Analyzing Crowdfunding of Public Projects Under Dynamic Beliefs

Extended Abstract

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ABSTRACT

In the last decade, social planners have used crowdfunding to raise funds for public projects. As these public projects are non-excludable, the beneficiaries may free-ride. Thus, there is a need to design incentive mechanisms for such strategic agents to contribute to the project. The existing mechanisms, like PPR or PPRx, assume that the agent's beliefs about the project getting funded do not change over time, i.e., their beliefs are static. Researchers highlight that unless appropriately incentivized, the agents defer their contributions in static settings, leading to a "race" to contribute at the deadline. In this work, we model the evolution of agents' beliefs as a random walk. We study PPRx – an existing mechanism for the static belief setting – in this dynamic belief setting and refer to it as PPRx-DB for readability. We prove that in PPRx-DB, the project is funded at equilibrium. More significantly, we prove that under certain conditions on agent's belief evolution, agents will contribute as soon as they arrive at the mechanism. Thus, we believe that by incorporating dynamic belief evolution in analysis, the social planner may mitigate the concern of race conditions in many mechanisms.

KEYWORDS

Crowdfunding, Public Projects, Martingale Theory

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1 INTRODUCTION

The process of raising funds for *public or private projects* through voluntary contributions is known as *crowdfunding*. As the contributors may be *strategic agents*, researchers analyze crowdfunding game-theoretically [1, 8, 11, 12]. We focus on crowdfunding public projects such as parks, libraries, and community services.

Provision Point mechanism for Public projects (PPP). Bagnoli and Lipman [2] introduce PPP, wherein a project issuer (PI) sets up the project's crowdfunding by announcing a target threshold, $H_0 \in \mathbb{R}_{\geq 0}$, known as the *provision point*. PI seeks voluntary contributions from interested agents towards this project before a known *deadline*, T. If the net contribution crosses the provision point by the deadline, PI funds the public project through them. If the target is not met, PI returns the contributions. Thus, in PPP, an agent i's quasi-linear utility with its project's valuation $\theta_i \in \mathbb{R}_{\geq 0}$ and contribution $x_i \in \mathbb{R}_{\geq 0}$ is $\theta_i - x_i$ if funded, and zero otherwise.

As public projects are non-excludable, strategic agents in PPP may choose not to contribute and *free-ride*. Moreover, PPP also admits several inefficient equilibria [2, 9]. The primary challenge in crowdfunding of public projects is thus the lack of *incentives* for strategic agents to contribute. Zubrickas [13] addresses this challenge with the introduction of *refund bonus schemes*.

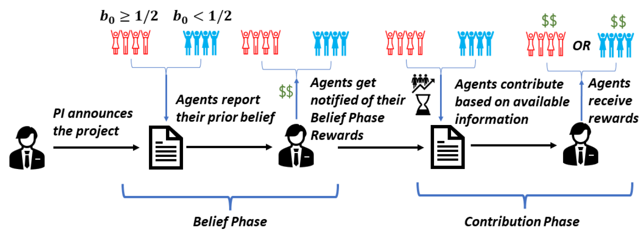

Figure 1: PPRx-DB: Protocol Illustration

Provision Point mechanism with Refunds (PPR). With PPR, if the project is not funded, the agents receive their contribution and an additional refund proportional to their contribution. Formally, each agent i's refund is $\frac{x_i}{C_0} \cdot B$, where x_i is its contribution, $C_0 = \sum_i x_i$ the net contribution, and $B \in \mathbb{R}_{>0}$ the bonus budget. This incentive structure avoids free-riding by incentivizing the agents to contribute. PPR also overcomes inefficient equilibria as Zubrickas [13] proves that at equilibrium $C_0 = H_0$ holds – when the total valuation $\vartheta = \sum_i \theta_i$ is more than the threshold H_0.

Modes of Crowdfunding. The following two settings are possible for a project's crowdfunding. (i) *Offline*: in which the participating agents are not aware of the history of the contributions and the net contribution at any epoch. (ii) *Online*: where the net and the history of contributions are visible to each participating agent (e.g., online platforms like `kickstarter.com` and `spacehive.com`). We refer to crowdfunding over online settings as *sequential crowdfunding*.

Particularly for sequential crowdfunding, *blockchain*-based online platforms are becoming popular. More concretely, crowdfunding is now being deployed as *smart contracts* over public blockchains such as the *Ethereum blockchain* (e.g., `weifund.io` and `starbase.co`).

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Carrying out transactions in Ethereum incurs *gas* (a form of payment). Damle et al. [7] introduce several refund schemes and show these schemes consume fewer gas units, and therefore, the corresponding crowdfunding mechanisms are *efficient* to deploy as smart contracts over blockchains.

For an offline setting, PPR is an excellent choice. However, PPR induces a simultaneous game [4]. In sequential crowdfunding, such a game results in the agents deferring their contribution until the deadline, which in turn may result in the project not getting funded [3, 4], i.e., a "race" condition (RC). Chandra et al. [4] introduce *Provision Point mechanism with Securities* (PPS), which employs a temporal refund scheme to avoid the race condition. Damle et al. [7] study various aspects of refund schemes to avoid the race condition and for efficient deployment in blockchain-based online settings.

Information Structure [6]. We define the tuple consisting of each agent's (i) valuation and (ii) belief as its *information structure*. The existing literature majorly assumes that each agent is interested in the funding of the public project, i.e., $\theta \geq 0$. The literature also assumes that each agent has *symmetric* belief, i.e., they believe that the public project will be funded with probability 1/2 and not with 1/2. Note that in the real world, the beliefs may be asymmetric. Damle et al. [6] present PPRx (which leverages PPR) for public projects when information structure allows positive valuation with asymmetric, yet *static*, beliefs.

