Decentralized Safe Control for Multi-Robot Navigation in Dynamic Environments with Limited Sensing

Extended Abstract

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ABSTRACT

Our research addresses the challenging multi-agent safe control problem where agents must reach their goals while avoiding collisions. Avoidance constraints are enforced within a limited sensing field, adding practical relevance to the problem. We propose a novel approach based on tractable Control Lyapunov Function (CLF)-based Quadratic Programs (QPs) for individual agents, enabling goal tracking while considering the dynamics of the obstacles in their limited sensing range. Our framework is highly adaptable, accommodating a large number of agents and ensuring scalability. Extensive experiments with differential drive robots illustrate the computational efficiency and scalability of our approach, even in highly occluded environments with large number of robots.

KEYWORDS

Decentralized Multi-Robot Navigation; Autonomous Navigation; Control Lyapunov Functions; Sequential Convex Programming; Decentralized Collision Avoidance

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1 INTRODUCTION

CLFs and control barrier functions (CBFs) are used to formulate control problems as optimization problems, typically modeled as quadratic programs (QPs) [8]. In the context of a nonlinear system, CLFs [7] are associated with achieving the objective of stability, while CBFs [16] are focused on ensuring safety. Primarily designed for single-agent systems, such methods seem to exploit specific structure in dynamical systems [4], or do not exhibit scalability [9]. Despite lack of formal guarantees and generalization capabilities, learning-based methods in autonomous systems have gained significant attention over the last few years (see [10, 12, 15] for example).

Traditionally, collision avoidance in multi-agent systems has been tackled by restricting agent velocities, as seen in previous works such as [1, 13]. These approaches were later extended to the design of multi-agent Control Barrier Functions (CBF) when either perfect knowledge of system dynamics [2] or worst-case uncertainty bounds were available [14]. However, developing a *decentralized* multi-agent CBF controller that can scale to handle a potentially unlimited number of agents has proven challenging due to the need for online integration of system dynamics under a computationally demanding backup strategy, especially in complex systems, as highlighted in [5].

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2 PRELIMINARIES

We consider control affine systems of the form,

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$ and $u \in \mathcal{U} \subset \mathbb{R}^m$, $f(x) : \mathcal{X} \mapsto \mathbb{R}^n$ denotes the drift vector field and $g(x) : \mathcal{X} \mapsto \mathbb{R}^{n\times m}$ denotes the control vector field.

Control Lyapunov Functions (CLFs): A continuously differentiable function $h : \mathcal{X} \mapsto \mathbb{R}$ is a Control Lyapunov Function [11] for the system (1) with respect to a set $\mathcal{X}_G \subset \mathcal{X}$ if the following conditions are satisfied :

$$h(x) \leq 0 \quad \forall x \in \mathcal{X}_G \tag{2a}$$

$$h(x) > 0 \quad \forall x \in \mathcal{X} \setminus \mathcal{X}_G, \tag{2b}$$

$$\exists u \in \mathcal{U} \; s.t. \; L_f h(x) + L_g h(x)u \leq -\alpha h(x) \quad \forall x \in \mathcal{X}, \tag{2c}$$

where $\alpha : \mathbb{R} \mapsto \mathbb{R}$ is an extended class $\mathcal{K}_\infty$ function, $L_v h(x) := \frac{\partial h(x)}{\partial x} \cdot v$ is the Lie derivative of the scalar function $h : \mathbb{R}^n \mapsto \mathbb{R}$ along a vector field $v : \mathbb{R}^n \mapsto \mathbb{R}^n$.

Sequential Convex Programming (SCP) [3] offers an efficient strategy for tackling optimization problems that lack convexity. This approach leverages well-established techniques such as Taylor expansion to iteratively approximate non-convex elements of the optimization problem. Consequently, it lacks theoretical guarantees regarding convergence to an optimal solution. Nonetheless, as suggested in [3], SCP frequently demonstrates practical efficacy, often delivering feasible solutions, if not necessarily optimal ones.

