Hybrid Participatory Budgeting: Divisible, Indivisible, and Beyond

Extended Abstract

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ABSTRACT

Participatory budgeting (PB) has been receiving significant attention lately both in theory and practice. PB is broadly classified into two categories: divisible PB and indivisible PB. Divisible PB imposes no constraint on the amount allocated to each project, whereas the indivisible PB assumes that each project is associated with a cost and the project must either be funded in full or not funded. In this work, we propose a rich PB model that encompasses many settings of PB as special cases. Some of such settings include the case where some projects are divisible and some are indivisible and the case where the cost of each project may belong to a continuum range of values. We propose various welfare and fairness objectives and verify the computational complexity of each of them. We prove experimentally that even the computationally hard objectives become tractable in practice. Also, we propose greedy approximation algorithms for such objectives and prove that our algorithms achieve nearly optimal solutions on real world PB datasets.

KEYWORDS

Participatory Budgeting, Approval Votes, Computational Complexity

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1 MOTIVATION

Participatory Budgeting (PB) is a democratic process that aggregates the preferences of stakeholders to determine an allocation of a divisible resource (aka *budget*) to a set of alternatives (aka *projects*). Depending on the nature of projects, PB is classified into two categories: divisible PB and indivisible PB. Divisible PB allows for the allocation of any portion of the budget to each project. Conversely, in indivisible PB, each project is associated with a fixed cost and must receive an allocation equal to either its fixed cost or zero.

Both divisible PB [1, 6, 8] and indivisible PB [2, 4, 5, 7, 9, 11–15, 17, 18, 20, 23] are well-studied in the social choice literature. In this paper, we introduce the model of *Hybrid Participatory Budgeting*, which encapsulates divisible PB and indivisible PB, among many others, as special cases. In our model, we go beyond assigning a single fixed cost to each project and instead accommodate a continuous range of values known as the *cost domain* of the project. We assume that every project p is associated with a cost domain denoted by $[\mathrm{l}_p, \mathrm{h}_p]$. The project p must either receive no amount or must receive an amount belonging to the interval $[\mathrm{l}_p, \mathrm{h}_p]$. Each voter i approves a set A_i of projects that she likes.

Hybrid PB can be employed to represent many existing PB scenarios as well as novel ones. Few such examples include the ones where: (i) some projects are divisible and some are indivisible (an open question posed by [3]) (ii) a minor deviation from the estimated cost of a project is allowed (iii) fairness bounds are imposed on the allocations to divisible projects (e.g., lower bound on a donation value to ensure tax relief).

In this work, we introduce hybrid PB, formulate mathematical objectives related to utilitarian welfare and fairness in hybrid PB, and propose PB rules which calculate solutions that are either optimal or nearly optimal with respect to these objectives. Note that Sreedurga [19] also permits each project to assume multiple possible costs, but the possible values form a discrete set. This work, on the other hand, assumes the cost domain to be a continuum range of values, thereby becoming incomparable with [19].

2 FORMAL MODEL

Let N denote the set $\{1, 2, \dots, n\}$ of n voters. We use $\mathcal{P}$ to denote the set $\{p_1, p_2, \dots, p_m\}$ of m projects and $b \in [1, \infty)$ to denote the available budget. Each project $p \in \mathcal{P}$ is associated with an interval $\mathrm{D}_p = [\mathrm{l}_p, \mathrm{h}_p] \subseteq [0, \infty)$ referred to as the *cost domain* of p. Every voter $i \in N$ approves a subset of projects $A_i \subseteq \mathcal{P}$. A hybrid PB instance $\langle N, \mathcal{P}, b, (\mathrm{D}_p)_{p\in\mathcal{P}}, (A_i)_{i\in N}\rangle$ is usually denoted by I.

An *allocation* is a m-sized vector $x \in \mathbb{R}_+^m$ denoting the amount allocated to each project. An allocation x is said to be ***feasible*** if $\sum_{p\in\mathcal{P}} x(p) \leq b$ and $x(p) \in \mathrm{D}_p \cup \{0\}$ for every $p \in \mathcal{P}$. For the sake of convenience, with a slight abuse of notation, we use $x(S)$ to denote $\sum_{p\in S} x(p)$ for a subset of projects $S \subseteq \mathcal{P}$. Let $\mathcal{F}$ denote the set of all feasible allocations.

Definition 1 (**Hybrid PB Rule**). *Given a hybrid PB instance I, a hybrid PB rule $\mathcal{R}$ outputs a feasible allocation $x \in \mathcal{F}$.*

3 WELFARE

In this section, we propose two utilitarian hybrid PB rules, each based on a distinct utility notion. The first rule, $\mathcal{R}_c$, assumes that every approved funded project yields a unit utility to the voter. In

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contrast, the second rule $\mathcal{R}_a$ considers the utility from a project to be equal to the amount allocated to the project.

