Veto Core Consistent Preference Aggregation

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ABSTRACT

The proportional veto principle is the notion that a coalition of x% of the voters should be able to block roughly x% of the outcomes. This is in opposition to the majority principle, which holds that 51% of the voters should have all the decision power and the remaining 49% zero; or the utilitarian principle, which focuses selecting an outcome that is best on average, even if that outcome is inadmissible for certain individuals or groups. Originating in public choice, the proportional veto principle found rich application in the theory of social multi-criteria evaluation and bilateral choice, but to date the family of voting rules which are consistent with this principle have not been subject to the same axiomatic scrutiny as majoritarian or positional approaches to voting. In this paper we address this gap by analysing two broad families of such rules, and six concrete examples, with respect to the properties of monotonicity, participation, and independence of unanimous losers.

KEYWORDS

preference aggregation; rank aggregation; voting rule; voting by veto; participation axiom; independence of irrelevant alternatives

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1 INTRODUCTION

Consider a choice situation with the following preference profile:

- $c \succ_1 b \succ_1 d \succ_1 e \succ_1 a \succ_1 f$.
- $c \succ_2 b \succ_2 f \succ_2 a \succ_2 d \succ_2 e$.
- $c \succ_3 f \succ_3 d \succ_3 e \succ_3 a \succ_3 b$.
- $b \succ_4 f \succ_4 a \succ_4 d \succ_4 e \succ_4 c$.
- $b \succ_5 f \succ_5 a \succ_5 d \succ_5 e \succ_5 c$.

In the abstract, it is meaningless to ask which letter is best suited for an unspecified purpose. We need some context. If we assume these are stand-ins for presidential candidates then a good case can be made for c – with a clear majority behind him, c's victory is likely to be accepted by society, and if he does not alienate his voters over the course of his tenure he is less likely to face parliamentary deadlock or have his policies overturned by referendum. The principle of *majoritarianism* has intuitive appeal, and is deeply rooted in the theory of democracy [4, part I, and 52, part I, II]. In social choice, the study of the majority principle goes back to Condorcet, who was perhaps the first to look at the mathematics behind such rules [4, part II, and 52, chapter 4].

If the letters stand for economic policies and the numbers for sectors of society, then b looks attractive – c is a terrible policy for almost half the society, whereas b seems likely to maximise aggregate income. It is a poor outcome for 3, but if that bothers us too much we can compensate 3 by levying a tax on the other sectors [25]. The principle at work here is *utilitarianism*, well known in political theory and especially in economics, which argues for quantifying and maximising the collective welfare of society [40, chapter 3]. In social choice, this goes back to Borda and Laplace who, unlike Condorcet were not interested in the will of the majority, but in finding the candidate with the most "merit" [4, part II, and 52, chapter 4].

But there are other choice situations we could think of. If we were choosing the healthiest food, then the high protein and vitamin content of b would not compensate for it being laced with mercury. If we propose an environmental treaty c to an international committee where participation is voluntary, diplomats 4 and 5 will pack their suitcases and leave. There are situations where for ethical or strategic purposes we cannot allow the outcome to be inadmissible to any voter, or by any evaluation criteria. This brings us to the *minority principle* and the idea of *voting by veto* [38, chapter 6]. Such voting rules are designed to pick a compromise not in the sense of an arithmetic average, but an outcome which insofar as possible does not harm overmuch the interests of any individual or coalition. The properties of such rules is the topic of this paper.

Related work

Voting by veto was first studied by Mueller [41] in the context of determining the funding of public goods. A group of n agents each submitted a spending proposal, to which the status quo was added for a total of $n + 1$. Agents took turns to cross out one of the proposals until one was left. Mueller studied the incentives such a procedure gave agents in deciding which proposal to submit, and the welfare properties of the chosen outcome.

Moulin continued the study of voting by veto schemes, allowing a voter to cast multiple [36], or even a fractional number of vetoes [38, chapter 6], which allowed him to extend the rule beyond the restriction of n voters and $n + 1$ outcomes. His main results were about the dominance solution of the procedure [36], and the functions it can implement [34, 37]. In [35] Moulin introduced the *proportional veto core*, which characterised all outcomes that can be attained while respecting a proportional distribution of veto power over the voters (see Definition 1 below).

Mueller and Moulin argued in favour of voting by veto and the proportional veto core by the protection these rules give to

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minorities (for a quantitative measure of minority protection of veto and other rules, see Kondratev and Nesterov [30]). This protection made proportional veto power attractive in the literature on social multi-criteria evaluation, where the focus is on situations where it is infeasible or unethical to choose an outcome that is terrible for some agents but good for others. A desirable voting rule should ensure that "minorities represented by criteria with smaller weights can still be very influent" [42]. Thus Gamboa and Munda [17] applied the proportional veto core in the choice of a windfarm location, and Diaz-Maurin et al. [10] for spent nuclear fuel disposal. A similar dynamic is present in bilateral choice, where no decision is possible without the approval of both parties; in this field almost every procedure in the literature selects from the veto core (for recent developments in bilateral choice, see Barberà and Coelho [3], Bol et al. [5], Laslier et al. [31], and the references therein).

In the computational literature, Bouveret et al. [6] noted the low communication complexity of voting by veto, while Ianovski and Kondratev [23] showed that the proportional veto core can be computed in polynomial time. Kim and Roush [26] were the first to suggest a continuous version of voting by veto and Ianovski and Kondratev [23] showed that this rule, which they called *veto by consumption* (see Definition 3 below), selects from the proportional veto core.

A surprising connection between voting by veto and distortion was discovered by Kizilkaya and Kempe [27], who found that a rule combining voting by veto and plurality scores achieved the optimal metric distortion rate, and in [28] showed the same for a plurality version of veto by consumption. Peters [45] further showed that all voting rules for selecting from the plurality version of the veto core achieve the optimal metric distortion rate.

2 PRELIMINARIES

We operate in the standard voting model [11, 14, 38, 48]. A voting situation consists of a set $\mathcal{V}$ of n *voters*, a set C of m *candidates*. We assume $\mathcal{V}$ and C are drawn from some well-ordered set such as the natural numbers; in particular, it is always possible to order $\mathcal{V}$ and C from first to last. Every voter is associated with a linear order over the candidates, which we term the voter's *preferences*. We use $\succ_i$ to denote the preference order of voter i. When brevity demands it, we may refer to a voter's preferences as his *type*, and use abc to denote the type of voter i with preference order $a \succ_i b \succ_i c$. An n-tuple of preferences/types is called a *profile*. A mapping that takes a profile to one or more candidates is called a *voting rule*.

