Facility Location Games with Task Allocation

Extended Abstract

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ABSTRACT

Facility location games have been studied extensively but most are about locating facilities given agents' profiles. However, in some real-life scenarios, the facility's location may be fixed already. When there are multiple facilities the strategic agents will always go to the closest one, resulting in the remote facilities unused. In this paper, we introduce the model that includes two facilities and n rational agents. There is one task at each facility to be done. Each agent will select one task and aims to minimize the amount of work assigned to her. Our goal is to design the allocation rules to achieve social optimality, i.e., every Nash equilibrium guarantees that every task can be completed. We show that no allocation rule can achieve social optimality without positive/negative incentives. For negative incentives, we propose a class of allocation rules with dummy work, where social optimality can be achieved, and no worker does the dummy work. For positive incentives, we first give a simple rule that achieves social optimality and propose a more complex rule to achieve the minimum subsidy.

KEYWORDS

Facility location games, Nash equilibrium, Task allocation

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1 INTRODUCTION

The classic facility location problem has been well studied by researchers since Moulin's work in 1980 [6]. Procaccia and Tennenholtz [7] initiate the approximate mechanism design for various facility location problems. In most of the studies on facility location problems, there is a set of agents located in a metric space and the goal is to locate k facilities in the space. Each agent has a cost to be served by the facilities. They can report some information and aim to minimize their own cost. The objective is usually to design mechanisms that output the location of facilities and minimize the total cost or maximum cost. At the same time, the mechanism should not incentivize agents to misreport information. In these settings, the objective of the mechanism and agents are close to some extent, both minimizing the cost.

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But in some real-life scenarios, if the facility's location is fixed already, then the goal may not be to choose a location to minimize the cost but rather sacrifice some cost to achieve other objectives. For example, we consider that there is a company assigning its employees to work in different branches such as convenience stores. Each branch has an amount of work to be done. Some branches are in the city center, but some are far away in the suburbs. But most employees live in the city. If the company lets employees choose a branch to work by themselves, then no one will go to the far away branches. In this case, the work at those far away branches cannot be done, and the company may get a loss. So the objectives of the company and employees are incompatible. One possible way is to assign each employee to a certain branch and then allocate the work. However, this method is a dictatorship and does not consider the employees' will. Therefore, there is a need to design a mechanism that allows the employees to select the branch by themselves and achieve the company's goal at the same time. This idea also coincides with [4], which studies scientific credit allocation problems. They found that if the scientific credit is divided equally among all individuals who joined the project, then the total weight of completed projects cannot be maximized. This occurs because individuals are rational and tend to select projects with higher expected credit, which limits the total weight of completed projects. Therefore, they proposed two types of mechanisms that re-weight the projects or give different amounts of credit to the individuals to ensure every Nash equilibrium to achieve social optimality.

When the allocation rule is given, this problem can be characterized by non-cooperative games and the Nash equilibrium plays an important role in such a game. For instance, as Figure 1 shows, suppose all branches and all employees are on a line. There are two branches with locations 0 and 1 and two employees with locations 1/4. If the company allocates work equally to each employee in the same branch, all employees will select the leftmost branch, implying that there is a branch with all work undone and the Nash equilibrium cannot always achieve the social optimal. Hence, that problem leads to a general question: We need to design some allocation rules so that each facility can attract a share of agents while the Nash equilibrium can achieve social optimality.

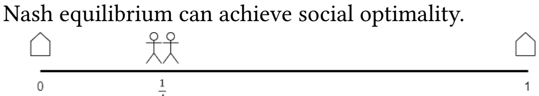

Figure 1: Example of employees and company branches

Therefore, our agenda can be divided into three parts.

- Is there an allocation rule under which every Nash equilibrium guarantees that every task can be completed?
- If we increase the amount of work for some tasks, is there a rule under which every Nash equilibrium guarantees that

every task can be completed and no one does the unnecessary work?
- If we leverage subsidies, how to use the minimum subsidy to make every Nash equilibrium guarantee that every task can be completed?

In this paper, we study the case where two tasks are going to be allocated, which is already very challenging.

1.1 Our Contribution

We study facility location games with two-task allocation by proposing a game theoretic model. Our objective is to design allocation rules (with subsidy or dummy work) to achieve social optimality, i.e., every Nash equilibrium guarantees that the task at both facilities can be completed. We mainly consider the setting where the facilities and the agents are in the same location (two-task allocation games). Based on the model, we study three types of allocation rules:

Pure Allocation (PA) We show that no allocation rule without leveraging subsidies or adding dummy work to the tasks can ensure that every Nash equilibrium achieves social optimality.