3 METHOD DESCRIPTION

Consider the following CLF-QP:

$$\min_{u \in \mathcal{U}} \; ||u||^2 \tag{3a}$$

$$s.t. \; L_f h(x) + L_g h(x)u \leq -\alpha h(x) \tag{3b}$$

This point-wise optimization problem (3) drives the state x guides state x to goal set $\mathcal{X}_G$ with CLF $h(\cdot)$. Let $\mathcal{S} = \{x_1, x_2, \ldots, x_N\}$ be state vectors for N robots with control affine systems like (1), and $\mathcal{G} = \{\mathcal{X}_{G1}, \mathcal{X}_{G2}, \ldots, \mathcal{X}_{GN}\}$ their goal sets. Define the propagated

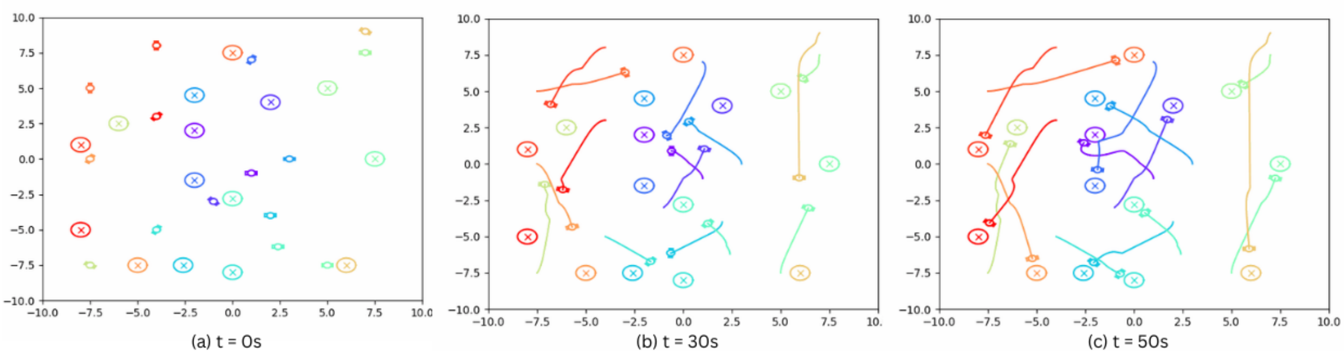

Figure 1: *Safe Multi-Robot Navigation* :- Application of MA-CLF-QP to a densely populated and dynamic environment featuring 15 robots. Our approach effectively guides each robot towards its respective goal while avoiding other robots.

state $\tilde{x} = x + [f(x) + g(x)u]dt$, *i.e.* state x updated by one discrete time-step dt along its trajectories.

Collision avoidance between x_i and x_j is enforced when they enter sensing fields, expressed as a lower bound on the Euclidean distance between their propagated states, *i.e.* $\Delta\tilde{x}_{ij} := \|\tilde{x}_i - \tilde{x}_j\| \geq r$, where r is the minimum avoidance radius. However this explicit constraint is non-convex. We leverage SCP to sequentially convexify this constraint around previous solutions. For the i^{th} robot, at the k^{th} iteration of SCP, the explicit avoidance constraint can be linearized using Taylor expansion:

$$\left\|\Delta\tilde{x}_{ij}^{k}\right\| = \left\|\Delta\tilde{x}_{ij}^{k-1}\right\| + \left(\frac{\partial\tilde{x}_i^{k-1}}{\partial u}\right)^T \frac{\Delta\tilde{x}_{ij}^{k-1}}{\left\|\Delta\tilde{x}_{ij}^{k-1}\right\|}\left(u^k - u^{k-1}\right) \tag{4}$$

We denote this linearized constraint (4) as $lin(||\Delta\tilde{x}_{ij}||)$.