A voting rule φ is *anonymous* if $\varphi(P) = \varphi(\pi P)$ for each permutation of the voters π and each profile P. It is *neutral* if $\varphi(\tau P) = \tau\varphi(P)$ for each permutation of the candidates τ and each P.

In this paper we are interested in voting rules based on the proportional veto principle, the idea of giving each coalition the power to block a number of candidates proportional to the size of the coalition [35].

Definition 1. Consider the mapping $v : 2^{\mathcal{V}} \to \mathbb{N}$ such that:

$$v(T) = \left\lceil m\frac{|T|}{n} \right\rceil - 1.$$

The value $v(T)$ is called the *veto power* of T. Intuitively, veto power is the number of candidates a coalition of voters T can veto.

A candidate c is *blocked* by a coalition T if there exists a *blocking set* of candidates, B, such that:

$$b \succ_i c \ \forall b \in B, \forall i \in T, \tag{1}$$

$$m - |B| \leq v(T). \tag{2}$$

Intuitively, condition (1) means that every voter in T considers every candidate in B to be better than c, and condition (2) means that the coalition T can guarantee that the winner will be among B by vetoing all the other candidates.

The set of all candidates that are not blocked is called the *veto core*. We shall use $\mathrm{VCore}(P)$ to denote the veto core of P. ∎

Example 2. Consider the profile from the introduction.

We have 5 voters and 6 candidates, and with $m = n + 1$ the veto power of a coalition simplifies to $v(T) = |T|$. Thus any individual voter has enough veto power to block their least preferred candidate, which eliminates $\{ b, c, e, f \}$. No voter individually can block d, but the coalition of 2, 4, and 5 can agree that the blocking set $\{ b, f, a \}$ is better than d. Since the coalition has enough veto power to cover the remaining three candidates, together they block d.

This leaves a in the veto core. ∎

The veto core is always non-empty [35], so it satisfies our definition of a voting rule – it maps a profile to a non-empty set of candidates. However, the veto core can be very large – under impartial culture, the size of the veto core tends to $m/2$ as $n \to \infty$ [23].

The large expected size of the veto core underlines the need for rules selecting subsets (preferably single-valued) of the veto core if we are to reap the benefits of veto power in practice. The first such rule was presented by Moulin [35], and operates by giving each voter r tokens and making t copies of each candidate. The voters take turns using their tokens to eliminate a copy of a candidate. If r and t are chosen appropriately, the candidates with a copy remaining at the end are guaranteed to be in the veto core. However, such a procedure assumes an order of voting, and is thus inherently non-anonymous. Kim and Roush [26] and Ianovski and Kondratev [23] introduced a continuous version of the procedure which recovered anonymity:

Definition 3. *Veto by consumption* is the voting rule that is computed by an algorithm that has voters eat the candidates from the bottom of their order up. Every candidate starts with capacity 1, and is being eaten by the voters who rank it last. Each voter eats at the speed of one candidate per time unit.

The outcome can be computed as follows. In round k, let c_i be the capacity of candidate i and n_i the number of voters eating i. The round lasts until some candidate is fully eaten. To move to round $k + 1$, do the following:

(1) Find an i which minimises c_i/n_i. Let r_k be this minimum ratio – this is the duration of the round.
(2) Update all capacities, $c_j = c_j - r_k n_j$.
(3) For all candidates who reached capacity 0, reallocate the voters eating these to their next worst candidates.

The last candidate to be eaten is the winner. In the case of two or more candidates being eaten simultaneously, a tie is declared among those candidates. ∎

Example 4. Consider the profile from the introduction. In the first round we have one voter each eating b, e, f and two voters eating c. In $1/2$ a time unit c is eaten, and b, e, f have $1/2$ capacity remaining. In the second round, voters 4 and 5 start eating e, so e is eaten by three voters. After $1/6$ a time unit, e is eaten and b, f have $1/3$ capacity left. In the third round, voters 2, 4, 5 start eating d, and after $1/3$ a time unit all candidates but a are eaten. In the last round a is eaten, so a is the winner. ■

The rule selects from the veto core, is neutral and anonymous, can be computed with a polynomial number of arithmetic operations, and empirically it tends to be single-valued [23]. However, other properties of this rule, or other rules that select from the veto core, have not been studied, which is the subject of this paper.

3 THE VETO CORE AS A VOTING PRINCIPLE

We begin by arguing that veto core consistency is something fundamentally novel, and the ethical principles incorporated in veto core consistency are not simple corollaries of, and in fact are often mutually exclusive with, standard desiderata commonly studied in voting theory. As a first step we will show that the class of veto core consistent voting rules is disjoint from two of the most famous principles in voting: the utilitarian principle of selecting the candidate with the highest overall "quality", usually traced to Borda and Laplace; and the majoritarian principle of respecting the will of the majority of the voters, associated with the work of Condorcet [4, part II, and 52, chapter 4]. A comprehensive overview of voting rules and their properties can be found in [11, 14, 38, 48].

Definition 5. A candidate c is a *majority winner* in a profile P if it is ranked first by over half the voters.

A voting rule φ is *majority-consistent* if for each P where c is a majority winner we have $\varphi(P) = c$. This includes the class of Condorcet methods as well as plurality, single transferable vote (STV), and Bucklin.

A candidate *a permutedly dominates b* in P if a has at least as many first positions, first + second positions, first + second + third positions, etc, as b, and at least one of these inequalities is strict [14, p. 166, and 13]. If all of these inequalities are strict, then *a strictly permutedly dominates b* [15].

A voting rule φ is *positional-consistent* if for each P where a permutedly dominates b and $b \in \varphi(P)$ we have $a \in \varphi(P)$. Positional-consistent rules include OWA-based scoring rules (which include the well-known positional scoring rules) [1, 9, 18, 19], DEA-based rules [51, chapter 4-5], and median-based rules such as Bucklin and convex median [30].

