted in Figure 1. Each white circle corresponds to an agent. Agent a_1 is located on 0 and agent a_2 on x, where x will be specified later in the proof. Without loss of generality we assume that $\ell = 1$.

On instance I the preferences of a_1 are $t_1 = (-1, 1)$, while a_2 has preferences $t_2 = (0, 1)$. It is not hard to see that the optimal locations for the facilities are $\mathrm{y}_1 = 1$ and $\mathrm{y}_2 = \frac{x}{2}$ where each agent gets utility $2 - \frac{x}{2}$. The optimal locations are depicted by black circles in the figure.

On instance I' agent a_1 has the same preferences as on instance I while the preferences of agent a_2 are $t_2' = (-1, 1)$. The optimal locations for the facilities in this instance are $\mathrm{y}_1 = 1$ and $\mathrm{y}_2 = x$ where each agent gets utility $2 - x$.



(a) Instance I (b) Instance I'

Figure 1: Example for preferences in $\{-1, 0, 1\}^2$.

Instances I and I' show that the mechanism that locates the facilities on the optimal locations is not strategy proof. On instance I

agent a_2 can declare $t'_2 = (-1, 1)$ and increase his utility from $2 - \frac{x}{2}$ to 2.

So, let M be a strategy proof mechanism. Firstly, we argue that on instance I mechanism M should locate facility f_1 on 1. In order to see this, suppose that M locates f_1 in $z < 1$. Then, there is another mechanism M' locating f_1 in 1 that achieves a better approximation than M. At instance I the utility of a_1 decreases by moving the facility f_1 to z while the utility of a_2 does not change. At instance I the utility of both agents decreases by moving the facility f_1 from 1 to z. Hence, under mechanism M that locates f_1 on z, both agents on both instances get weakly lower utility than the mechanism M' that locates f_1 on 1. Next, suppose that M locates facility f_2 on $y_2 \leq x$ on instance I. Similarly as above, if $x < y_2$, then the approximation of M on instance I could be improved.

Since M is strategy proof, facility f_2 cannot be located on any $y'_2 > y_2$ on instance I'. If $y'_2 > y_2$, then agent a_2 from I could declare preferences $t'_2 = (-1, 1)$ and increase his utility. We consider the following two cases concerning the location y'_1 in which M locates facility f_1 on I':

- $y'_1 \geq x$. Then, obviously $y_1 = 1$ since otherwise the utility of both agents in I' is decreasing and thus M does not achieve the maximum approximation. So, under M agent a_2 on instance I' gets utility at most $2 - 2x + y_2$ and thus M achieves approximation $\frac{2-2x+y_2}{2-x}$. Furthermore, on instance I agent a_1 gets utility $2\ell - y_2$, since as explained earlier, M locates f_1 on 1. Thus, the approximation of M on instance I is $\frac{4-2y_2}{4-x}$. Observe that the approximation guarantee of M on I is decreasing with y_2 while on I' it is increasing with y_2. So, if we optimize the approximation guarantee and solve for y_2 we get that $y_2 = \frac{6x-2x^2}{8-3x}$. Thus, if $y'_1 > x$, the approximation of M is at most

$$\frac{4 - 2 \cdot \frac{6x-2x^2}{8-3x}}{4-x} = \frac{4x^2 - 24x + 32}{3x^2 - 20x + 32}. \tag{2}$$

- If M on instance I' locates f_1 on $y'_1 < x$, then observe that there is no location y'_2 for f_2 such that both agents get utility strictly larger than 1. Thus, in this case M achieves approximation at most

$$\frac{1}{2-x}. \tag{3}$$

Observe that the approximation guarantee in (2) increases with x while in (3) it decreases with x. So if we optimize on the approximation guarantee of M, we have to solve for x the equation $-4x^3 + 29x^2 - 60x + 32 = 0$. The unique solution in $[0, 1]$ is $x = \frac{13-\sqrt{41}}{8}$. Using this value in (2) and (3) we get that any deterministic strategy proof mechanism on instances I and I' achieves approximation less than 0.851. □

The inapproximability bound can be extended to strategy proof in expectation mechanisms.

Theorem 4.2. *There is no α-approximate strategy proof in expectation mechanism for the 2-facility game with $\alpha \geq 0.851$.*

5 DETERMINISTIC MECHANISMS

In this section we propose deterministic strategy proof mechanisms. An initial approach would be to consider each facility independently and place it to its optimal location. This mechanism is strategy proof. As we already proved, placing one facility on its optimal position is a strategy proof mechanism. Furthermore, since we locate the facilities *independently* no agent has an incentive to lie. However, this mechanism achieves poor approximation if the agents want to be away from the facilities. Consider the case where there are n agents on the locations $0, \frac{2\ell}{n}, \frac{3\ell}{n}, \ldots, \frac{(n-1)\ell}{n}, \ell$ each of whom has preferences $(-1, -1)$. Observe that the optimal location for one facility is to place it on $\frac{\ell}{n}$ since this location maximizes the minimum distance between any agent and the facility. Thus, both facilities will be placed on the same location $\frac{\ell}{n}$. Then the agent located in 0 has utility $\frac{2\ell}{n}$, the minimum over all the agents. It is not hard to see that an optimal solution is to locate the facility f_1 on 0 and the facility f_2 on ℓ where each agent gets utility ℓ. Hence, the mechanism that locates the facilities independently in their optimal locations is $\frac{2}{n}$-approximate.

