# TaxAI: A Dynamic Economic Simulator and Benchmark for Multi-Agent Reinforcement Learning

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# ABSTRACT

Taxation and government spending are crucial tools for governments to promote economic growth and maintain social equity. However, the difficulty in accurately predicting the dynamic strategies of diverse self-interested households presents a challenge for governments to implement effective tax policies. Given its proficiency in modeling other agents in partially observable environments and adaptively learning to find optimal policies, Multi-Agent Reinforcement Learning (MARL) is highly suitable for solving dynamic games between the government and numerous households. Although MARL shows more potential than traditional methods such as the genetic algorithm and dynamic programming, there is a lack of large-scale multi-agent reinforcement learning economic simulators. Therefore, we propose a MARL environment, named TaxAI, for dynamic games involving N households, government, firms, and financial intermediaries based on the Bewley-Aiyagari economic model. Our study benchmarks 2 traditional economic methods with 7 MARL methods on TaxAI, demonstrating the effectiveness and superiority of MARL algorithms. Moreover, TaxAI's scalability in simulating dynamic interactions between the government and 10,000 households, coupled with real-data calibration, grants it a substantial improvement in scale and reality over existing simulators. Therefore, TaxAI is the most realistic economic simulator for optimal tax policy, which aims to generate feasible recommendations for governments and individuals.

# **KEYWORDS**

Multi-agent reinforcement learning; optimal tax policy; dynamic economic simulator; benchmark; tax evasion behavior

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# **1 INTRODUCTION**

The invisible hand [69, 70] of the market is not omnipotent, and in reality, all countries rely on government intervention to promote economic development and maintain social fairness. The extent of government intervention varies from country to country, such as a free market economy [8, 73], planned economies [57, 61] or mixed economies [44, 45, 58]. However, determining the optimal government intervention degree is challenging for several reasons. Firstly, extracting relevant and actionable information from the complex society is arduous. Secondly, governments face difficulties in effectively modeling a vast and heterogeneous population with diverse preferences and characteristics. Lastly, the behavioral response of individuals to incentives remains highly unpredictable.

In this intricate matter of government intervention, we opt to investigate a crucial and efficacious tool, tax policy, which is commonly studied using agent-based modeling (ABM) [13, 23] in economics. ABM is an effective approach to simulate individual behaviors and show the relationship between micro-level decisions and macro-level phenomena. However, traditional ABM suffers from simplicity and subjectivity in setting model parameters and behavior rules, making it difficult to simulate realistic scenarios [13, 37, 51]. While Multi-Agent Reinforcement Learning (MARL) surpasses traditional ABM settings by offering optimal actions based on evolving state information. Some MARL algorithms perform well in partially observable environments and adaptively learn to reach equilibrium solutions [33, 59, 72]. Hence, MARL is well-suited for addressing dynamic game problems involving the government and a large population. However, despite the significant advantages of MARL, there is currently a shortage of large-scale MARL economic simulators designed specifically for the study of tax policies. In the existing literature, the AI Economist [84] and

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Simulator	AI Economist	RBC Model	<b>TaxAI</b> (ours)
Households'	10	100	10000
Maximum Number			
Tax Schedule	Non-linear	Linear	Non-linear
Тах Туре	Income	Income	Income&
			Wealth&
			Consumption
Social Roles' Types	2	3	4
Saving Strategy	×	$\checkmark$	$\checkmark$
Heterogenous Agent	$\checkmark$	$\checkmark$	$\checkmark$
Real-data Calibration	×	×	$\checkmark$
Open source	$\checkmark$	×	$\checkmark$
MARL Benchmark	×	×	$\checkmark$

Table 1: A comparison of MARL simulators for optimal taxation problems.

the RBC model [21] emerge as the most closely related simulators to TaxAI. However, these models exhibit certain limitations, notably a partial grounding in economic theory, limited scalability in simulating a significant number of agents, and the absence of calibration using real-world data (as detailed in Table 1). Therefore, proposing a more realistic MARL environment to study optimal tax policies and solve dynamic games between the government and the population holds significant research and practical value.