A voting rule φ is *veto core consistent* if $\varphi(P) \subseteq \mathrm{VCore}(P)$ for each P. The veto core and veto by consumption are veto core consistent, and we define two more families in Definition 9 below. ■

Proposition 6. *Veto core consistent and majority-consistent voting rules are disjoint.*

Veto core consistent and positional-consistent voting rules are disjoint.

Proof. We can verify both claims on the profile from the introduction. The veto core is $\{a\}$, so any veto core consistent rule must elect a uniquely. However, the majority winner is c, and d permutedly dominates a (observe that d has two third places, two fourth places, and one fifth place, while a has two third places, one fourth place, and two fifth places). Thus a majority-consistent rule must elect c, and a positional-consistent rule cannot elect a without d. □

However, every veto core consistent vote does satisfy weaker notions of utilitarianism and majoritarianism.

Definition 7. A candidate c is a *majority loser* in a profile P if it is ranked last by more than half the voters. A voting rule φ satisfies the *majority loser criterion* if for each P where c is a majority loser we have $c \notin \varphi(P)$.

A candidate a is *Pareto-dominated* by c in P if $c \succ_i a$ for every voter i. If a candidate is not Pareto-dominated in P, we say that it is *Pareto-efficient*. A voting rule satisfies *Pareto efficiency* if every election winner is a Pareto-efficient candidate. ■

Proposition 8. *Every veto core consistent voting rule satisfies:*

(1) *Majority loser criterion (indeed, such a rule will never elect a candidate ranked last by more than n/m voters).*
(2) *Pareto efficiency.*

Proof. (1) Suppose c is ranked last by more than n/m the voters. The coalition that ranks c last has a veto power of at least $\left\lceil m\frac{\lfloor n/m\rfloor+1}{n}\right\rceil - 1 \geq \lceil 1+\epsilon\rceil - 1 \geq 1$, so at the very least they can guarantee that c is blocked.
(2) The coalition of all voters will block every Pareto-dominated candidate, so the veto core consists only of Pareto-efficient candidates.

□

Proposition 8 provides an easy way to verify that some candidates are definitely blocked, and we rely on this a lot in the following proofs. In general, however, there is no easy way to demonstrate membership in the veto core, and one must either consider all possible blocking sets or rely on the polynomial-time algorithm [23].

The above results dealt with veto core consistent rules as a class. In the sequel we consider concrete rules and families in this class.

4 AXIOMATIC PROPERTIES

We have already defined two veto core consistent voting rules – the veto core and veto by consumption. To have something to compare these rules against, we can define two very broad families of veto core consistent rules.

Definition 9. Given a voting rule φ, define $\mathcal{S}_\varphi$ to be the voting rule that, on profile P, first computes $\mathrm{VCore}(P)$, then computes φ on the profile restricted to the candidates in $\mathrm{VCore}(P)$:

$$\mathcal{S}_\varphi(P) = \varphi(P\mid_{\mathrm{VCore}(P)}).$$

To define the second family, recall that a mapping that takes a profile to one or more rankings of the candidates from best to worst is called a *ranking rule*. Given ranking R, let $\mathrm{top}(R)$ be the top ranked element of R. Given a ranking rule ρ, define $\mathcal{R}_\rho$ to be the voting rule that, on profile P, computes $\rho(P)$, then selects the top ranked candidate(s) in $\mathrm{VCore}(P)$ with respect to rankings in $\rho(P)$:

$$\mathcal{R}_\rho(P) = \bigcup_{R\in\rho(P)} \mathrm{top}(R\mid_{\mathrm{VCore}(P)}).$$

■

Ranking rules are closely related to voting rules, and with some care properties of voting rules can be extended to ranking rules.

Definition 10. A ranking rule ρ is *anonymous* if $\rho(P) = \rho(\pi P)$ for each permutation of the voters π and each profile P. It is *neutral* if $\rho(\tau P) = \tau\rho(P)$ for each permutation of the candidates τ and each P.

Let $\underline{R}(c)$ be the set of candidates ranked no higher than c under R. A ranking rule ρ is *positional-consistent* if for each P where a permutedly dominates b (Definition 5) we have that for each $R \in \rho(P)$ there exists $R' \in \rho(P)$ with $\underline{R}(b) \subseteq \underline{R'}(a)$. It is *strongly positional-consistent* if for each P where a permutedly dominates b we have that a is ranked higher than b in each $R \in \rho(P)$.

Let $\overline{R}(c)$ be the set of candidates ranked no lower than c under R. A ranking rule ρ is *iterative positional-consistent* if for each triple of profile P, ranking $R \in \rho(P)$, and candidate c, this candidate does not strictly permutedly dominate any other candidate in the restricted profile $P\mid_{\overline{R}(c)}$.

Given a ranking rule ρ, define the voting rule $\mathrm{top}(\rho)$ by $\mathrm{top}(\rho)(P) = \bigcup_{R\in\rho(P)} \{\, \mathrm{top}(R) \,\}$. Let X be any property defined for voting rules. We say that ρ satisfies X *for top-ranked candidates* if $\mathrm{top}(\rho)$ satisfies X. ■

Example 11. Recall *generalised antiplurality* is the ranking rule that ranks candidates with the least last place positions first, breaking first-order ties by the number of second-to-last positions, second-order ties with the number of third-to-last positions, and so on.

Let P be obtained from the profile from the introduction by adding g to the bottom of each voter's preference order. The veto core is $\{\, a, b, f \,\}$ and the generalised antiplurality ranking is $dabfecg$. Hence, $\mathcal{R}_{\mathrm{GA}}(P) = a$, since a is the highest ranked candidate in $\{\, a, b, f \,\}$. In the restricted profile $P\mid_{\mathrm{VCore}(P)}$, we have three voters of type bfa, one voter each of types baf, fab. Candidates b and f have the least third places and b has the least second places, thus generalised antiplurality selects b and $\mathcal{S}_{\mathrm{GA}}(P) = b$. ■

Monotonicity

Monotonicity is meant to capture the notion that increased support for a candidate should not harm that candidate [14, p. 160]. It can be strengthened to strict monotonicity, which holds that new winners should not be added to the winning set [49], or positive responsiveness, which captures the idea that the smallest extra support for a tied candidate is enough to break the tie [14, p. 155, and 32].