The example above provides evidence that a mechanism with good approximation ratio should not put both facilities on the same location if there are agents who have preference -1 for both facilities; in the worst case the agent that is closest to the facilities might have preference -1 for both of them and thus get low utility. On the other hand, the facilities should not be far away from each other. This is because, in the worst case again, an agent might have preference -1 for the facility that is close to his location and preference 1 for the facility that is far from his location.

Using the intuition gained from the discussion above we propose a mechanism for the 2-facility game that comprises these ideas and places the facilities symmetrically away from the endpoints of the segment.

Mechanism `Fixed` depicts our approach. It does not use any information from the agents, thus it is de facto strategy proof.

Definition 5.1 (`Fixed` *Mechanism*). Let $z_f = 1 - \frac{\sqrt{2}}{2}$. `Fixed` mechanism sets $y_1 = z_f \cdot \ell$ and $y_2 = (1 - z_f) \cdot \ell$.

Theorem 5.2. `Fixed` *is $z_f \simeq 0.292$-approximate.*

Proof sketch. Tables 1 and 2 show the utility the agent located on x_i gets under $y = (z \cdot \ell, (1 - z) \cdot \ell)$ and the corresponding ratio. Our goal is to find a $z \in [0, \ell]$ that maximizes the minimum ratio. Thus, the optimal guarantee for Mechanism 2 is achieved when $\frac{z}{\ell} = \frac{\ell-2z}{2\ell-2z}$. If we solve for z, the feasible solution is $z_f = (1 - \frac{\sqrt{2}}{2})\ell$.

t_i	$u_i(x_i, t_i, y)$	$u^*_i(x_i, t_i)$	Ratio
1, 1	$\ell + 2x_i$	2ℓ	$\geq 1/2$
-1, 1	$2z \cdot \ell$	$2\ell - x_i$	$\geq z$
1, -1	$(2 - 2z) \cdot \ell$	$2\ell - x_i$	$\geq 1/2$
-1, -1	$\ell - 2x_i$	$2\ell - 2x_i$	$\geq (1 - 2z)/(2 - 2z)$

Table 1: Case analysis when $x_i \leq z \cdot \ell$ or $x_i \geq (1 - z) \cdot \ell$.

□

t_i	$u_i(x_i, t_i, \mathbf{y})$	$u_i^*(x_i, t_i)$	Ratio
1, 1	$(1+2z)\cdot\ell$	2ℓ	$\geq 1/2$
-1, 1	$2x_i$	$2\ell - x_i$	$\geq 2z/(2-z)$
1, -1	$2\ell - 2x_i$	$2\ell - x_i$	$\geq 2/3$
-1, -1	$(1-2z)\cdot\ell$	$2\ell - 2x_i$	$\geq (1-2z)/(2-2z)$

Table 2: Case analysis when $z\cdot\ell < x_i < (1-z)\cdot\ell$.

Theorem 5.2 shows the sharp contrast between 1-facility and 2-facility games where both locations and preferences are private. Recall that [8] proved that for 1-facility games there is no deterministic strategy proof mechanism with bounded approximation guarantee. Observe furthermore that `Fixed` does not require any information from the agents. Next we prove that it is optimal when no communication is allowed.

Theorem 5.3. `Fixed` *is optimal when no communication is allowed.*

Proof. Let M be any deterministic mechanism that locates the facilities with no communication. Since M is deterministic, it locates the facilities on the same locations for any instance. So, let $\mathbf{y}_1\cdot\ell$ and $\mathbf{y}_2\cdot\ell$ be the locations of the first and the second facility respectively. Without loss of generality assume that $0 \leq \mathbf{y}_1 \leq \mathbf{y}_2 \leq 1$. We will prove our claim by contradiction. So, for the sake of contradiction assume that the approximation ratio of M is strictly better than $z = (1-\frac{\sqrt{2}}{2})$. Without loss of generality we assume that $\mathbf{y}_1 \leq \frac{1}{2}$. Consider the following two instances. On the first instance there is only one agent on $\mathbf{y}_1\cdot\ell$ with preferences $(-1,-1)$. The utility of the agent under M is $(\mathbf{y}_2-\mathbf{y}_1)\cdot\ell$. The optimal solution locates both facilities on ℓ and the agent gets utility $(2-2\mathbf{y}_1)\cdot\ell$. So, the approximation ratio of M is $\frac{\mathbf{y}_2-\mathbf{y}_1}{2-2\mathbf{y}_1}$. Since the approximation of M is strictly greater than z, we get that

$$\mathbf{y}_1 < \frac{\mathbf{y}_2 - 2z}{1-2z}. \quad (4)$$

Now, consider the instance where there is only one agent on 0 with preferences $(-1,1)$. Under M, the agent gets utility $(1+\mathbf{y}_1-\mathbf{y}_2)\cdot\ell$. The optimal solution for this instance locates the first facility on ℓ, the second one on 0, and the agent gets utility 2ℓ. Hence, the approximation guarantee of M on this instance is $\frac{1+\mathbf{y}_1-\mathbf{y}_2}{2}$. Again, since we assume that the approximation is strictly greater than z, we get that