Therefore, we introduce a dynamic economic simulator, TaxAI, based on the Bewley-Aiyagari economic model [1, 2], which is widely used to study capital market frictions, wealth distribution, economic growth issues. By incorporating the Bewley-Aiyagari model, TaxAI benefits from robust theoretical foundations in economics and models a broader range of social roles (shown in Figure 1). Based on TaxAI, we benchmark 2 economic methods and 7 MARL algorithms, optimizing fiscal policy for the government, working and saving strategies for heterogeneous households. In our experiments, we compared 9 baselines across four distinct tasks, evaluating them from both macroeconomic and microeconomic perspectives. Our results reveal the tax-avoidance behavior of MARLbased households and the varying saving and working strategies among households with different levels of wealth. Finally, we test the TaxAI environment using 9 baselines with households' number ranging from 10, 100, 1000, and even up to 10,000, demonstrating its capability to simulate large-scale agents. In summary, our work encompasses the following three contributions:

**1.** A dynamic economic simulator TaxAI. The simulator incorporates multiple roles and main economic activities, employs real-data calibration, and facilitates simulations of up to 10,000 agents. These features provide a more comprehensive and realistic simulation than existing simulators.

2. Validation of MARL feasibility in optimizing tax policies. We implemented 2 traditional economic approaches and 7 MARL methods to solve optimal taxation for the social planner, and optimal saving and working strategies for households. The results obtained through MARL methods surpass those achieved by traditional methods. **3. Economic analysis of different policies**. We conducted assessments from both macroeconomic and microeconomic perspectives, uncovering tax-avoidance behaviors among MARL-based households in their pursuit of maximum utility. Furthermore, we observed distinct strategies among households with differing levels of wealth.

Codes for the TaxAI simulator and algorithms are shown in the GitHub repository https://github.com/jidiai/TaxAI.

#### 2 RELATED WORKS

Classic Tax Models. Economic models provide powerful tools for modeling economic activities and explaining economic phenomena. In microeconomics, the Supply and Demand model [35, 68] reveals the mechanism behind market price formation, while the marginal utility theory [46] underscores the significance of consumption decisions. In macroeconomics, the Keynesian Aggregate Demand-Aggregate Supply Model [6, 31] addresses shortterm fluctuations and policy effects [30]. The Comparative Advantage Theory [20, 43] in international trade explains collaborations across nations. The Quantity Theory of Money [34, 55] investigates the relationship between money supply and price levels. Regarding the optimal tax problem, the Ramsey-Cass-Koopmans (RCK) model [16, 48] studies the consumption and savings decisions of representative agents but ignoring individual heterogeneity. The Diamond-Mirrlees model [26, 27] considers the role of taxes and labor supply in social welfare but overlooks income and asset taxes. The Overlapping Generations (OLG) model [64] emphasizes intergenerational inheritance and resource transfers [25, 36]. In contrast, the Bewley-Aiyagari model [3, 11] can assess the impact of taxation on growth, wealth distribution and welfare while simulating realworld income disparities and risk-bearing capacity of individuals. This makes the Bewley-Aiyagari model an ideal choice for studying optimal taxation and household strategies.

Traditional Economic Methods. The optimal tax policy and wealth distribution [10] have been extensively studied in economics. Existed works [3, 18] have utilized mathematical programming methods to address decision-making processes related to governments and households [5, 12]. However, these approaches oversimplify decision-makers rationality and fail to consider autonomous learning abilities and environmental uncertainties. In contrast, dynamic programming-based approaches [28, 32] consider long-term consequences and environmental dynamics but struggle to model nonrational behaviors [15]. Alternative approaches, such as empirical rules [40, 53] and Agent-Based Modeling (ABM) [71, 77], have emerged to address these limitations. ABM enables the exploration of micro-level behaviors and their impact on macro-level phenomena, showing in the ASPEN model [7], income distribution [29] and transaction development [47]. Despite the abundance of research in economics based on ABM, this approach often involves relatively simplistic and subjective specifications of individual behavior, making it challenging to investigate the dynamic optimization of individual strategies.

MARL and Simulators for Economy. MARL aims to address issues of cooperation [14] and competition [83] among multiple decision-makers [74]. The simplest MARL method Independent Learning [56, 76], including IPPO [24], and IDDPG [52], involves each agent learning and making decisions independently, disregarding the presence of other agents [75]. In the Centralized Training Decentralized Execution (CTDE) algorithms, like MADDPG [54], QMIX [62], and MAPPO [81], agents share information during training but make decentralized decisions during execution to enhance collaborative performance. To address significant computational and communication overhead posed by a growing number of agents [80], Mean-Field Multi-Agent Reinforcement Learning (MF-MARL) [86] simplifies the problem by treating homogeneous agents as distributed particles [38]. On the other hand, Heterogeneous Agent Reinforcement Learning (HARL) [85], HAPPO and HATRPO, is designed to achieve effective cooperation in a general setting involving heterogeneous agents.