Definition 12. Let P' be obtained from profile P by having one voter raise candidate c one position in his voting order. A voting rule φ is *monotonic* if for each P where $c \in \varphi(P)$ we have $c \in \varphi(P')$ [14, p. 160]. If it furthermore holds that $\varphi(P') \subseteq \varphi(P)$, then φ is *strictly monotonic* [49]. A voting rule φ satisfies *positive responsiveness* if for each P where $c \in \varphi(P)$ we have $\varphi(P') = c$ [14, p. 155, and 32].

Let $\underline{R}(c)$ be the set of candidates ranked no higher than c under R. A ranking rule ρ is *monotonic* if for each $R \in \rho(P)$ there exists $R' \in \rho(P')$ such that $\underline{R}(c) \subseteq \underline{R'}(c)$. A ranking rule ρ is positively responsive if $\underline{R}(c) \subseteq \underline{R'}(c)$ for all $R \in \rho(P)$ and $R' \in \rho(P')$. ■

Monotonicity is a very basic property satisfied by many voting rules. The exceptions are typically iterative rules such as STV, plurality with run-off, or sequential majority elimination [50, theorem 2, and 15], which can lead to controversy since such rules are used in many real elections [20, 21].

It turns out that it is quite easy to combine monotonicity and veto core consistency: it is satisfied by the veto core, $\mathcal{R}_\rho$ for suitable choice of ρ, and veto by consumption (Propositions 13, 15, 16). Since $\mathcal{S}_\varphi$ is essentially a two-stage elimination rule, it should be not surprising that it almost always fails monotonicity (Proposition 14).

Positive responsiveness is harder to satisfy than monotonicity. It is intuitively a positional notion, and thus it is not surprising that strongly monotonic scoring rules are positively responsive, but it is failed by many majority-consistent rules such as Copeland, Simpson's maxmin rule, and ranked pairs. We see below that veto core consistency is consistent with positive responsiveness, but we achieve this only by combining a veto core consistent rule with a positively responsive tie-breaking mechanism (Propositions 15, 18).

Proposition 13. *The veto core is strictly monotonic.*

Proof. Suppose $c \in \mathrm{VCore}(P)$. Let P' be obtained from P by having voter i raise c one position in his voting order.

We first show that $c \in \mathrm{VCore}(P')$. We proceed by contradiction. If $c \notin \mathrm{VCore}(P')$, then that must mean there exists a coalition T and a blocking set B such that:

$$m - |B| \le v(T) \ \text{ and } \ bP'_j c \ \forall b \in B, \forall j \in T.$$

Since in P c does not improve his position vis-à-vis any other candidate, c is blocked by T in P.

Next we show that $\mathrm{VCore}(P') \subseteq \mathrm{VCore}(P)$. Observe that if $a \notin \mathrm{VCore}(P)$, then that must mean there exists a coalition T and a blocking set B such that:

$$m - |B| \le v(T) \ \text{ and } \ bP_j a \ \forall b \in B, \forall j \in T.$$

Since in P' a does not improve his position vis-à-vis any other candidate, a is still blocked by T in P'. □

Proposition 14. *Let φ be a voting rule that is majority-consistent when the number of candidates is two. Then $\mathcal{S}_\varphi$ is not monotonic.*

Let φ be a neutral and anonymous voting rule. Then $\mathcal{S}_\varphi$ is not positively responsive.

Proof. Let φ be a voting rule that is majority-consistent when the number of candidates is two. Consider a profile with one voter each of types cba, bac, acb, and two voters each of types abc, cab, bca. The veto core is $\{\, a, b, c \,\}$. Observe that a is preferred by the majority to b, b to c, and c to a.

Without loss of generality, suppose that a is a (possibly tied) winner. Now suppose the voter of type cba changes their preferences to cab, moving a up one position. The veto core is now $\{\, a, c \,\}$, since b is ranked last by four voters. However c is the unique winner, since φ is the majority rule for $m = 2$.

Let φ be a neutral and anonymous voting rule. Consider the profile with a voter of type $dabc$ and a voter of type $cbad$. The veto core is $\{\, a, b \,\}$, and φ must declare a tie because in the reduced profile we have one voter of type ab and one of type ba. After changing the type of the first voter to $adbc$ the veto core is still $\{\, a, b \,\}$, and the reduced profile is still ab, ba, so the candidates are tied, which violates positive responsiveness. □

The requirements on φ in the above proposition are very weak, but we cannot relax them further. There exist monotonic $\mathcal{S}_\varphi$ if φ does not reduce to the majority rule on $m = 2$, for example with

φ being dictatorial, imposed (according to some fixed ranking of candidates), or indecisive (electing all candidates). Likewise, if we do not require φ to be neutral and anonymous, then $\mathcal{S}_\varphi$ is positively responsive with a dictatorial or imposed φ.

By Proposition 14, the veto core fails positive responsiveness, because the φ that elects every candidate is anonymous and neutral, and $\mathcal{S}_\varphi = \text{VCore}$.

Proposition 15. *If a ranking rule ρ is monotonic, then $\mathcal{R}_\rho$ is a monotonic voting rule.*

If a ranking rule ρ is positively responsive, then $\mathcal{R}_\rho$ is a positively responsive voting rule.

Proof. Consider a candidate $c \in \mathcal{R}_\rho(P)$. This means that c is top-ranked in $\text{VCore}(P)$ by some ranking $R \in \rho(P)$. The set of candidates ranked no higher than c, $\underline{R}(c)$, is such that $\text{VCore}(P) \subseteq \underline{R}(c)$. Let P' be obtained from P by having a voter raise c one position in his voting order.

Since the veto core is strictly monotonic, $c \in \text{VCore}(P') \subseteq \text{VCore}(P)$. Since ρ is monotonic (positively responsive), $\underline{R}(c) \subseteq \underline{R'}(c)$ for some (every) ranking $R' \in \rho(P')$. Hence,

$$c \in \text{VCore}(P') \subseteq \text{VCore}(P) \subseteq \underline{R}(c) \subseteq \underline{R'}(c).$$

For monotonicity, this establishes that $c \in \text{VCore}(P')$ and is top-ranked in at least one ranking in $\rho(P')$, and is thus a winner. For positive responsiveness, this establishes that $c \in \text{VCore}(P')$ and is top-ranked in every ranking in $\rho(P')$, and is thus the unique winner. □

The proof that veto by consumption is strictly monotonic is a bit involved, and can be found in the supplementary material. However, it is easy to see that it is not positively responsive.