$$\mathbf{y}_1 > 2z + \mathbf{y}_2 - 1. \quad (5)$$

The combination of Equations (4) and (5) dictates that $\mathbf{y}_2 > 3 - \frac{1}{2z} - 2z > 1 - z$. Using another two instances we can prove that $\mathbf{y}_1 < z$. More specifically, we use the instance where there is only one agent on $\mathbf{y}_2\cdot\ell$ with preferences $(-1,-1)$ and the instance where there is only one agent on ℓ with preferences $(1,-1)$. Finally, consider again the instance where there is only one agent on 0 with preferences $(-1,1)$. Recall that the approximation guarantee of the mechanism on this instance is $\frac{1+\mathbf{y}_1-\mathbf{y}_2}{2}$. So, since $\mathbf{y}_1 < z$ and $\mathbf{y}_2 > 1-z$ we get that the approximation guarantee is strictly less than z which is a contradiction. Our claim follows. □

5.1 `Fixed`$^+$ mechanism

In order to describe `Fixed`$^+$, we need to introduce the following events:

- L_j: Every agent wants facility j below $\ell/2$. Formally, for every agent i with $x_i \leq \frac{\ell}{2}$ it holds that $t_{ij} \in \{0,1\}$ and for every agent i with $x_i > \frac{\ell}{2}$ it holds that $t_{ij} \in \{0,-1\}$.
- H_j: Every agent wants facility j above $\ell/2$. Formally, for every agent i with $x_i \leq \frac{\ell}{2}$ it holds that $t_{ij} \in \{0,-1\}$ and for every agent i with $x_i > \frac{\ell}{2}$ it holds that $t_{ij} \in \{0,1\}$.

`Fixed`$^+$ mechanism

Input: Locations $x_1,\ldots,x_n$ and preferences $p_1,\ldots,p_n$.
Output: Locations $\mathbf{y}_1$ and $\mathbf{y}_2$.
Set $z_d = \frac{17-\sqrt{161}}{16}$.

1. If events L_1 and L_2 occur, then set $\mathbf{y}_1 = \mathbf{y}_2 = z_d\cdot\ell$.
2. Else if events L_1 and H_2 occur, then set $\mathbf{y}_1 = z_d\cdot\ell$ and $\mathbf{y}_2 = (1-z_d)\cdot\ell$.
3. Else if events H_1 and H_2 occur, then set $\mathbf{y}_1 = \mathbf{y}_2 = (1-z_d)\cdot\ell$.
4. Else if events H_1 and L_2 occur, then set $\mathbf{y}_1 = (1-z_d)\cdot\ell$ and $\mathbf{y}_2 = z_d\cdot\ell$.
5. Else set $\mathbf{y}_1 = z_d\cdot\ell$ and set $\mathbf{y}_1 = (1-z_d)\cdot\ell$.

Lemma 5.4. `Fixed`$^+$ *is strategy proof.*

Proof sketch. Mechanism `Fixed`$^+$ is strategy proof due to the combination of the symmetric locations it locates the facilities and the way it chooses how to locate them. If the locations were not symmetric, or if z_d was varying through the Steps, then the mechanism would not be strategy proof. The intuition behind the strategy proofness is as follows. The first four steps of the mechanism trigger when the preferences of all the agents align in some way. On the other hand, Step 5 triggers only when agent's preferences conflict each other. Since there are only two plausible positions for each facility and they are symmetric, no agent can increase his utility by misreporting his location or his preferences. If the preferences are aligned any change on the facilities can only weakly decrease his utility. If Step 5 is triggered, then any kind of misreporting will either have no effect on the outcome, i.e., the conflicts will still be present, or it will make the preferences aligned, but the new location for the facilities will be worse for the agent that misreported his type. □

Theorem 5.5. `Fixed`$^+$ *is* $\frac{2z_d}{2-z_d} \simeq 0.311$*-approximate.*

Proof. In order to prove our claim, we will focus on the agent that gets the minimum utility under `Fixed`$^+$. We will prove that for every possible combination of his preferences and his location the agent gets at least $\frac{2z_d}{2-z_d}$ fraction of the utility he would get under an optimal solution. So, let i be an agent that gets minimum utility under `Fixed`$^+$. Without loss of generality we will assume that he is located below $\frac{\ell}{2}$. Observe that for the preference combinations $(0,1),(1,0),(0,-1),(-1,0)$ the agent gets utility at least ℓ, while the maximum utility he can get is trivially bounded by 2ℓ. Hence, if the agent's preferences are one of these combinations, then under any location for the facilities the agent gets at least half of his maximum utility and the mechanism is at least $\frac{1}{2}$-approximate.

