

Multiple Levels of Importance in Matching with Distributional Constraints

Extended Abstract

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ABSTRACT

In this paper, we study the two-sided matching problem with soft diversity constraints in which each student belongs to one type and each school imposes soft targets on each type. We first identify limitations of type-specific quotas in a previous model and introduce a new general model that takes different levels of importance of types into account. Then we propose a new algorithm that yields a non-wasteful and fair outcome with respect to different levels of importance.

KEYWORDS

Two-sided matching; Diversity constraints; Soft quotas

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1 INTRODUCTION

Diversity concerns are pervasive in real-life matching markets. In the problem of *school choice with diversity constraints*, each student is associated with a set of types that capture traits such as being from a disadvantaged group. To achieve a balanced integration of students from diverse backgrounds, each school typically imposes a maximum quota and minimum quota on each type [2].

If diversity constraints are viewed as rigid bounds, then there may not exist any outcome that fulfills all type-specific minimum quotas and it may lead to undesirable waste of school seats. It is also impossible to design any mechanism that satisfies desirable properties such as fairness and non-wastefulness [2]. In addition, it is NP-hard to check the existence of feasible and stable outcomes [1]. Recent literature treat diversity constraints as *soft diversity constraints* such that a school may admit more students of some type than its maximum quota or fewer students of some type than its minimum quota [4].

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Ehlers et al. [2] proposed a fairness concept for soft diversity constraints that captures a natural idea called *dynamic priorities*: Schools give higher precedence to students whose types have not met their minimum quotas, and lower precedence to students whose types have reached the minimum quotas. Such dynamic priorities have been incorporated into mechanism design [3, 6].

There are two main concerns about the limitations of this concept. The first concern is whether one should indeed treat the status of being undersubscribed equally. Consider the case that one type is severely undersubscribed while the other type almost fills its minimum quota. It is reasonable to give higher precedence to some student of the former type, who will help more to achieve a diversity balance. The second concern is whether we should treat all types equally. It is common that some types are more important than others, such as gender over the districts where students are from. However, neither of these two concerns was taken into consideration in previous work.

In this paper, we propose a new model on school choice with soft diversity constraints that takes different levels of importance of types into account. The new model allows schools to specify the importance of each student in terms of achieving diversity constraints. Then we propose a new algorithm that yields a non-wasteful and fair outcome with respect to different levels of importance.

2 SCHOOL CHOICE WITH SOFT DIVERSITY CONSTRAINTS

In this section, we describe the previous model of the *school choice problem with soft diversity constraints*. An instance of the school choice problem with soft diversity constraints is composed of a tuple $(S, C, q_C, T, \eta, \bar{\eta}, \mathcal{X}, \succeq_S, \succeq_C)$. There is a set of students S and a set of schools C . A capacity vector $q_C = (q_c)_{c \in C}$ consists of each school c 's capacity q_c . Let T denote the type space and let $T(s)$ represent the set of types to which student s belongs. We followed the setting of [2] in which each student is associated with one type, i.e., $|T(s)| = 1$.

Each school c imposes a minimum quota $\underline{\eta}_c^t$ and a maximum quota $\bar{\eta}_c^t$ on each type $t \in T$. All type-specific minimum quotas of school c constitute a minimum vector $\underline{\eta}_c = (\underline{\eta}_c^t)_{t \in T}$, and all

minimum vectors of schools constitute a minimum matrix $\underline{\eta}$. Similarly let $\bar{\eta}$ denote the maximum matrix consisting of all schools' type-specific maximum quotas.

Each contract $x = (s, c)$ is a student-school pair denoting that student s is matched with school c . An *outcome* (or a matching) is a set of contracts. Let $\mathcal{X} \subseteq S \times C$ denote the set of available contracts. Given any outcome $X \subseteq \mathcal{X}$, let X_s denote the set of contracts involving student s , and X_c denote the set of contracts involving school c and $X_{c,t}$ denote the set of contracts involving type t and school c .