Proposition 16. *Veto by consumption is strictly monotonic.*

Proposition 17. *Veto by consumption fails positive responsiveness.*

Proof. Consider the profile with a voter of type *abcd* and *cdab*. The winners are $\{ a, c \}$. Change the type of the second voter to *cadb*. The winners are still $\{ a, c \}$. □

However, do note that ties are crucial to the above proof. If we combine veto by consumption with a suitable tie-breaking mechanism, such as generalised antiplurality (see Example 11), positive responsiveness is recovered.

Proposition 18. *Veto by consumption with generalised antiplurality tie-breaking satisfies positive responsiveness.*

Proof. For a profile P, let B be the set of winners under veto by consumption and $A \subseteq B$ the set of winners under veto by consumption with generalised antiplurality tie-breaking. Let a profile P' be obtained from P by having one voter raise a candidate $c \in A$ one position in his voting order. Because generalised antiplurality ranks c no lower than any other candidate from B in P, it ranks c higher than any other candidate from B in P'. Denote B' the set of winners under veto by consumption in P'. By strict monotonicity of veto by consumption, $c \in B'$ and $B' \subseteq B$. Hence, generalised antiplurality ranks c higher than any other candidate from B' in P' and c is the unique winner under veto by consumption with generalised antiplurality tie-breaking. □

Independence of unanimous losers

A unanimous loser is a candidate that is ranked last by every single voter. Independence of unanimous losers formalises the notion that such candidates should have no effect on the election [12].

Definition 19. A candidate is a *unanimous loser* if the candidate is ranked last in every voter's preference order. A voting rule satisfies *independence of unanimous losers* if removing the unanimous loser does not change the set of elected outcomes. ■

The practical relevance of such independence is protection from spoilers – in some choice situations it is relatively easy to add a terrible outcome to the agenda, such as the ranking of wines [2], arbitrators [3], and athletes [29], and we should hope that the decision is not that easy to manipulate. By itself independence of unanimous losers is a fairly weak axiom, yet it is violated by a number of well-known rules such as antiplurality (without tie-breaking) [12, theorem 4.1], Nanson's procedure [29, appendix B], and the shortlisting procedure [3, example 4].

Veto by consumption clearly satisfies independence of unanimous losers, since all these candidates are eaten simultaneously by all the voters, before moving on to the real candidates. On the other hand it is easy to see that the veto core fails this property – once we add $m(n-1)$ unanimous losers,

$$v(T) = \left\lceil mn\frac{|T|}{n} \right\rceil - 1 = m|T| - 1,$$

and only the coalition of all voters will have enough veto power to block any real candidate. The veto core will consist of all Pareto-efficient candidates.

The other veto core consistent rules also fare poorly.

Proposition 20. *For every voting rule φ, $\mathcal{S}_\varphi$ fails independence of unanimous losers.*

Proof. Suppose, for contradiction, that $\mathcal{S}_\varphi$ satisfies independence of unanimous losers. Consider the profile P^{-d} with voter 1 of type *bca* and voter 2 of type *abc*. The veto core is $\{ b \}$, so b is the unique winner. Let P be the profile with voter 1 of type *bcad* and voter 2 of type *abcd*. It is obtained by adding the unanimous loser d, so by independence of unanimous losers $\mathcal{S}_\varphi(P) = b$. Consider another profile Q^{-d} with voter 1 of type *bac* and voter 2 of type *acb*. The veto core is $\{ a \}$, so a is the unique winner. Let Q be the profile with voter 1 of type *bacd* and voter 2 of type *acbd*, which is obtained by adding the unanimous loser d. It must be the case that $\mathcal{S}_\varphi(Q) = a$, but that is a contradiction because $\text{VCore}(P) = \text{VCore}(Q) = \{ a, b \}$ and the two profiles coincide on the relative ranking of a and b, so φ must choose the same winner in both cases. □

Proposition 21. *If ρ is majority-consistent for top-ranked candidates, positional-consistent, or iterative positional-consistent, then $\mathcal{R}_\rho$ fails independence of unanimous losers.*

Proof. Suppose ρ is majority-consistent for top-ranked candidates. Consider the profile with three voters of type *abc* and two voters of type *cba*. The veto core is $\{ b \}$, so b is the winner. Now add the unanimous loser d to obtain three voters of type *abcd* and two of type *cbad*. The veto core is $\{ a, b \}$, a is the majority winner, and hence $\mathcal{R}_\rho$ must select only a.

Suppose ρ is positional-consistent. Recall the profile from the introduction, where the veto core was $\{ a \}$. Now add two unanimous losers g and h at the bottom of every preference order to obtain profile P. The veto core is $\{ a, b, d, f \}$. Because d permutedly dominates a, there exists $R \in \rho(P)$ such that d is ranked higher than a. Hence, $\mathcal{R}_\rho$ cannot select only a.

Suppose ρ is iterative positional-consistent. Consider the profile with 2 voters of type acb, 4 of type bca, 5 of type abc. The veto core is $\{ b \}$. Now add the the unanimous loser d. Then d is obviously blocked, and c is blocked by the coalition of nine voters who prefer the blocking set $\{ b \}$ to c. No coalition blocks a or b, and the veto core is $\{ a, b \}$. For each $R \in \rho(P)$, d is ranked last, because all other candidates strictly permutedly dominate d. Then c is ranked second to last, because both a and b strictly permutedly dominate c in the profile restricted to $\{ a, b, c \}$. Then b is ranked below a, because a strictly permutedly dominates b in the profile restricted to $\{ a, b \}$. Hence, a is the unique winner. □

The above proposition shows that $\mathcal{R}_\rho$ fails independence of unanimous loser for many reasonable choices of ρ. We cannot prove such a proposition for *all* choice of ρ, since veto by consumption satisfies independence of unanimous losers, so we could choose ρ to be the elimination order of candidates in a run of veto by consumption. However, as the following proposition shows, this is essentially all we can do: if ρ satisfies some minimal requirements but is not itself veto core consistent, then $\mathcal{R}_\rho$ fails independence of unanimous losers.