- $p_i = (1, 1)$. Observe that if there exists an agent with preferences $(1, 1)$, then Fixed$^+$ will locate the facilities either through Step 1, or through Step 5. We will consider each case separately and we will identify the worst case for each one. Observe that under any of these steps, agent i gets utility at least $(1 + 2z_d) \cdot \ell$, while the maximum utility he can get is bounded by 2ℓ. So, the mechanism is $(\frac{1}{2} + z_d)$-approximate in any of these steps. Hence, the mechanism is $\frac{2z_d}{2-z_d}$-approximate.
- $p_i = (1, -1)$. When there exists an agent below $\frac{\ell}{2}$ with preferences $(-1, 1)$, then Fixed$^+$ will locate the facilities either through Step 2, or through Step 5. Observe that both steps locate the facilities in the same way. Since Step 2 allows more freedom in order to construct the instance where Fixed$^+$, we will study only this step. If we check Tables 1 and 2 we can see that in any case the ratio of the mechanism is greater than $\frac{1}{2}$.
- $p_i = (-1, 1)$. When there exists an agent below $\frac{\ell}{2}$ with preferences $(-1, 1)$, then Fixed$^+$ will locate the facilities either through Step 4, or through Step 5. When Step 4 is used, we can check Tables 1 and 2 and can see that in any case the ratio of the mechanism is greater than $\frac{1}{2}$. When Step 5 is used, the utility of agent i is at least $2z_d \cdot \ell$. For the chosen value of z_d, the worst case for this step is when agent i is located on $z_d \cdot \ell$ and there is another agent on $\frac{\ell}{2} + \epsilon$ with preferences $(0, 1)$. Then, the optimal solution locates the first facility on ℓ and the second one on z where the utility for agent i is $(2 - z_d) \cdot \ell$. Hence, the approximation guarantee of the mechanism for this case is $\frac{2z_d}{2-z_d}$.
- $p_i = (-1, -1)$. When there exists an agent below $\frac{\ell}{2}$ with preferences $(-1, -1)$, then Fixed$^+$ will locate the facilities either through Step 3, or through Step 5. When Step 3 is used by the mechanism, then agent i gets utility $(1 - z_d - x_i) \cdot 2\ell$ while the optimal value is trivially bounded by $(1 - x_i) \cdot 2\ell$. Hence, since $x_i \leq \frac{\ell}{2}$, the approximation guarantee of the mechanism is bounded by $1 - \frac{z_d}{2} > \frac{2z_d}{2-z_d}$. When Step 5 is used, the worst case instance for the mechanism is when agent i is located on z_d there is another one agent on $\frac{\ell}{2} - \epsilon$ with preferences $(1, 0)$. Then, the utility of agent i in the optimal solution is bounded by $\frac{7\ell}{4} - z_d \cdot \ell$ while the utility he gets under Fixed$^+$ is $(1 - 2z_d) \cdot \ell$. Hence, the approximation ratio of Fixed$^+$ is $\frac{1-2z_d}{\frac{7}{4}-z_d} = \frac{2z_d}{2-z_d}$, for the chosen value of z_d. □

Observe that since Fixed$^+$ asks for the exact location of every agent, it requires arbitrarily large communication; this happens for example when the location x_i of an agent i is irrational. However, a closer look shows that this is not necessary. Fixed$^+$ only needs to know whether an agent is located below or $\frac{\ell}{2}$. One bit suffices for this piece of information; an agent transmits 0 if he is below $\frac{\ell}{2}$ and 1 if he is above $\frac{\ell}{2}$. Furthermore, his preference for each facility requires two bits, so Fixed$^+$ requires only five bits per agent.

An interesting question is whether there exists a mechanism that achieves better approximation when every agent communicates $O(1)$ bits.

6 RANDOMISED MECHANISMS

In this section we propose two randomised mechanisms, Random and Random$^+$ that are universally strategy proof and achieve constant approximation ratio. Random requires zero communication and Random$^+$ can be implemented using five bits per agent.

Definition 6.1 (Random mechanism). Random sets $\mathbf{y}_1 = \mathbf{y}_2 = 0$ with probability $\frac{1}{2}$ and $\mathbf{y}_1 = \mathbf{y}_2 = \ell$ with probability $\frac{1}{2}$.

THEOREM 6.2. Random *is universally strategy proof and achieves* $\frac{1}{2}$ *approximation.*

PROOF. Firstly, it is easy to see that the the mechanism is universally strategy proof since in each case the mechanism chooses a fixed location, which is strategy proof. We will prove that every agent gets utility at least $\frac{\ell}{2}$ in expectation from every facility. Suppose that agent $i \in N$ is located on x_i and has preferences t_i. Let us study the expected utility the agent gets from the facility j. If $t_{ij} = 1$, then the agent's utility is $\ell - x_i$ when $\mathbf{y}_j = 0$ and x_i when $\mathbf{y}_j = \ell$. If $t_{ij} = -1$, then the agent gets utility is x_i if $\mathbf{y}_j = 0$ and $\ell - x_i$ if $\mathbf{y}_j = \ell$. If $t_{ij} = 0$, then the agent gets utility ℓ irrespectively from $\mathbf{y}_j$. It is not hard to see that the agent gets utility at least $\frac{\ell}{2}$ in expectation from each facility. So the agent in expectation gets utility at least ℓ. The maximum utility the agent can get is trivially bounded by 2ℓ. Hence, every agent gets at least half of the maximum utility he could get and the theorem follows. □

Although Random seems naive, it achieves the best approximation so far, using zero communication as well. However, we should note that Random can be extended for k-facility games, for any k, and achieve $\frac{1}{2}$ approximation. Furthermore, we use the intuition obtained from it in order to construct Random$^+$. The first four steps of Random$^+$ are the same as in Fixed$^+$, so again we will use the events L_j and H_j introduced in the previous section.