Currently, there are not many efforts employing MARL methods to determine optimal tax policies and individual strategies. The closest works to our paper include AI economist [84] and the RBC model [21]. While they both account for fundamental economic activities, they lack large-scale agent simulation, realdata calibration, and MARL benchmarks. These limitations make practical implementation challenging, which is why we introduce TaxAI. Besides, prior research has already explored reinforcement learning-based approaches in some subproblems. For instance, in addressing optimal savings and consumption problems, some studies [4, 63, 67] have utilized single-agent RL to model the representative agent or a continuum of agents. Meanwhile, others have employed MARL to solve rational expectations equilibrium [41, 50], optimal asset allocation and savings strategies [60]. Regarding optimal government intervention problems, research has explored the application of RL in investigating optimal monetary policy [19, 42], market prices [22, 66], international trade [65], redistribution systems [49, 79], and the cooperative relationship between central and local governments under COVID-19 [78].

#### **3 BEWLEY-AIYAGARI MODEL**

In comparison to classical economic models in Section 2, we contend that the Bewley-Aiyagari model serves as the most suitable theoretical foundation for investigating optimal tax policy for the government and optimal savings and labor strategies for households. In this section, we provide a brief overview of the Bewley-Aiyagari model. This model encompasses four key societal roles: N households, a representative firm, a financial intermediary, and the government. The interactions among them are illustrated in Figure 1. All model variables and their corresponding symbols are organized in Appendix Table 8. More details on model assumptions and dynamics are shown in Appendix A.3, A.4.

#### 3.1 N Households

To avoid differences in age, gender, and personality, individuals are modeled as households, whose main activities include production, consumption, saving, and tax payments. At timestep t, households' income  $i_t$  is derived from two sources. They **work** in the technology firm for the labor income  $W_t e_t h_t$ , which depends on the wage rate  $W_t$ , the individual labor productivity levels  $e_t$  and the working hours  $h_t$ . On the other hand, households can only engage in **savings** and are assumed not to borrow  $(a_t \ge 0)$ . They earn interest income  $r_{t-1}a_t$  from savings, which depends on household asset  $a_t$  and the

return to savings  $r_{t-1}$ .

$$i_t = W_t e_t h_t + r_{t-1} a_t \tag{1}$$

In this model, N households are heterogeneous in terms of labor productivity levels  $e_t$  and initial asset  $a_0$ , and we model  $e_t$  as either being in a super-star ability or a normal state. In the normal state, it follows an AR(1) process [12], a model commonly used for analyzing and forecasting time series data.

$$\log e_t = \rho_e \log e_{t-1} + \sigma_e u_t \tag{2}$$

where  $\rho_e$  is the persistence and  $\sigma_e$  is the volatility of the standard normal shocks  $u_t$ . In the super-star state, the labor market ability is  $\bar{e}$  times higher than the average. The transition of households from the normal state to the super-star state occurs with a constant probability p while remaining in the super-star state has a constant probability q.

In addition, each household seeks to maximize lifetime utility (3) depends on **consumption**  $c_t$  and **working hours**  $h_t$ , subject to budget constraint, where  $\beta$  is the discount factor,  $\theta$  is the coefficient of relative risk aversion (CRRA), and  $\gamma$  represents the inverse Frisch elasticity.  $T_N$  denotes the maximum steps.

$$\max \quad \mathbb{E}_0 \sum_{t=0}^{T_N} \beta^t \left( \frac{c_t^{1-\theta}}{1-\theta} - \frac{h_t^{1+\gamma}}{1+\gamma} \right)$$
  
s.t.  $(1+\tau_s)c_t + a_{t+1} = i_t - T(i_t) + a_t - T^a(a_t)$  (3)

The households are required to **pay taxes** to the government, including consumption taxes  $\tau_s$ , income taxes  $T(i_t)$ , and asset taxes  $T^a(a_t)$ , the last two are expressed by a nonlinear HSV tax function [9, 40],

$$T(i_t) = i_t - (1 - \tau) \frac{i_t^{1-\xi}}{1-\xi}, \quad T^a(a_t) = a_t - \frac{1 - \tau_a}{1 - \xi_a} a_t^{1-\xi_a}$$
(4)

where  $\tau$ ,  $\tau_a$  determine the average level of the marginal income and asset tax, and  $\xi$ ,  $\xi_a$  determine the slope of the marginal income and asset tax schedule. It presents a free market economy when all taxes are equal to 0.