Proposition 22. *Let ρ be a ranking rule that satisfies independence of unanimous losers and Pareto efficiency, but is not veto core consistent (all three properties for top-ranked candidates). Then $\mathcal{R}_\rho$ fails independence of unanimous losers.*

Proof. Suppose candidate c is top-ranked by some ranking in $\rho(P)$, Pareto-efficient, but not in the veto core. Hence c is not selected by $\mathcal{R}_\rho(P)$. In the profile P' that results from P by adding $m(n-1)$ unanimous losers, the veto core consists of all Pareto-efficient candidates, including c. Because ρ satisfies independence of unanimous losers for top-ranked candidates, c is still top-ranked by some ranking in $\rho(P')$ and hence is selected by $\mathcal{R}_\rho(P')$. □

Participation

Voter adaptability [46], positive involvement [50, proposition 2(iii)], and negative involvement [33, p. 93] are variations of the participation principle that a voter should not regret turning up to the polling booth. The properties formalised by these axioms are conceptually similar, but none of the three imply another.

Definition 23. For a profile P, we denote P_{-i} the profile obtained by removing a voter i from P.

A voting rule φ satisfies *voter adaptability* if for each profile P where voter i ranks a candidate c first and $\varphi(P_{-i}) = c$ we have $\varphi(P) = c$ [46].

A voting rule φ satisfies *positive involvement* if for each profile P where voter i ranks a candidate c first and $c \in \varphi(P_{-i})$ we have $c \in \varphi(P)$ [50, proposition 2(iii)].

A voting rule φ satisfies *negative involvement* if for each profile P where voter i ranks a candidate c last and $c \in \varphi(P)$ we have $c \in \varphi(P_{-i})$ [33, p. 93].[1] Rule φ satisfies *strong negative involvement* if in addition $\varphi(P_{-i}) = c$. ■

Participation is usually defined to require i to prefer the outcome with him than without him, $\varphi(P) \succeq_i \varphi(P_{-i})$, but considerable differences arise as to how authors deal with ties. It is a very strong property and among well-known rules it is only satisfied by the scoring rules, and is failed by every iterative scoring rule [47, corollary 6.1, and 43, theorem 7] and Condorcet method [39].[2] The three properties we consider are much weaker than standard participation, and rules satisfying voter adaptability, positive or negative involvement are somewhat more common. Scoring rules satisfy all three, as do certain Condorcet methods such as Simpson's maxmin rule, whereas Young's rule satisfies negative involvement but not positive involvement or voter adaptability [8, 44] and split cycle satisfies positive and negative involvement [22], but not voter adaptability. The Coombs rule satisfies negative involvement but not positive involvement or voter adaptability [11, p. 60, and 8], plurality with runoff and STV satisfy voter adaptability, positive but not negative involvement [33, p. 93, and 16, p. 208, and 8].[3]

We have been unable to devise a veto core consistent rule that satisfies voter adaptability, but neither could we show that veto core consistency and voter adaptability are inconsistent; the problem remains open. Negative involvement seems easier for a veto core consistent rule to satisfy than positive involvement, but the veto core satisfies both.

Proposition 24. *The veto core satisfies both positive involvement and negative involvement, but fails strong negative involvement.*

Proof. Let P be a profile with n voters and P_{-i} be obtained from P by removing voter i.

For positive involvement, suppose $c \in \mathrm{VCore}(P_{-i})$ and voter i ranks c first. Assume, for contradiction, that c is blocked in P. This means there exists a coalition $T \subseteq \mathcal{V}$ and blocking set B satisfying:

$$\forall b \in B, \forall j \in T : \; b \succ_j c,$$
$$m - |B| \leq \left\lceil m \frac{|T|}{n} \right\rceil - 1 \leq \left\lceil m \frac{|T|}{n-1} \right\rceil - 1.$$

Because c is voter i's top candidate, $i \notin T$ and c is blocked in P_{-i}.

For negative involvement, suppose $c \in \mathrm{VCore}(P)$ and voter i ranks c last. For contradiction, suppose $c \notin \mathrm{VCore}(P_{-i})$. This means there exists a coalition $T \subseteq \mathcal{V} \setminus \{ i \}$ and blocking set B satisfying:

$$\forall b \in B, \forall j \in T : \; b \succ_j c,$$
$$m - |B| \leq \left\lceil m \frac{|T|}{n-1} \right\rceil - 1.$$

[1] If a voting rule satisfies negative involvement, then it is resistant to the no-show paradox in Fishburn and Brams [16].

[2] Saari [47, corollary 6.1] considers resolute (i.e. single-valued) election outcomes and participation of two voters of the same type. Núñez and Sanver [43, theorem 7] assume that ties are broken lexicographically. Moulin [39] and Brandt et al. [7, theorems 3,4] show that every resolute Condorcet method fails participation. Jimeno et al. [24, proposition 2] and Brandt et al. [7, theorems 5-8] show that every irresolute Condorcet method fails participation for optimists or pessimists.

[3] STV is the only single-winner iterative scoring rule (with one by one elimination) that satisfies positive involvement [48, p. 258]. However, Fishburn and Brams [16, p. 212] show that STV does not satisfy positive involvement as a multiwinner voting rule and acknowledge that it has been known at least since 1910.

Voter i ranks c last, so $b \succ_i c$ for all $b \in B$. Thus there exists a coalition $T \cup \{ i \}$ for which:

$$\forall b \in B, \forall j \in T \cup \{ i \} : \ b \succ_j c,$$
$$m - |B| \leq \left\lceil m \frac{|T|}{n-1} \right\rceil - 1 \leq \left\lceil m \frac{|T|+1}{n} \right\rceil - 1.$$

As such, $T \cup \{ i \}$ blocks c in P.

To show that the veto core does not satisfy strong negative involvement consider a profile with voters of type cba, cab, abc, bac. The veto core remains $\{ a, b \}$ even if we remove the voter that ranks a last. □

However the veto core fails voter adaptability, as does every rule from the $\mathcal{S}_\varphi$ family.

Proposition 25. *For every voting rule φ, $\mathcal{S}_\varphi$ fails voter adaptability.*

Let φ be a majority-consistent voting rule when the number of candidates is two. Then $\mathcal{S}_\varphi$ fails positive involvement and negative involvement.