Definition 2 ($\mathcal{R}_c$ rule). *Given a hybrid PB instance I, $\mathcal{R}_c$ rule outputs an allocation $x \in \mathcal{F}$ that maximizes $\sum_{i \in N} |p \in A_i : x(p) \geq \mathrm{l}_p|$.*

Definition 3 ($\mathcal{R}_a$ rule). *Given a hybrid PB instance I, $\mathcal{R}_a$ rule outputs a feasible allocation $x \in \mathcal{F}$ that maximizes $\sum_{i \in N} \sum_{p \in A_i} x(p)$.*

Proposition 1. *Given a hybrid PB instance I, an allocation output by $\mathcal{R}_c$ can be computed in polynomial time. Conversely, it is* NP*-hard to compute an allocation output by $\mathcal{R}_a$.*

3.1 Fixed Parameter Tractability of $\mathcal{R}_a$ Rule

$$\max \sum_{p \in \mathcal{P}} |i \in N : p \in A_i| \cdot x_p$$

$$\text{subject to} \sum_{p \in \mathcal{P}} x_p \leq b \tag{1}$$

$$\mathrm{l}_p\, y_p - x_p \leq 0 \quad \forall p \in \mathcal{P} \tag{2}$$

$$x_p - \mathrm{h}_p\, y_p \leq 0 \quad \forall p \in \mathcal{P} \tag{3}$$

$$x_p \geq 0 \quad \forall p \in \mathcal{P} \tag{4}$$

$$y_p \in \{0,1\} \quad \forall p \in \mathcal{P} \tag{5}$$

Theorem 3.1. *Given a hybrid PB instance I, computing an allocation output by $\mathcal{R}_a$ is in FPT with respect to m.*

The above theorem follows from the fact that a mixed integer linear program is fixed parameter tractable with respect to the number of variables [16]. Despite being computationally hard theoretically, many times in practice, MILP is solvable very quickly in a matter of seconds, or at most a few minutes, using one of the many available MILP solvers. On executing $\mathcal{R}_a$ on 300 real-world PB datasets [10], it took 26.6 seconds on an average for each dataset. Note that we modified indivisible PB instance to be a hybrid PB instance by allowing the allocation to deviate from the estimated cost by 10%.

3.2 Greedy Algorithm for $\mathcal{R}_a$ rule

We now propose a polynomial-time greedy algorithm for $\mathcal{R}_a$. The algorithm simply sorts the projects in the non-increasing order of their approval scores and sequentially allocate an amount as high as possible to each project in that order. Our algorithm is found to perform remarkably well on the datasets by achieving an output that yields a welfare of at least 90% of the optimal value on all the 300 datasets. Particularly, it achieves at least 96% of the optimal welfare on 275 datasets and more than 99% for 231 of them.

4 FAIRNESS

In this section, we introduce a family of fair hybrid PB rules. Fairness in PB is typically based on the concept of *share*, which is a lower threshold on the amount that needs to be allocated in favor of a voter or a group of voters. Share of a voter, α, is assumed to be the least amount required to satisfy a voter. That is, we say that a voter i is satisfied if $x(A_i) \geq \alpha$. Existing literature on PB study various specific values of share such as $\frac{b}{n}$ [1, 6] and $\frac{|A_i|}{m} b$ [8] for each $i \in N$. Moreover, several works in PB [21, 22] consider share to be a parameter of the rule. Following them, we take α as a parameter.

Definition 4 ($\mathcal{R}_{f,\alpha}$ rule). *Given a hybrid PB instance I, $\mathcal{R}_{f,\alpha}$ rule outputs a feasible allocation $x \in \mathcal{F}$ that maximizes $\sum_{i \in N} \mathbb{1}_{x(A_i) \geq \alpha}$.*

Proposition 2. *Given a hybrid PB instance I, it is* NP*-hard to compute an allocation output by $\mathcal{R}_{f,\alpha}$.*

4.1 Fixed Parameter Tractability

$$\max \sum_{i \in N} z_i$$

$$\text{subject to} \sum_{p \in \mathcal{P}} x_p \leq b \tag{6}$$

$$\mathrm{l}_p\, y_p - x_p \leq 0 \quad \forall p \in \mathcal{P} \tag{7}$$

$$x_p - \mathrm{h}_p\, y_p \leq 0 \quad \forall p \in \mathcal{P} \tag{8}$$

$$z_i \cdot \alpha \leq \sum_{p \in A_i} x_p \quad \forall i \in N \tag{9}$$

$$x_p \geq 0 \quad \forall p \in \mathcal{P} \tag{10}$$

$$y_p \in \{0,1\} \quad \forall p \in \mathcal{P} \tag{11}$$

$$z_i \in \{0,1\} \quad \forall i \in N \tag{12}$$

Theorem 4.1. *Given a hybrid PB instance I, computing an allocation output by $\mathcal{R}_{f,\alpha}$ is in FPT with respect to m + n.*

Proof of the above is similar to that of Theorem 3.1. On executing $\mathcal{R}_{f,\alpha}$ on the PB datasets [10] with $\alpha = \frac{tb}{n}$ (t is a constant input by the user), it took 42.3 seconds on an average for each dataset. Share value in the code can easily be modified to any other value without effecting the time complexity of the execution.

4.2 Greedy Algorithm

We propose a greedy algorithm for $\mathcal{R}_{f,\alpha}$ which iteratively proceeds as follows: at each iteration, we compute the project w_p that satisfies most number of previously unsatisfied voters, allocate to w_p the minimum amount required to satisfy all those new voters, mark it, and repeat until all the projects are marked. Our algorithm is found to perform remarkably well on the datasets by achieving an output that yields a fair coverage of at least 90% of the optimal value on all the 300 datasets. Particularly, it achieves at least 95% of the optimal value on 274 datasets and more than 98% for 226 of them.

5 SUMMARY

We introduced the hybrid PB model and proposed welfare and fairness objectives. We modeled our computationally hard problems as MILP and verified their fixed parameter tractability. We also proposed greedy algorithms and proved empirically that they give nearly optimal solutions on the real-world PB datasets. It needs to be highlighted that, although all the results in Section 4 assume the voters to have same share parameter, they can easily be extended to the case where every voter i has a different share parameter α_i. Such a change does not affect the computational complexity or the execution time in our empirical results.

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