#### 3.2 Technology Firm

As the representative of all firms and industries, the firm converts capital and labor into goods and services. We assume it produces a homogeneous good with technology, which can meet the consumption need of households, following the Cobb–Douglas production function,

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \tag{5}$$

where  $K_t$  and  $L_t$  are capital and labor used for production,  $\alpha$  is capital elasticity, and we normalize the output price to 1. The firm rent capital at a rental rate  $R_t$  and hires labor at a wage rate  $W_t$ . The produced output is used for all households' gross consumption  $C_t$ , government spending  $G_t$ , and physical capital investment  $X_t = K_{t+1} - (1 - \delta)K_t$ , with the depreciation rate  $\delta$ , so the aggregate resource constraint is

$$Y_t = C_t + X_t + G_t \tag{6}$$

Suppose the firm takes the marginal income from labor as households' wage rate  $W_t$  and the marginal income from capital as the



Figure 1: Model Dynamics in the Bewley-Aiyagari Model. A: Economic activities among the government, the firm, the financial intermediary, and households. B: The influence of households' saving and labor strategies on current utility and wealth. Households must strike a balance between consumption and savings, as well as work and leisure, to optimize lifetime utility. Increasing consumption enhances current utility but reduces current wealth, affecting future utility. Longer working hours yield higher labor income, thereby increasing wealth, but simultaneously result in disutility. C: The effect of government taxation on households' wealth. The social planner employs a nonlinear taxation, applying varying tax rates based on different assets. As tax rates rise, the taxes paid by households increase, with wealthier household contributing more. This narrows the gap in households' post-tax wealth, leading to a reduction in the Gini coefficient for wealth distribution.

rental rate  $R_t$ .

$$W_t = \frac{\partial Y_t}{\partial L_t} = (1 - \alpha) \left(\frac{K_t}{L_t}\right)^{\alpha}, \quad R_t = \frac{\partial Y_t}{\partial K_t} = \alpha \left(\frac{K_t}{L_t}\right)^{\alpha - 1}$$
(7)

Market clearing on labor and goods is an important assumption for simplification, which means there is an equilibrium between supply and demand. The goods market clears by Walras' Law, and the labor market clearing condition is  $L_t = \sum_{i}^{N} e_t^i h_t^i$ .

#### 3.3 Government

The government has multiple goals, such as promoting economic growth, maintaining social fairness and stability, and maximizing social welfare. To optimize these objectives, the government typically employs three tools, including government spending  $G_t$ , taxation  $T_t$ , and debt  $B_t$  with the interest rate  $r_{t-1}$ . For instance, when maximizing economic growth, the government's objective and the budget constraint are as follows:

$$\max \quad J = \mathbb{E}_0 \sum_{t=0}^{T_N} \beta^t \left( \frac{Y_t - Y_{t-1}}{Y_{t-1}} \right)$$
  
s.t.  $(1 + r_{t-1}) B_t + G_t = B_{t+1} + T_t$  (8)

where taxes  $T_t$  are derived from personal income taxes, wealth taxes, and consumption taxes.

$$T_t = \sum_{i}^{N} \left( T(i_t^i) + T(a_t^i) + \tau_s c_t^i \right)$$
(9)

In addition to the task of maximizing social welfare, the government also has the objectives of maximizing economic growth, optimizing social equity, and multi-objective optimization. The corresponding mathematical objective functions are shown in the Appendix A.2.

#### 3.4 Financial Intermediary

We posit a financial intermediary where households can deposit their savings and the intermediary uses these funds to purchase capital and government bonds. Its budget constraint is defined as:

$$K_{t+1} + B_{t+1} - A_{t+1} = (R_t + 1 - \delta) K_t + (1 + r_{t-1}) (B_t - A_t)$$
(10)

where  $A_t$  are the gross deposits from the households. No-arbitrage implies that  $R_{t+1} = r_t + \delta$ .