Random$^+$ mechanism

Input: Locations $x_1, \ldots, x_n$ and preferences $p_1, \ldots, p_n$.
Output: Locations y_1 and y_2.
Set $z_r = \frac{13-\sqrt{161}}{8}$

(1) If events L_1 and L_2 occur, then set $y_1 = y_2 = z_r \cdot \ell$.
(2) Else if events L_1 and H_2 occur, then set $y_1 = z_r \cdot \ell$ and $y_2 = (1 - z_r) \cdot \ell$.
(3) Else if events H_1 and H_2 occur, then set $y_1 = y_2 = (1 - z_r) \cdot \ell$.
(4) Else if events H_1 and L_2 occur, then set $y_1 = (1 - z_r) \cdot \ell$ and $y_2 = z_r \ell$.
(5) Else with probability $\frac{1}{2}$ set $y_1 = y_2 = z_r \cdot \ell$ and with probability $\frac{1}{2}$ set $y_1 = y_2 = (1 - z_r) \cdot \ell$.

LEMMA 6.3. Random$^+$ *is universally strategy proof.*

The proof of Lemma 6.3 uses similar arguments as Lemma 5.4.

THEOREM 6.4. Random$^+$ *is* $(\frac{1}{2} + z_r) \simeq 0.538$*-approximate.*

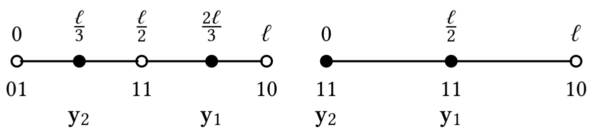
The proof of Theorem 6.4 uses similar arguments as Theorem 5.5. Furthermore, using the same technique as for $\texttt{Fixed}^+$, $\texttt{Random}^+$ can be implemented in a communication efficient way where each agent sends only five bits to the planner.

7 TWO-PREFERENCE INSTANCES

In this section we study k-facility games where all the agents have preferences in $\{0,1\}^k$, $\{1,-1\}^k$, or in $\{0,-1\}^k$, which we call two-preference instances. The non existence of optimal deterministic strategy proof mechanisms can be extended even on two-preference instances with three agents.

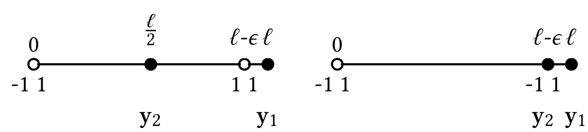
Theorem 7.1. *For any $k \geq 2$, there is no optimal deterministic strategy proof mechanism for k-facility games even on two-preference instances with three agents and known locations.*

The proof of the theorem follows from the instances below. As in Theorem 4.1, white circles correspond to agents and black circles to the optimal locations.



(a) Instance I (b) Instance I'

Figure 2: Example for preferences in $\{0,1\}^2$. The agent located on 0 in the instance I can declare preferences $(1,1)$ and increase his utility by moving the facility f_2 closer to 0.



(a) Instance I (b) Instance I'

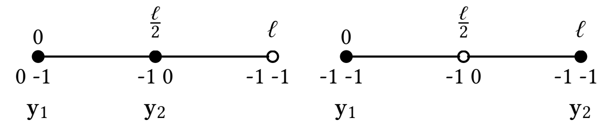
Figure 3: Example for preferences in $\{-1,1\}^2$. The agent located on $\ell-\epsilon$ in the instance I can declare preferences $(-1,1)$ and increase his utility by moving the facility f_2 closer to $\ell-\epsilon$.

We now show how we can modify $\texttt{Fixed}^+$ by changing the value of z_f and achieve better approximation guarantees. We denote the mechanisms as $\texttt{Fixed}^{\{0,1\}}$, for preferences in $\{0,1\}^k$, and $\texttt{Fixed}^{\{0,-1\}}$, for preferences in $\{-1,0\}^k$. Furthermore, for $k=2$ we derive a new deterministic mechanism termed OPT^2, for the the case where all agents have preferences in $\{0,1\}^2$ and their locations are known.

Definition 7.2. $\texttt{Fixed}^{\{0,1\}}$ sets $\mathrm{y}_1 = \ldots = \mathrm{y}_k = \frac{\ell}{2}$.

Theorem 7.3. $\texttt{Fixed}^{\{0,1\}}$ *is $\frac{1}{2}$-approximate.*

Proof. Observe that for every agent i and any facility j it holds that $u_{ij}(x_i, t_{ij}, \mathrm{y}_j) \geq \ell - |x_i - \frac{\ell}{2}| \geq \frac{\ell}{2}$. Hence, $u_i(x_i, t_i, \mathrm{y}) \geq \frac{k \cdot \ell}{2}$. Observe, however, that $\max_{\mathrm{y}} u_i(x_i, t_i, \mathrm{y}) \leq k \cdot \ell$. Hence, agent i under y gets at least half of his maximum utility. □



(a) Instance I (b) Instance I'

Figure 4: Example for preferences in $\{-1,0\}^2$. The agent located on 0 in the instance I can declare preferences $(-1,-1)$ and increase his utility by moving the facility f_2 away from 0. Observe that for the Instance I' there are two optimal solutions ($\mathrm{y}_1 = 0, \mathrm{y}_2 = \ell$ and $\mathrm{y}_1 = \ell, \mathrm{y}_2 = 0$). However, this does not affect the correctness of our example assuming that the mechanism chooses a solution *deterministically*.