Each student s has a preference ordering \succsim_s over $\mathcal{X}_s \cup \{\emptyset\}$ where \emptyset is a null contract indicating that student s is unmatched. A contract (s, c) is *acceptable* to student s if $(s, c) \succsim_s \emptyset$ holds. Let $\succsim_S = \{\succsim_{s_1}, \dots, \succsim_{s_n}\}$ denote the preference profile of all students. Each school c has a priority ordering \succsim_c over $\mathcal{X}_c \cup \{\emptyset\}$. A contract (s, c) is *acceptable* to school c if $(s, c) \succsim_c \emptyset$ holds. Let $\succsim_C = \{\succsim_{c_1}, \dots, \succsim_{c_m}\}$ denote the priority profile of all schools.

An outcome X is *feasible* if i) each student s is matched to at most one school, and ii) each school c admits at most q_c students. A feasible outcome X is *individually rational* if each contract $(s, c) \in X$ is acceptable to both student s and school c . Without loss of generality, we focus on individually rational outcomes only.

Given a feasible outcome X , student s claims an empty seat of school c if $(s, c) \succ_s X_s$, $(s, c) \succ_s \emptyset$ and $|X_c| < q_c$. A feasible outcome is *non-wasteful* if no student claims an empty seat.

3 MULTIPLE LEVELS OF IMPORTANCE

Next, we introduce a function $f(\cdot)$ that brings flexibility to specify different levels of importance of each type to each school under different outcomes. In our new model of school choice with soft diversity constraints, an instance I^F consists of a tuple $(S, C, q_C, T, f(\cdot), \mathcal{X}, \succsim_S, \succsim_C)$ in which the two matrices $\underline{\eta}$ and $\bar{\eta}$ are replaced by the function $f(\cdot)$.

Formally, given an instance I^F and a feasible outcome X , the function $f(c, t, |X_{c,t}|)$ returns a real number that quantifies the importance of type t to school c , depending on the number of contracts involving type t that have already been assigned to school c in the outcome X . When there is no ambiguity, we simply $f(c, t, |X_{c,t}|)$ as $f(|X_{c,t}|)$. For each school c , there are at most $|S| \times |T|$ different values of $f(\cdot)$, where each distinct value specifies one particular level of importance. We assume that $f(\cdot)$ is a decreasing function, s.t., the value of $f(|X_{c,t}|)$ decreases as the number of contracts involving type t and school c increases.

Example 3.1. Take *dynamic priorities* considered in the introduction, for instance. One way to capture it is to define the following function $f(\cdot)$ that specifies three levels of importance.

$$f(|X_{c,t}|) = \begin{cases} 1 & \text{if } |X_{c,t}| < \underline{\eta}_c^t \\ 0 & \text{if } \underline{\eta}_c^t \leq |X_{c,t}| < \bar{\eta}_c^t \\ -1 & \text{if } |X_{c,t}| \geq \bar{\eta}_c^t \end{cases} \quad (1)$$

The function $f(|X_{c,t}|)$ returns 1 if type t is undersubscribed, returns 0 if type t has reached the minimum quota but not the maximum quota, and returns -1 if type t is oversubscribed.

We can resolve the two issues mentioned in the introduction by modifying the values of $f(|X_{c,t}|)$. For example, when type t is

severely undersubscribed, say $|X_{c,t}| < 0.5 \cdot \underline{\eta}_c^t$, school c can increase the value of $f(|X_{c,t}|)$ to 3 to emphasize the importance of type t . If some type t' is less important even though it is undersubscribed, school c can reduce the value of $f(|X_{c,t'}|)$ to 0.5.