## **4 TAXAI SIMULATOR**

In this section, we model the above problem of optimizing tax policies for the government and developing saving and working strategies for households as multiplayer general-sum Partially Observable Markov Games (POMGs). In the POMGs  $\langle N, S, O, \mathcal{A}, P, \mathcal{R}, \gamma \rangle$ .  $\mathcal{N} = \{2, ..., N\}$  denotes the set of all agents, S represents the state space,  $O^i$  denotes the observation space for agent *i*, and  $\mathcal{O} := \mathcal{O}^1 \times ... \times \mathcal{O}^N$ .  $\mathcal{A}^i$  signifies the action space for agent *i*, and  $\mathcal{A} := \mathcal{A}^1 \times ... \times \mathcal{A}^N$ .  $P : S \times \mathcal{A} \to \Omega(S)$  denotes the transition probability from state  $s \in S$  to next state  $s' \in S$  for any joint action  $a \in \mathcal{A}$  over the state space  $\Omega(S)$ . The reward function  $\mathcal{R} := \{R^i\}_{i \in \mathcal{N}}$ , here  $R^i : S \times \mathcal{A} \times S \to \mathbb{R}$  denotes the reward function of the agent *i* for a transition from (s, a) to s'. The discount factor  $\gamma \in [0, 1)$  keeps constant across time. The specific details of POMGs is shown in the Figure 2 and following paragraphs.

Observation Space O. In the real world, households can observe their own asset  $a_t$  and productivity ability  $e_t$ , and acquire statistical data about the population from the news. While the government can collect data from all households and access current market prices  $W_t$ . However, the presence of a large number of heterogeneous households results in a considerably high-dimensional state space. To mitigate the dimensionality challenge, TaxAI categorizes households based on their wealth into two groups [17]: the top 10% richest and



Figure 2: The Markov game between the government and household agents. In the center of the figure, we display the Lorenz curves of households' wealth distribution. The global observation consists of the average assets  $\bar{a}_t$ , income  $\bar{i}_t$ , and productivity level  $\bar{e}_t$  of the 50% poorest households and 10% richest households, along with the wage rate  $W_t$ . For the government agent, it observes the global observation and takes tax and spending actions  $\{\tau_t, \xi_t, \tau_{a,t}, \xi_{a,t}, r_t^G\}$  through the actor-network. For household agents, they observe both global and private observation, including personal assets  $\{a_t^i\}$  and productivity level  $\{e_t^i\}$ , and generate savings and workings actions  $\{p_t^i, h_t^i\}$  through the actor-network. The actor-network structure in the figure is just an example.

the bottom 50% poorest households. The average wealth  $\{\bar{a}_t^r, \bar{a}_t^P\}$ , income  $\{\bar{i}_t^r, \bar{i}_t^P\}$ , and labor productivity levels  $\{\bar{e}_t^r, \bar{e}_t^P\}$  of these two groups are incorporated into the global observation. Therefore, the government's observation space  $O_g = \{W_t, \bar{a}_t^r, \bar{i}_t^r, \bar{e}_t^r, \bar{a}_t^P, \bar{i}_t^P, \bar{e}_t^P\}$ , while the household agent *i* can observe the global and its private information  $O_h^i = \{W_t, \bar{a}_t^r, \bar{i}_t^r, \bar{e}_t^r, \bar{a}_t^P, \bar{i}_t^P, \bar{e}_t^P\}$ , Moreover, the initialization of state information is calibrated by statistical data from the 2022 Survey of Consumer Finances (SCF)<sup>1</sup>.

Action Space  $\mathcal{A}$ . The decision-making of household and government agents needs to adhere to budget constraints at every step; however, the abundance of constraints often renders many MARL algorithms ineffective. Therefore, we have introduced proportional actions to alleviate these constraints. For household agents, the optimization of savings  $a_{t+1}$  and consumption  $c_t$  has been transformed into optimizing savings ratio  $p_t \in (0, 1] \cdot h_{max}$ , where  $h_{max}$  is calibrated by the real wealth-to-income ratio data.

$$p_t = \frac{a_{t+1}}{i_t - T(i_t) + a_t - T^a(a_t)} \tag{11}$$

For the government agent, the fiscal policy tools include optimizing tax parameters { $\tau_t$ ,  $\xi_t$ ,  $\tau_{a,t}$ ,  $\xi_{a,t}$ }, and the ratio of government spending to GDP  $r_t^G = G_t/Y_t$ . Thus, the action space of the government  $\mathcal{A}_g$  is { $\tau_t$ ,  $\xi_t$ ,  $\tau_{a,t}$ ,  $\xi_{a,t}$ ,  $r_t^G$ }, while the action space of each household  $\mathcal{A}_h^i$  is { $p_t^i$ ,  $h_t^i$ }.