Definition 7.4. $\texttt{Fixed}^{\{0,-1\}}$ sets $\mathrm{y}_1 = \ldots = \mathrm{y}_{\lceil \frac{k}{2} \rceil} = 0$ and $\mathrm{y}_{\lfloor \frac{k}{2} \rfloor} = \ldots = \mathrm{y}_k = \ell$.

Theorem 7.5. $\texttt{Fixed}^{\{0,-1\}}$ *is $\frac{\lfloor \frac{k}{2} \rfloor}{k}$-approximate.*

Proof. Observe that since $t_i \in \{0,-1\}^k$ it holds that $u_i(x_i, t_i, \mathrm{y}) = \sum_j \lceil \frac{k}{2} \rceil \cdot x_i + \lfloor \frac{k}{2} \rfloor \cdot (\ell - x_i) \geq \lfloor \frac{k}{2} \rfloor \cdot \ell$. Observe though that $\max_{\mathrm{y}} u_i(x_i, t_i, \mathrm{y}) \leq k \cdot \ell$. Hence, $\texttt{Fixed}^{\{0,-1\}}$ is at least $\frac{\lfloor \frac{k}{2} \rfloor}{k}$-approximate. □

Definition 7.6. OPT^2 locates each one of the two facilities independently on its optimal location.

It is not hard to see that OPT^2 is strategy proof. This is because we know that when agents' locations are known, the mechanism that locates one facility on the leftmost optimal location is strategy proof. So, since the mechanism locates each facility independently no agent can increase his utility by lying.

Theorem 7.7. *OPT^2 is $\frac{3}{4}$-approximate.*

Proof. Before we analyze the approximation guarantee of the mechanism, let us first study the locations in which the mechanism places the facilities. Since the preferences of each agent are in $\{0,1\}^2$, it is not hard to see that the optimal location for each facility is the median point between the locations of the leftmost and the rightmost agents that want to be close to the facility.

Without loss of generality we can assume that the agent with the minimum utility under OPT^2, denoted by a_1, has preferences $(1,1)$. If $t_i = (1,0)$, then the agent would have utility at least $\frac{3}{2}\ell$ since any other agent who wants to be close to the first facility is located in distance at most ℓ from a_1's location. The maximum utility the agent can get is 2ℓ, so the mechanism is then $\frac{3}{4}$-approximate.

Assume that a_1 is located on $x \leq \frac{\ell}{2}$. Then, without loss of generality we can assume that he is located on 0, since for any other location the agent would be closer to the facilities and thus his utility would increase. Then, observe that agent a_1, alongside with the rightmost agents, will define the locations of the facilities. Observe that if the rightmost agent has preferences $(1,1)$, then OPT^2 is optimal. So, we can assume that the rightmost agent, denoted by a_{r1}, has preferences $(0,1)$. In the worst case a_{r1} will be located on ℓ, since for every other location the utility of agent a_1 will be lower.

We have to consider the two possible preferences for the second rightmost agent with preference 1 for the first facility and prove that OPT^2 achieves the desired approximation. We will use a_i to denote this agent and x_i to denote his location.

Firstly, we consider the case where agent a_i has preferences $(1, 1)$ and $x_i \geq \frac{\ell}{2}$. The utilities of the agents for the facilities under the locations (y_1, y_2), where $y_2 \leq x_i$, are $u_1 = 2 - y_1 - y_2$, $u_i = 2 - 2x_i + y_1 + y_2$ and $u_{r_1} = 1 + y_2$. OPT^2 will locate the facilities to $y_1 = \frac{x_i}{2}$ and $y_2 = \frac{\ell}{2}$ and the utility of agent a_1 will be $u_1 = \frac{3-x_i}{2}$. Observe that the locations of the facilities that make the utilities of these three agents equal provide an upper bound on the utility the agent a_1 gets under the optimal solution, since any other solution would yield lower utility for at least one of these agents. If we find the locations of the facilities that equalize the utilities for the agents we get $y_1 = 2x_i - \ell$ and $y_2 = \ell - x_i$ and thus the optimal utility for agent a_1 is bounded by $2\ell - x_i$. Hence, OPT^2 is $\alpha = \frac{3-x_i}{4-2x_i} \geq \frac{3}{4}$-approximate.

In the case where $x_i < \frac{\ell}{2}$, it is not difficult to see that agent a_1 gets utility at least $\frac{5}{4}\ell$ under OPT^2. Observe that under the optimal solution the utility of the agents is bounded by $\frac{3}{2}\ell$, since there are no locations for the facilities where both a_1 and a_{r1} get more than $\frac{3}{2}\ell$. Thus, in this case the mechanism is $\frac{5}{6}$-approximate.

If the preferences of a_i are $(1, 0)$, then similar analysis can be applied. □

8 UTILITARIAN AND HAPPINESS

In this section we show that `Fixed`, $\texttt{Fixed}^{\{0,1\}}$, $\texttt{Fixed}^{\{0,-1\}}$, and `Random` achieve the same approximation guarantees for Utilitarian and Happiness objectives as Egalitarian. All mechanisms remain strategy proof since they do not require any information from the agents. Recall, Utilitarian is the sum of the utilities of the agents, formally $\sum_i u_i(x_i, t_i, y)$ and Happiness is $\min_i \frac{u_i(x_i,t_i,y)}{u_i^*(x_i,t_i)}$, where $u_i^*(x_i, t_i) = max_y u_i(x_i, t_i, y)$.