Proof. Suppose, for contradiction, that $\mathcal{S}_\varphi$ satisfies voter adaptability. Consider two groups of voters:

• $a \succ_1 b \succ_1 c$.	• $a \succ_1 b \succ_1 c$.
• $a \succ_2 b \succ_2 c$.	• $a \succ_2 c \succ_2 b$.
• $a \succ_3 b \succ_3 c$.	• $a \succ_3 c \succ_3 b$.
• $b \succ_4 c \succ_4 a$.	• $b \succ_4 a \succ_4 c$.
• $b \succ_5 c \succ_5 a$.	• $b \succ_5 a \succ_5 c$.
• $b \succ_6 a \succ_6 c$.	• $b \succ_6 a \succ_6 c$.

Consider the profile consisting of voters 1–5 on the left. The veto core is $\{ b \}$. Once we add 6, the veto core becomes $\{ a, b \}$, and by voter adaptability b should remain the unique winner.

Now consider the profile consisting of voters 2–6 on the right. The veto core is $\{ a \}$. Once we add 1, the veto core becomes $\{ a, b \}$, and by voter adaptability a should remain the unique winner. However, that is impossible because the left and right profiles coincide on their restriction to $\{ a, b \}$, so φ must select the same outcome in both cases.

Next, suppose φ is majority consistent for $m = 2$ and consider profile P_{-i} with five voters of type abc and three of type bca. The veto core is $\{ b \}$. If we add voter i with type bac the veto core becomes $\{ a, b \}$, and every majority-consistent φ selects only a. Hence, $\mathcal{S}_\varphi$ fails positive involvement.

Finally, consider a profile Q with four voters (1–4) of type abc, four (5–8) of type cab, four (9–12) of type bca. Without loss of generality, suppose a is among the winners according to φ, $a \in \varphi(Q)$. Now let P be a profile with four voters (1–4) of type $abcd$, one (voter 5) $cabd$, three (6–8) $cadb$, three (9–11) $bcad$, and voter i (voter 12) of type $bcda$. The veto core is $\{ a, b, c \}$, and the restriction of P to $\{ a, b, c \}$ is Q, so $a \in \mathcal{S}_\varphi(P) = \varphi(Q)$. However, if we remove i, then the veto core is $\{ a, c \}$ in P_{-i}, and only c wins by majority, $\mathcal{S}_\varphi(P_{-i}) = c$, violating negative involvement. □

Whether or not a rule of the family S_φ can satisfy strong negative involvement remains open.

Open Problem 26. *Does there exist a voting rule φ for which S_φ satisfies strong negative involvement?*

Veto by consumption, meanwhile, satisfies strong negative involvement (see the supplementary material), but fails both voter adaptability and positive involvement.

Proposition 27. *Veto by consumption satisfies strong negative involvement.*

Proposition 28. *Veto by consumption fails voter adaptability and positive involvement.*

Proof. Consider the voters:

- $a \succ b \succ c \succ d \succ e \succ f \succ g \succ h \succ i \succ j \succ k$.
- $k \succ j \succ i \succ h \succ g \succ f \succ e \succ d \succ c \succ b \succ a$.
- $f \succ a \succ b \succ c \succ d \succ e \succ g \succ h \succ i \succ j \succ k$.

In the profile with the first two voters the winner under any veto core consistent voting rule is candidate f. With all three voters the winner is d, but the third voter prefers f to any other candidate. (Candidates a, b, k, j, i, h are eaten first. Then g is eaten and the capacity of c is reduced to $1/2$. Then c is eaten and the capacity of f, e are reduced to $1/2$. Then f, e are eaten and the capacity of d is reduced to $1/2$.) □

The $\mathcal{R}_\rho$ family fares no better with respect to these properties.

Proposition 29. *Let ρ be a positional-consistent ranking rule. Then $\mathcal{R}_\rho$ fails voter adaptability. If, in addition, ρ is strongly positional-consistent, then $\mathcal{R}_\rho$ fails positive involvement.*

Suppose ρ is majority-consistent for top-ranked candidates, or iterative positional-consistent. Then $\mathcal{R}_\rho$ fails voter adaptability and positive involvement.

Proof. Suppose ρ is positional-consistent. Consider the profile:

- $a \succ d \succ e \succ f \succ b \succ c$.
- $a \succ c \succ e \succ f \succ b \succ d$.
- $a \succ c \succ d \succ f \succ b \succ e$.
- $b \succ c \succ e \succ d \succ a \succ f$.
- $b \succ c \succ e \succ d \succ a \succ f$.

The veto core is $\{ b \}$. Let P be a profile consisting of these five voters and a voter of type $bacdef$. The veto core is now $\{ a, b, c, d, e \}$. Observe that a permutedly dominates b. Because ρ is positional-consistent, there exists some $R \in \rho(P)$ such that a is ranked higher than b. So, the outcome is not $\{ b \}$, violating voter adaptability. If ρ is strongly positional-consistent, then a is always ranked higher than b and hence b is not selected under $\mathcal{R}_\rho$, violating positive involvement.

Suppose ρ is majority-consistent for top-ranked candidates, or iterative positional-consistent. Consider a profile with five voters of type abc and three of type bca. The veto core is $\{ b \}$. If we add a voter of type bac, the veto core becomes $\{ a, b \}$. The majority ranks a first, so if ρ is majority-consistent for top-ranked candidates then a is the unique winner under $\mathcal{R}_\rho$. If ρ is iterative positional-consistent, then c is ranked last, because both a and b strictly permutedly dominate c. Then b is ranked below a, because a strictly permutedly dominates b when only these two candidates remain. In either case voter adaptability and positive involvement are violated. □

The below proposition may not sound surprising, but it covers almost all ρ not covered by the previous propositions.

Table 1: Properties of specific veto core consistent voting rules.