 $\mathit{Reward\ function\ }\mathcal{R}.$  The reward function for each household is denoted as:

$$r_{h,t}(s_t, a_{h,t}^i) = \frac{c_t^{i\,1-\theta}}{1-\theta} - \frac{h_t^{i\,1+\gamma}}{1+\gamma}$$
(12)

On the other hand, the government's objectives are more diverse, and we have defined four distinct experimental tasks within TaxAI: (1) Maximizing GDP growth rate. (2) Minimizing social inequality. (3) Maximizing social welfare. (4) Optimizing multiple tasks. For more details about reward function see Appendix A.2. For example, the government's reward function when maximizing GDP growth rate is denoted as:

$$r_{g,t}(s_t, a_{g,t}) = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$
(13)

In summary, we outline three key improvements made in constructing the **TaxAI** simulator: (1) To bridge the gap between economic models and the real world, we opt to calibrate TaxAI using 2022 SCF data. (2) To mitigate the curse of dimensionality associated with high-dimensional state information, we draw inspiration from the World Inequality Report 2022 [17] and employ grouped statistical averages for households as a representation of this highdimensional state information. (3) In response to the abundance of constraints, we introduce the concept of proportional actions, facilitating control over the range of actions to adhere to these constraints. More details about environment setting are shown in Appendix A, including model assumptions, terminal conditions, parameters setting, and timing tests in Appendix B.

#### **5 EXPERIMENTS**

This section will begin by introducing **9** baseline algorithms 5.1, followed by conducting the following three sub-experiments: **Firstly** 5.2,

<sup>&</sup>lt;sup>1</sup>https://www.federalreserve.gov/econres/scfindex.htm

we aim to illustrate the superior performance of MARL algorithms over traditional methods from both macroeconomic and microeconomic perspectives. In the **second** part 5.3, we conduct an economic analysis of the optimization process for government and heterogeneous household strategies. **Lastly** 5.4, we assess the scalability of TaxAI by comparing the simulation results for different numbers of households, specifically N=10, 100, 1000, and even 10000. Additional results on full training curves, economic evolution, and hyperparameters, are shown in Appendix E.

#### 5.1 Baselines

We compare 9 different baselines, including traditional economic methods and 4 distinct MARL algorithms, providing a comprehensive MARL benchmark for large-scale heterogeneous multi-agent dynamic games in a tax revenue context. Additional experimental settings are shown in the Appendix E.1.

(1) Traditional Economic Methods: Free Market Policy, Genetic Algorithm (GA) [32].

(2) Independent Learning: Independent PPO [84].

(3) Centralized Training Distributed Execution: MADDPG [54], MAPPO [82], both with parameter sharing.

(4) Heterogeneous-Agent Reinforcement Learning: HAPPO, HA-TRPO, HAA2C [85].

(5) Mean Field Multi-Agent Reinforcement Learning: Bi-level Mean Field Actor-Critic (BMFAC), shown in Appendix D.

## 5.2 Comparative Analysis of Multiple Baselines

We benchmark 9 baselines on 4 distinct tasks, with the training curves of macroeconomic indicators in 3 tasks shown in Figure 3 and test results shown in Table 2 (Figure 8 in Appendix E presents the training curves for 6 macro-indicators in 4 tasks). In Figure 3, each row represents a task, including maximizing GDP, minimizing inequality, and maximizing social welfare. Each column represents a macroeconomic indicator, where longer years indicate longer economic stability, higher GDP represents a higher level of economic development, and a lower wealth Gini coefficient indicates fairer wealth distribution. The X-axis of each subplot represents training steps.

*Macroeconomic Perspectives.* From Figure 3, it can be observed that in each macro-indicator, most MARL algorithms outperform traditional economic methods. In the GDP optimization task, HA-TRPO achieves the highest per capita GDP, while BMFAC performs best in the tasks of minimizing inequality and maximizing social welfare. Different algorithms also differ in terms of convergence solutions. MADDPG excels in optimizing GDP but at the cost of reducing social welfare for higher GDP. The BMFAC algorithm excels in optimizing social welfare and the Gini coefficient. HARL algorithms, including HAPPO, HATRPO, and HAA2C, can simultaneously optimize all four macroeconomic indicators, but while achieving the highest GDP, social welfare is not maximized. On the other hand, MAPPO excels in optimizing social welfare.