Theorem 8.1. *For Utilitarian and Happiness objectives the following hold.* `Fixed` *is* z_f*-approximate.* $\texttt{Fixed}^{\{0,1\}}$ *is* $\frac{1}{2}$*-approximate.* $\texttt{Fixed}^{\{0,-1\}}$ *is* $\frac{\lfloor \frac{k}{2} \rfloor}{k}$*-approximate.* `Random` *is* $\frac{1}{2}$*-approximate.*

Proof. In the proofs of Theorems 5.2, 7.3, 7.5, and 6.4 it is proven that for every agent i holds that $\frac{u_i(x_i,t_i,y)}{u_i^*(x_i,t_i)} \geq \alpha$, where α is the approximation ratio of the corresponding mechanism. Hence, the claim for Happiness already follows from those proofs since they capture the definition of Happiness. For Utilitarian, observe that $OPT_w = \max_y \sum_i u_i(x_i, t_i, y) \leq \sum_i u_i^*(x_i, t_i)$. So, from the proofs of the aforementioned theorems we get that $u_i(x_i, t_i, y) \geq u_i^*(x_i, t_i) \cdot \alpha$ for every i. So, if we sum over i we get that $\sum_i u_i(x_i, t_i, y) \geq \alpha \cdot \sum_i u_i^*(x_i, t_i) \geq \alpha \cdot OPT_w$ and the theorem follows. □

The observing reader may wonder whether the approximation guarantee of `Fixed` for Utilitarian contradicts the result of [32]. Recall, [32] proved that there is no deterministic strategy proof mechanism for Utilitarian with approximation ratio better than $\frac{2}{n}$. However, a closer look will reveal that in order to establish that result the following assumptions must be made. Firstly, that every agent wants to be close to the first facility and away from the second facility. Furthermore, they defined the utility of an agent located on x_i to be $u_i(x_i, y) = |x_i - y_1| - |x_i - y_2|$. This different definition of utility is crucial for deriving those negative results and this is the reason why our results do not contradict theirs.

9 DISCUSSION

In this paper we studied heterogeneous facility locations on the line segment. To the best of our knowledge, this is the first systematic study of this model for the Egalitarian objective. We derived inapproximability results for strategy proof mechanisms for Egalitarian even for instances with known locations and two agents. Furthermore, we derived strategy proof mechanisms that achieve constant approximation for Egalitarian, some of which also achieve the same guarantee for Utilitarian and Happiness objectives.

All of our mechanisms are simple and can be implemented in a communication efficient way. More specifically, every mechanism needs zero or five bits of information from every agent. Communication efficiency is crucial for real life scenarios. Consider the example of the factory and the school discussed in the introduction. If thousand of citizens live on this street, then our mechanisms require only their preferences and whether they live on the west part of the street or on the east one and not their full address saving huge amount of time to the planner. To the best of our knowledge, this is the first time that communication complexity is studied for facility location problems. We strongly believe that there is much to be said about facility location mechanisms and communication complexity. Firstly, it would be really interesting to understand how limited communication affects the approximation guarantee of mechanisms. Is there a better randomised mechanism than `Random` when no communication is allowed? Are there better mechanism than $\texttt{Fixed}^+$ and $\texttt{Random}^+$ when every agent is allowed to communicate $O(1)$ bits? Can $\texttt{Fixed}^+$ and $\texttt{Random}^+$ be extended for $k \geq 3$ facilities?

Another intriguing avenue of research is to use communication complexity in order to define "simple" mechanisms. Recently Li [19] defined the *obviously strategy proof* (OSP) mechanisms in order to capture the simplicity of mechanisms. Intuitively, a mechanism is obviously strategy proof if it remains incentive compatible even when some of the agents are not fully rational. The formal definition of OSP is quite technical, and thus we decided not to include it in our paper since it would deviate from its main theme. However, we strongly believe that some of our mechanisms, if not all of them, should be *obviously strategy proof* [19]. `Fixed` and `Random` do not use any information from the agents. In both $\texttt{Fixed}^+$ and $\texttt{Random}^+$, if an agent knows the declarations of the rest of the agents, then he can verify that he cannot increase his utility by misreporting his type using $O(1)$ space. We believe that this kind of mechanism are de facto simple and deserve further studying.

REFERENCES

[1] Noga Alon, Michal Feldman, Ariel D Procaccia, and Moshe Tennenholtz. 2010. Strategyproof approximation of the minimax on networks. *Mathematics of Operations Research* 35, 3 (2010), 513–526.

[2] Liad Blumrosen, Noam Nisan, and Ilya Segal. 2007. Auctions with Severely Bounded Communication. *J. Artif. Intell. Res.* 28 (2007), 233–266.

[3] Simina Brânzei and Ariel D. Procaccia. 2015. Verifiably Truthful Mechanisms. In *ITCS*. 297–306.

[4] Qingpeng Cai, Aris Filos-Ratsikas, and Pingzhong Tang. 2016. Facility Location with Minimax Envy. In *IJCAI*. 137–143.