Allocation with Dummy (DA) We propose a class of allocation rules, which add dummy work to each task. We show that a subclass of DA (DA-seq) can achieve social optimality. We also show that some of the DA-seq rules can guarantee that no one will be allocated dummy work in any Nash equilibrium.

Allocation with Subsidy (SA) We introduce a class of allocation rules that subsidize some workers. We first propose a special subclass of SA (SA-1), which only subsidizes one worker to achieve social optimality. Then we design a more complex allocation rule (SA-X), which first calculates the amount of subsidy that will be introduced to make a state a Nash equilibrium, and then assigns and tunes the subsidies cleverly to make the state that uses the minimum subsidy the only Nash equilibrium.

Besides, We also extended the model to a more general setting where the agents and facilities can be at any location on a line. We found that allocation rules DA-Seq and SA-X still work when the distance between two facilities is small.

1.2 Related Work

The classic facility location problem arises in the combinatorial optimization field. Moulin [6] proposed the median-point mechanism which places the facility at the median interval of all agents and shows it is strategy-proof and optimal for the social cost objective. After that, Procaccia and Tennenholtz [7] studied the approximate mechanism design without money for facility location problems. More works on facility location games can be found in a recent survey [1].

Kleinberg and Ore [4] studied the scientific credit allocation problems. In their model, each individual in scientific communities can choose some projects to work on. However, they found that if the credit is divided equally among all individuals who joined the project after succeeding, then the total weight of completed projects cannot be maximized. Therefore, they propose two types of mechanisms that re-weight the projects so that every Nash equilibrium achieves social optimality.

If we consider two-task allocation games, our problem shares some similarities with the crowdsourcing problem. Crowdsourcing was introduced by [3] and has become a popular research topic within the artificial intelligence research community. There are many crowdsourcing models that use monetary incentives such as cash or coupons to encourage workers ([2] [5] [10] [8] [9]). The monetary incentive is similar to the subsidy given to workers in our model.

2 PRELIMINARIES

We first develop a model with two facilities. Let $N = \{1, \ldots, n\}$ be a set of workers. There are two different facilities, and each has an amount of task p_j to be allocated to the workers where $j \in \{1, 2\} = S$. Let $P = \{p_1, p_2\}$ be the task profile. The facilities and workers are located on a line interval $(0, L)$. Let $\mathbf{X} = \{x_1, x_2, ...x_n\}$ be the location profile of workers and $Y = \{y_1, y_2\}$ be the location profile of facilities. When $x_1 = x_2 = ...x_n = y_1 = y_2$, we call it a two-task allocation game.

Each worker has a strategy $s_i \in S$, corresponding to the task she will select. We denote the strategies of all workers by a strategy profile $\mathbf{s} = \{s_1, ..., s_n\}$ and denote the strategies of all workers except i by $\mathbf{s}_{-i}$. We use $K_j(\mathbf{s})$ to denote the number of workers with $s_i = j$ in strategy profile $\mathbf{s}$. We consider three types of allocation rules $\mathcal{R}$. Pure Allocation (PA) Given a task profile P and a strategy profile $\mathbf{s}$, output an allocation $A = \{A_1, \cdots, A_n\}$ where A_i is the amount of work allocated to worker i. Allocation with Dummy (DA) Given a task profile P and a strategy profile $\mathbf{s}$, output a dummy profile $D = \{d_1, d_2\}$, an allocation $A = \{A_1, \cdots, A_n\}$ and a dummy allocation $A^+ = \{A_1^+, \cdots, A_n^+\}$ where d_j is the amount of dummy work introduced to task j, A_i is the amount of work allocated to worker i, A_i^+ is the amount of dummy work allocated to worker i. Subsidy Allocation (SA) Given a task profile P and a strategy profile $\mathbf{s}$, output an allocation $A = \{A_1, \cdots, A_n\}$ and a subsidy allocation $A^- = \{A_1^-, \cdots, A_n^-\}$ where A_i is the amount of work allocated to worker i, and A_i^- is the amount of subsidy allocated to worker i.

In a facility location game with two tasks, which is formed by $(N, P, \mathcal{R})$, the cost of worker $i \in N$ is defined as

$$c_i(\mathcal{R}(P, \mathbf{s})) = \begin{cases} |y_{s_i} - x_i| + A_i & \mathcal{R} \in PA \\ |y_{s_i} - x_i| + A_i + A_i^+ & \mathcal{R} \in DA \\ |y_{s_i} - x_i| + A_i - A_i^- & \mathcal{R} \in SA \end{cases}.$$

Each worker is rational and aims to minimize her cost. Our goal is designing allocation rules $\mathcal{R}^*$ so that in any facility location game with two tasks $(N, P, \mathcal{R}^*)$, every Nash equilibrium can achieve social optimality, i.e., both tasks can be completed.

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