*Microeconomic Perspectives*. During the testing phase, we conduct experiments on households following random, GA, and MAD-DPG policies within the same environment at each step. We utilize 10 distinct random seeds to simulate an economic society spanning 300 timesteps. In Figure 4, these subplots present various microeconomic indicators, including the average tax revenue, average utility, average labor supply, and average consumption for all households at each time step. The random policy represents a strategy unaffected by government tax policies, while the GA policy represents a conventional economics approach. We observe that households under the MADDPG strategy pay the lowest taxes, indicating **tax evasion behavior**, while simultaneously achieving utility levels significantly surpassing those of the GA and random policies. Labor supply and consumption are statistical measures of household microbehavior. We find that MADDPG-based households tend to opt for low consumption and reduced labor supply strategies.

Table 2: Test results for 9 baselines on 5 economic indicators under maximizing social welfare task (N = 100 households).

Baselines	Years	Average Social Welfare	Per Capita GDP	Wealth Gini	Income Gini
Free market	1.0	2.9	4.3 <i>e</i> 6	0.79	0.39
GA	200.0	6.9	1.2e7	0.54	0.52
IPPO	162.7	1035.5	8.4 <i>e</i> 6	0.62	0.44
MADDPG	204.2	1344.6	1.0e6	0.61	0.58
MAPPO	274.5	3334.7	7.3 <i>e</i> 6	0.61	0.65
HAPPO	298.7	1986.0	1.6 <i>e</i> 7	0.52	0.54
HATRPO	300.0	1945.0	1.7e7	0.52	0.54
HAA2C	300.0	2113.3	1.4e7	0.51	0.53
BMFAC	292.8	3722.2	2.8 <i>e</i> 6	0.48	0.50

## 5.3 Economic Analysis of MARL Policy

Figure 5 illustrates the training curves of the MADDPG algorithm, aiming to maximize GDP on TaxAI. We utilize the experimental results to analyze the coordination of government actions (tax) and households' actions (labor and consumption) and their impact on economic indicators. The X-axis represents steps, while the Y-axis represents various economic indicators. In the subplots for income tax, wealth tax, total tax, labor, consumption, and households' utility, we categorize households into three groups based on wealth: the wealthy (top 10% richest), the middle class (top 10-50% richest), and the poor (50-100% top richest). We display the average indicators for these three groups as well as the average for all households (purple line), other subplots represent macroeconomic indicators (blue line). In each subplot, the solid line represents the mean of experimental results under different seeds, and the shaded area represents the standard deviation. The results of the testing phase are presented in Table 3. The following are two intriguing findings:

1. MADDPG converges towards the highest GDP while compromising social welfare under maximizing GDP task. As observed from Figure 5, during the initial 2e5 steps, the government increases income and wealth tax, leading to a reduction in the income and wealth Gini, making the wealth distribution more equitable. Social equity is a prerequisite for increasing the duration of the economic system (measured by years). Simultaneously, households choose to increase labor and reduce consumption, leading to a decrease in their utility. As the total savings of households increase, financial intermediaries can provide more production capital to firms.



Figure 3: The training curves for 9 baselines on 4 macro-economic indicators under 3 different tasks (N = 100).



Figure 4: The micro-level behaviors of Random, GA, and MADDPG households while facing identical observations in an episode (300 steps). The subfigure illustrates the average values of labor provided, consumption, taxes paid, and utility for all households at each step. The results reveal that MADDPG households exhibit tax evasion behavior and attain the highest utility.

Table 3: The per capita economic indicators of the MADDPG algorithm during the testing phase at N = 100 for 3 household groups and the average level across all households.

Households' groups	Income tax	Wealth tax	Total tax	Labor supply	Consumption	Wealth	Income	Per year utility
The wealthy	1.9e6	1.0e7	1.2e7	2.3e6	4.4e7	5.3e7	5.5 <i>e</i> 6	8.7
The middle class	5.7e6	3.0 <i>e</i> 6	8.7 <i>e</i> 6	7.1e6	6.4e5	2.1 <i>e</i> 7	7.2e6	-24.1
The poor	2.8 <i>e</i> 6	1.2 <i>e</i> 6	4.0 <i>e</i> 6	4.9e6	2.3e5	9.2e6	4.6e6	-25.6
Mean value	3.8 <i>e</i> 6	2.9 <i>e</i> 6	6.7 <i>e</i> 6	5.5e6	4.8 <i>e</i> 6	1.8 <i>e</i> 7	5.7 <i>e</i> 6	-22.7

The additional production labor and capital leads to an increase in output (GDP). The wage rate tends to decrease with an increase in labor and increase with a capital increase, exhibiting a trend of initially decreasing and then increasing.