Let $\mathcal{S}_i = \{j \in \{1, 2, \ldots, i-1, i+1, \ldots, N\} \quad s.t. \quad \|\Delta\tilde{x}_{ij}\| \leq F_i\}$ denote the set of all robot indices that are within the sensing field (F_i) of the i^{th} robot. The MultiAgent CLF-QP is formulated as the following point-wise optimization problem:

Definition 1 (***MA-CLF-QP***). *$\forall i \in \{1, 2, \ldots, N\}$ solve :*

$$\min_{u_i \in \mathcal{U}} \quad ||u_i||^2 \tag{5a}$$

$$s.t. \quad L_f h_i(x_i) + L_g h_i(x_i)u \leq -\alpha h_i(x_i) \tag{5b}$$

$$lin\left(\left\|\Delta\tilde{x}_{ij}\right\|\right) \geq r \quad \forall j \in \mathcal{S}_i \tag{5c}$$

MA-CLF-QP runs on each robot individually and has access to the state vectors and their derivatives corresponding to every other robot in its sensing field.

4 RESULTS & CONCLUSION

We validate Algorithm 1 on differential drive robots governed by unicycle kinematics:

$$\begin{bmatrix}\dot{x}_r \\ \dot{y}_r \\ \dot{\phi}_r\end{bmatrix} = \begin{bmatrix}\frac{R}{2}\cos(\phi_r) & 0 \\ \frac{R}{2}\sin(\phi_r) & 0 \\ 0 & \frac{R}{L}\end{bmatrix}\begin{bmatrix}u_1 \\ u_2\end{bmatrix}$$

where $X_r := [x_r, y_r]^T$ is the center-of-mass (COM) position, $\phi_r \in [-\pi, \pi]$ is the heading, R is the wheel radius, L is the track-width of the robot, u_1 and u_2 represent the linear and angular velocity control inputs. Goal sets are ellipses centered around the goal setpoint, with corresponding CLFs as the functional form of the ellipse [6]. For unicycle systems, the CLF must explicitly incorporate the robot's heading. This is achieved by parameterizing the CLF with respect to a point situated at a specified distance (l) ahead of the COM along the robot's heading.

Algorithm 1 Multi-Agent CLF-QP for the i^{th} robot

Require: $\mathcal{X}_{Gi}$, T, dt
- $M \leftarrow \lceil T/dt \rceil$;
- $\mathcal{S}_i \leftarrow$ sense(x_i);
- $u_i \leftarrow$ **MA-CLF-QP**($\mathcal{X}_{Gi}, \mathcal{S}_i$) without (5c);
- **for** $2 \leq k \leq M$ **do**
  - $x_i \leftarrow$ update_state(u_i);
  - $\mathcal{S}_i \leftarrow$ sense(x_i);
  - $u_i \leftarrow$ **MA-CLF-QP**($\mathcal{X}_{Gi}, \mathcal{S}_i$);
- **end for**

$$X_p = \begin{bmatrix}x_p \\ y_p\end{bmatrix} = \begin{bmatrix}x_r \\ y_r\end{bmatrix} + \begin{bmatrix}l\cos(\phi_r) \\ l\sin(\phi_r)\end{bmatrix}$$

The CLF with respect to X_p is now given as:

$$h(X_p) = \frac{(x_p - x_G)^2}{a^2} + \frac{(y_p - y_G)^2}{b^2} - 1$$

Here $X_G = [x_G, y_G]$ is the goal setpoint, a and b are the semi-major and semi-minor axes lengths respectively. Simulation results for 15 differential drive robots are presented in Figure 1. Our results demonstrate successful and safe navigation in highly occluded environments, with each robot requiring only $\sim$ 1ms to solve its QP (5). The results can be extended to accommodate even larger number of robots.

In this study, we introduce a comprehensive framework for achieving safe and efficient multi-agent control in complex, dynamic environments. Expanding the CLF-QP framework to the multi-agent context involves integrating SCP, resulting in a computationally efficient solution scalable for any number of agents. Importantly, this scalability is achieved without explicit knowledge of other agents' dynamics, simplifying barrier function design and avoiding generalization challenges with data-driven methods.

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