2 PPRX-DB: CROWDFUNDING UNDER DYNAMIC BELIEFS

This work incorporates *dynamic* beliefs in the analysis of incentive-based crowdfunding mechanisms. We study PPRx [6] under dynamic beliefs, and to distinguish our setting, we refer to it as *Provision Point Mechanism for agents With Dynamic Belief* (PPRx-DB). We first argue that the agent's beliefs will evolve as a random walk.

2.1 Belief as a Random Walk

Consider the following empirical observations (from [3, 10]).

(1) The probability of funding a project *decreases* with an *increase* in its duration [10].
(2) Agents prefer to contribute even in the absence of refunds [3].

These observations indicate a change in the agent's belief regarding the project's funding. With (1), agents become reluctant to fund projects with greater target deadlines. Moreover, from (2), it is natural to assume that the availability of critical information, such as net contribution and the remaining time, will also impact the agent's belief. We refer to such evolving beliefs as *dynamic* beliefs. We model this belief evolution as a *random walk*. We argue that each agent's step size, at any epoch, will be a posterior update depending on its prior belief and other auxiliary information (e.g., net contribution or the time remaining).

2.2 PPRx-DB

We now briefly introduce PPRx-DB and summarize its equilibrium analysis. We refer the reader to [5] for the formal protocol description and results.

PPRx-DB: Protocol. PPRx-DB comprises two separate phases: (i) *Belief Phase*: where each agent i reports its prior belief $b_{i,0} \in [0, 1]$. The PI sorts all agents with belief $\geq 1/2$ and $< 1/2$ into distinct sets and communicates the BBR reward m_i [5, Eq. 3] of each agent. Let $B_B \in \mathbb{R}_{>0}$ denote this phase's budget. (ii) *Contribution Phase*: The agents observe their (dynamic) belief, net contribution, time remaining, and BBR reward and contribute to the project's funding. Let $B_C \in \mathbb{R}_{>0}$ denote this phase's budget. The PI funds the project if the net contribution crosses the target before the deadline, and only agents with belief $\geq 1/2$ get the BBR reward. Otherwise, all agents get the PPR refund, their contributions are returned, and agents with a belief of $< 1/2$ get the BBR reward. Figure 1 illustrates the protocol, and [5, Protocol 1] provides the formal description.

Agent i's Prior Belief	Agent Belief	Equilibrium Contribution ($x_i^\star$)	Equilibrium Time ($t_{i,2}^\star$)	Race Condition
$b_{i,0} \geq 1/2$	Martingale	Closed-form	Deadline	✓
	Super-martingale		At Arrival†	✗/✓†
	Sub-martingale		At Arrival†	✗/✓†
$b_{i,0} < 1/2$	Martingale	Closed-form	Deadline	✓
	Super-martingale		At Arrival	✗
	Sub-martingale		Deadline	✓

†: Depends on the agent's prior belief and random walk's type (refer to Table 2 in [5])

Table 1: Summary of Our Results for PPRx-DB. Here, "✗" denotes that the mechanism avoids the race condition.

PPRx-DB: Equilibrium Analysis. We provide PPRx-DB's equilibrium analysis when the random walk evolves as a (i) martingale, (ii) super, and (iii) sub-martingale. The equilibrium analysis of PPRx-DB involves the following: (i) project status at equilibrium, (ii) equilibrium contribution, and (iii) equilibrium time of contribution. We show that in PPRx-DB, the project gets funded at equilibrium.

Theorem. In PPRx-DB, if $\vartheta > H_0$ and $B_B, B_C > 0$, then at equilibrium $C_0 = H_0$.

We derive the closed form of an agent i's equilibrium contribution $x_i^\star$ based on its belief b_t at an epoch t. Conditioning on the belief evolution, we derive the time of equilibrium contribution, $t_{i,2}^\star$, and present the scenarios when PPRx-DB avoids the race condition.

Theorem. In PPRx-DB, if $\vartheta > H_0$ and $B_B, B_C > 0$, we have:

- $x_i^\star \leq \frac{H_0 b_{i,t_{i,2^\star}} (\theta_i + m_i)}{B_C(1 - b_{i,t_{i,2^\star}}) + H_0 b_{i,t_{i,2^\star}}}$, $\forall i$ s.t. $b_{i,0} \geq 1/2$
- $x_i^\star \leq \frac{H_0 b_{i,t_{i,2^\star}} \theta_i + H_0 m_i (1 - b_{i,t_{i,2^\star}})}{B_C(1 - b_{i,t_{i,2^\star}}) + H_0 b_{i,t_{i,2^\star}}}$, $\forall i$ s.t. $b_{i,0} < 1/2$

as the set of sub-game perfect equilibrium contributions. Here, $b_{i,t_{i,2^\star}}$ depends on the type of random walk and $t_{i,2}^\star$ (refer to Lemma 4 and Lemma 5 in [5] for their formal definition).

Table 1 summarizes the results. By utilizing the evolution of the belief as a martingale, super/sub-martingale, we identify conditions wherein agents are incentivized to contribute as soon as they arrive (i.e., avoid the race condition). Thus, though theoretically sound, complex mechanisms such as PPS may not be warranted in practice for sequential crowdfunding (refer to [5, Table 1]).

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