2. Different wealth groups adopt distinct strategies. From the experimental curves (Figure 5) and results (Table 3) for the wealthy,

the middle class, and poor households, we find the following patterns: During 0 ~ 2*e*5 steps, the wealthy contribute significantly to taxation. To stabilize their wealth, they increase work hours and reduce consumption, leading to a decline in utility. In the second phase (2*e*5 ~ 4*e*5 steps), the wealthy maximize utility by reducing work hours and significantly increasing consumption, even though their wealth levels decrease. In the third phase (4*e*5 ~ 6*e*5 steps), the



Figure 5: Temporal evolution of economic indicators during MADDPG training under maximizing GDP task on TaxAI (N = 100).

wealthy simultaneously increase labor and consumption, resulting in increased wealth while maintaining relatively stable utility. On the other hand, the middle class and the poor slightly increase work hours and reduce consumption during all three phases, leading to modest growth in wealth but significantly lower utility compared to the wealthy.

Table 4: The per capita GDP achieved by 9 baselines for different numbers *N* of households under maximizing GDP task.

Algorithm	N=10	N=100	N = 1000	N=10000
Free Market	1.3e6	4.3 <i>e</i> 6	3.9e6	4.0 <i>e</i> 6
GA	1.7e8	1.5e7	NA	NA
IPPO	5.0e6	1.6 <i>e</i> 7	1.7 <i>e</i> 7	1.6 <i>e</i> 7
MADDPG	3.2e6	1.1 <i>e</i> 7	1.7 <i>e</i> 7	1.7e7
MAPPO	6.1 <i>e</i> 6	7.3e6	1.2e7	NA
HAPPO	1.8 <i>e</i> 7	1.6 <i>e</i> 7	1.5e7	NA
HATRPO	3.2e7	1.7e7	2.0e7	NA
HAA2C	1.6 <i>e</i> 7	1.4e7	1.6 <i>e</i> 7	NA
BMFAC	4.0e6	1.2e7	1.2e7	NA

## 5.4 Scalability of Environment

To showcase the scalability of TaxAI in simulating large-scale household agents, we conduct tests with varying numbers of households: 10, 100, 1000, and even 10,000 (as shown in Table 4). The table presents the average per capita GDP for each baseline. The results in Table 4 indicate that IPPO and the improved MADDPG algorithm successfully achieve the maximum GDP when N = 10,000, whereas traditional methods yield NA (not available). HATRPO achieves optimal strategies at N = 100 and N = 1000, respectively, while GA only achieves optimal GDP when N is small. The above results indicate that TaxAI is capable of simulating 10,000 household agents, surpassing other benchmarks by a significant margin. Moreover, MARL algorithms can successfully solve the optimal tax problem in large-scale agent scenarios. These two advantages are crucial for simulating real-world society.

#### **6** CONCLUSION

We introduce TaxAI, a large-scale agent-based dynamic economic environment, and benchmark 2 traditional economic methods and 7 MARL algorithms on it. TaxAI, in contrast to prior work, excels in modeling large-scale heterogeneous households, a wider range of economic activities, and tax types. Moreover, it is calibrated using real data and comes with open-sourced simulation code and MARL benchmark. Our results illustrate the feasibility and superiority of MARL in addressing the optimal taxation problem, while also revealing MARL households' tax evasion behavior.

In the future, we aim to expand and enrich the economic theory of TaxAI by incorporating a broader range of social roles and strategies. Furthermore, we will enhance the scalability of our simulator to accommodate one billion agents, enabling simulations that closely resemble real-world scenarios. By doing so, we aim to attract more researchers to explore complex economic problems using AI or RL techniques, thereby offering practical and feasible recommendations for social planners and the population.

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# ETHICS STATEMENT

This paper presents work whose goal is to advance the fields of AI for economics. Our work aims to offer suggestions and references for governments and the people, yet it must not be rashly applied to the real world. There are many potential societal consequences of our work, none of which we feel must be specifically highlighted here.

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