Property	VCore	$\mathcal{S}_{\mathrm{Iter}}$	$\mathcal{R}_{\mathrm{Lex}}$	$\mathcal{R}_{\mathrm{GA}}$	VbC	$\mathrm{VbC}_{\mathrm{GA}}$
Anonymity	Yes	Yes	Yes	Yes	Yes	Yes
Neutrality	Yes	Yes	No	Yes	Yes	Yes
Monotonicity	Yes	No	Yes	Yes	Yes	Yes
P. responsiveness	No	No	Yes	Yes	No	Yes
Independence of UL	No	No	No	No	Yes	Yes
Voter adaptability	No	No	No	No	No	No
P. involvement	Yes	No	No	No	No	No
N. involvement	Yes	No	No	No	Yes	Yes
S. n. involvement	No	No	No	No	Yes	Yes

VCore: the veto core itself, identical to $\mathcal{S}_\varphi$ where φ elects all candidates, or $\mathcal{R}_\rho$ where ρ elects all $m!$ rankings; $\mathcal{S}_{\mathrm{Iter}}$: the iterative veto core, obtained by finding the veto core of the veto core until a fixed point is reached; $\mathcal{R}_{\mathrm{Lex}}$: $\mathcal{R}_\rho$ where ρ ranks the candidates in lexicographical order, identical to $\mathcal{S}_{\mathrm{Lex}}$; $\mathcal{R}_{\mathrm{GA}}$: $\mathcal{R}_\rho$ where ρ ranks the candidates by their generalised antiplurality score; VbC: Veto by consumption; $\mathrm{VbC}_{\mathrm{GA}}$: Veto by consumption with generalised antiplurality tie-breaking.

Proposition 30. *Let ρ satisfy independence of unanimous losers and Pareto efficiency, but fail voter adaptability/positive involvement (all four properties for top-ranked candidates). Then $\mathcal{R}_\rho$ also fails voter adaptability/positive involvement.*

Proof. Let P be a profile with n voters and P_{-i} the profile with $n-1$ voters obtained by removing voter i from P. Choose P and P_{-i} such that they illustrate the failure of voter adaptability/positive involvement by ρ.

Consider the profiles P'_{-i} and P' that result from P_{-i} and P by adding $m(n-1)$ unanimous losers. The veto cores of P'_{-i} and P' consist of all Pareto-efficient candidates of P_{-i} and P respectively. As such, the same candidates win in $\mathcal{R}_\rho(P'_{-i})$ and $\mathcal{R}_\rho(P')$ as in $\rho(P_{-i})$ and $\rho(P)$ and the same property is violated. □

Propositions 25, 29, and 30 cover every way we could think of to select from the veto core using a known voting/ranking rule, and show that such approaches will fail positive involvement and voter adaptability. That does not necessarily mean these properties are inconsistent with veto core consistency, but that we have to consider rules that select from the veto core directly. For example, we have seen in Proposition 24 that the veto core itself satisfies positive involvement, so we know this is possible. However we have been unable to find a veto core consistent voting rule satisfying voter adaptability.

Open Problem 31. *Is voter adaptability consistent with veto core consistency?*

5 CONCLUSION

In Proposition 6 and 8 we established some properties common to all veto core consistent rules. To illustrate our results about the different families of rules, we compare six concrete examples in Table 1. For proofs that $\mathcal{R}_{\mathrm{Lex}}$ and $\mathcal{R}_{\mathrm{GA}}$ fail negative involvement, see the supplementary material; the other results follow from the propositions in this paper.

The $\mathcal{S}_\varphi$ family is not consistent with independence of unanimous losers or voter adaptability, so these properties are failed for any choice of φ (Propositions 20, 25). If we want a minimally "reasonable" φ – neutral, anonymous, and reducing to the majority rule on $m = 2$ – we fail the rest of the studied properties, as seen in the column for $\mathcal{S}_{\mathrm{Iter}}$ (Propositions 14, 25).

If we do not require φ to reduce to the majority on $m = 2$, the column for VCore demonstrates that we can recover many of the failed properties (Propositions 13, 24). However, VCore fails strong negative involvement, and it remains open whether in fact this property is consistent with the $\mathcal{S}_\varphi$ family.

If we are willing to part with neutrality, then $\mathcal{S}_{\mathrm{Lex}} = \mathcal{R}_{\mathrm{Lex}}$ can give us monotonicity and positive responsiveness. However, if we step outside the $\mathcal{S}_\varphi$ family, we can get the same properties with $\mathcal{R}_{\mathrm{GA}}$ without having to sacrifice neutrality. Indeed, $\mathcal{R}_{\mathrm{GA}}$ represents the most we can get from the $\mathcal{R}_\rho$ family without choosing a trivialising ρ (e.g., the order of veto by consumption to get independence of unanimous losers, or rank everyone equally to get positive involvement) – see Propositions 15, 21, 22, 29, 30.

$\mathrm{VbC}_{\mathrm{GA}}$ weakly improves over VbC, and performs very well with respect to the properties we studied – failing to satisfy only voter adaptability (which no veto core consistent rule we are aware of does) and positive involvement (only satisfied by VCore) – see Propositions 16, 17, 18, 27, 28. The key property that seems to separate VbC and $\mathrm{VbC}_{\mathrm{GA}}$ from the other rules is independence of unanimous losers, which is always failed by $\mathcal{S}_\varphi$ (Proposition 20) and can only be satisfied by a trivial $\mathcal{R}_\rho$ (Propositions 21, 22).

This paper is the first attempt to systematically study the axiomatic properties of veto core consistent voting rules. We introduced the families $\mathcal{S}_\varphi$ and $\mathcal{R}_\varphi$ and compared them to the veto core and veto by consumption with respect to the properties of monotonicity, participation, and independence of unanimous losers. In the future this analysis could be extended to other properties, and to new voting rules in the veto core consistent family.

Future directions

The main theme of this paper is that veto core consistent voting rules form a well-defined family around a solution concept that is distinct from the better known majoritarian or utilitarian approaches to voting, but is well worth studying. It seems to us that a natural next step is to populate this family with interesting rules. In this paper we provide the templates $\mathcal{S}_\varphi$ and $\mathcal{R}_\rho$ which can be used to define as many rules as one may want, but as we have seen these have limits – in particular, with respect to independence of unanimous losers. On the practical side, veto core consistent rules have already seen application in social multi-criteria evaluation, and further work in this direction must be encouraged to provide better decision making tools to societies facing a difficult choice. As a purely technical question, we have been unable to establish whether voter adaptability is compatible with veto core consistent rules. We look forward to discovering the answer.

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