[5] Yukun Cheng, Wei Yu, and Guochuan Zhang. 2011. Mechanisms for obnoxious facility game on a path. In *Combinatorial Optimization and Applications*. Springer, 262–271.

[6] Yukun Cheng, Wei Yu, and Guochuan Zhang. 2013. Strategy-proof approximation mechanisms for an obnoxious facility game on networks. *Theoretical Computer Science* 497 (2013), 154–163.

[7] Elad Dokow, Michal Feldman, Reshef Meir, and Ilan Nehama. 2012. Mechanism design on discrete lines and cycles. In *EC*. ACM, 423–440.

[8] Itai Feigenbaum and Jay Sethuraman. 2015. Strategyproof Mechanisms for One-Dimensional Hybrid and Obnoxious Facility Location Models. In *AAAI*.

[9] Itai Feigenbaum, Jay Sethuraman, and Chun Ye. 2013. Approximately Optimal Mechanisms for Strategyproof Facility Location: Minimizing L_p Norm of Costs. *CoRR* abs/1305.2446 (2013).

[10] Michal Feldman and Yoav Wilf. 2013. Strategyproof facility location and the least squares objective. In *EC*. ACM, 873–890.

[11] Diodato Ferraioli and Carmine Ventre. 2017. Obvious Strategyproofness Needs Monitoring for Good Approximations. In *AAAI*. 516–522.

[12] Aris Filos-Ratsikas, Minming Li, Jie Zhang, and Qiang Zhang. 2015. Facility Location with Double-Peaked Preferences. In *AAAI*, Blai Bonet and Sven Koenig (Eds.). AAAI Press, 893–899.

[13] Dimitris Fotakis and Christos Tzamos. 2010. Winner-imposing strategyproof mechanisms for multiple facility location games. In *Internet and Network Economics*. Springer, 234–245.

[14] Dimitris Fotakis and Christos Tzamos. 2014. On the power of deterministic mechanisms for facility location games. *ACM Transactions on Economics and Computation* 2, 4 (2014), 15.

[15] Iddan Golomb and Christos Tzamos. 2017. Truthful Facility Location with Additive Errors. *arXiv preprint arXiv:1701.00529* (2017).

[16] Yannai A. Gonczarowski and Noam Nisan. 2017. Efficient empirical revenue maximization in single-parameter auction environments. In *STOC*. 856–868.

[17] Eyal Kushilevitz and Noam Nisan. 1997. *Communication complexity*. Cambridge University Press.

[18] Shengwu Li. 2017. Obviously Strategy-Proof Mechanisms. *Available at SSRN* ssrn.com/abstract=2560028 (2017).

[19] Shengwu Li. Forthcoming. Obviously Strategy-Proof Mechanisms. *American Economic Review* (Forthcoming).

[20] Pinyan Lu, Xiaorui Sun, Yajun Wang, and Zeyuan Allen Zhu. 2010. Asymptotically optimal strategy-proof mechanisms for two-facility games. In *Proceedings of the 11th ACM conference on Electronic commerce*. ACM, 315–324.

[21] Pinyan Lu, Yajun Wang, and Yuan Zhou. 2009. Tighter bounds for facility games. In *Internet and Network Economics*. Springer, 137–148.

[22] Lili Mei, Minming Li, Deshi Ye, and Guochuan Zhang. 2016. Strategy-proof Mechanism Design for Facility Location Games: Revisited (Extended Abstract). In *AAMAS*. 1463–1464.

[23] Dilip Mookherjee and Masatoshi Tsumagari. 2014. Mechanism Design with Communication Constraints. *Journal of Political Economy* 122, 5 (2014), 1094–1129.

[24] Hervé Moulin. 1980. On strategy-proofness and single peakedness. *Public Choice* 35, 4 (1980), 437–455.

[25] Ariel D Procaccia and Moshe Tennenholtz. 2009. Approximate mechanism design without money. In *EC*. ACM, 177–186.

[26] Paolo Serafino and Carmine Ventre. 2016. Heterogeneous facility location without money. *Theoretical Computer Science* 636 (2016), 27–46.

[27] Akihisa Sonoda, Taiki Todo, and Makoto Yokoo. 2016. False-Name-Proof Locations of Two Facilities: Economic and Algorithmic Approaches. In *AAAI*. 615–621.

[28] Nguyen Kim Thang. 2010. On (Group) Strategy-Proof Mechanisms without Payment for Facility Location Games.. In *WINE*. Springer, 531–538.

[29] Deshi Ye, Lili Mei, and Yong Zhang. 2015. Strategy-Proof Mechanism for Obnoxious Facility Location on a Line. In *Computing and Combinatorics*. Springer, 45–56.

[30] Timothy Van Zandt. 2007. Communication Complexity and Mechanism Design. *Journal of the European Economic Association* 5, 2/3 (2007), 543–553.

[31] Qiang Zhang and Minming Li. 2014. Strategyproof mechanism design for facility location games with weighted agents on a line. *Journal of Combinatorial Optimization* 28, 4 (2014), 756–773.

[32] Shaokun Zou and Minming Li. 2015. Facility location games with dual preference. In *AAMAS*. International Foundation for Autonomous Agents and Multiagent Systems, 615–623.