We incorporate the function $f(\cdot)$ into our new fairness concept, which serves as a measurement of the contribution that some unmatched student s can make to school c in terms of achieving diversity goals. The higher value the function $f(\cdot)$ returns, the more the contribution that student will make. To simplify the presentation, we use another function $g(X_c, s, s')$ to compare the contribution of students s and s' made to school c in terms of achieving diversity goals. Given an outcome X and two students s, s' with $(s, c) \notin X, (s', c) \notin X$,

$$g(X_c, s, s') = f(|X_{c,t}|) - f(|X_{c,t'}|) \quad (2)$$

where student s and s' belong to type t and t' respectively.

Definition 3.2 (Fairness). Given an instance I^F and a feasible outcome X , student s has *justified envy* towards another student s' if i) $(s, c) \succ_s \{(s', c)\}$, $(s', c) \in X$ and ii) for the outcome $X' = X \setminus \{(s', c)\}$, one of the two following cases holds:

- a) $g(X'_c, s, s') > 0$; or
- b) $g(X'_c, s, s') = 0$ and $(s, c) \succ_c (s', c)$.

An outcome is fair if it admits no justified envy.

4 A NEW ALGORITHM

In this section, we present a new algorithm that yields a non-wasteful and fair outcome. We make use of the framework of the Generalized Deferred Acceptance algorithm (GDA) [5], which works as follows. Students first propose to their favorite schools. Each school then chooses a set of students based on its choice function and rejects others. Repeat this procedure until no more student is rejected. Different ways to determine the choice function of schools specify different implements of GDA algorithms.

Next, we define a choice function Ch_c^f of school c that takes multiple levels of importance into account, which generalizes the choice function in [2] to our new model in which there are multiple levels of importance of each type.

Input: A set of contracts X .
Output: A set of contracts $Y \subseteq X$.

- 1: $Y \leftarrow \emptyset$ % remove unacceptable contracts.
- 2: **while** $X_c \neq \emptyset$ **and** $|X_c| < q_c$ **do**
- 3: Scan over X_c based on school priority ordering \succ_c and select the first contract $y = (s, c)$ such that student s the greatest importance based on function $f(\cdot)$.
- 4: $Y \leftarrow Y \cup \{y\}, X \leftarrow X \setminus \{y\}$
- 5: **return** Y

Algorithm 1: Choice function Ch_c^f of school c

THEOREM 4.1. *The GDA algorithm with choice function Ch_c^f defined in Algorithm 1 always yields a non-wasteful and fair outcome.*

Note that Theorem 4.1 only holds when each student belongs to exactly one type, and the question on how to weaken fairness in a reasonable way to make it compatible with non-wastefulness when students have multiple types is still open.

REFERENCES

- [1] H. Aziz, S. Gaspers, Z. Sun, and T. Walsh. 2019. From Matching with Diversity Constraints to Matching with Regional Quotas. In *Proceedings of the 18th International Conference on Autonomous Agents and MultiAgent Systems*. International Foundation for Autonomous Agents and Multiagent Systems, 377–385.
- [2] L. Ehlers, I. E. Hafalir, M. B. Yenmez, and M. A. Yildirim. 2014. School choice with controlled choice constraints: Hard bounds versus soft bounds. *Journal of Economic Theory* 153 (2014), 648–683.
- [3] Y. A. Gonczarowski, N. Nisan, L. Kovalio, and A. Romm. 2019. Matching for the Israeli “Mechinot” Gap Year: Handling Rich Diversity Requirements. In *Proceedings of the 20th ACM Conference on Economics and Computation*. 321–321.
- [4] I. E. Hafalir, M. B. Yenmez, and M. A. Yildirim. 2013. Effective affirmative action in school choice. *Theoretical Economics* 8, 2 (2013), 325–363.
- [5] J. W. Hatfield and P. R. Milgrom. 2005. Matching with contracts. *American Economic Review* 95, 4 (2005), 913–935.
- [6] R. Kurata, N. Hamada, A. Iwasaki, and M. Yokoo. 2017. Controlled school choice with soft bounds and overlapping types. *Journal of Artificial Intelligence Research* 58 (2017), 153–184.