Optimal Diffusion Auctions

Extended Abstract

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ABSTRACT

Diffusion auction design is a new trend in mechanism design for which the main goal is to incentivize existing buyers to invite their neighbors on a social network, to join an auction. With more buyers, a diffusion auction will be able to receive higher revenue. Existing studies have proposed many diffusion auctions to attract more buyers, but the seller's revenue is not optimized. In this study, we investigate what optimal revenue the seller can achieve by attracting more buyers. Different from the traditional setting, the revenue can be achieved highly relies on the structure of the network. We propose a class of mechanisms, where for any given structure, an optimal diffusion mechanism can be found. Moreover, we show that an optimal mechanism that handles all structures does not exist. Therefore, we also propose mechanisms that have bounded approximations of the optimal revenue in all structures.

KEYWORDS

Mechanism Design; Diffusion Auctions; Revenue Maximization

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1 INTRODUCTION

Single-item auction is a classic mechanism design problem, where a seller sells an item to a fixed group of buyers [13]. Recently, to further improve the seller's revenue, researchers started to utilize the connections between buyers to incentivize existing buyers to invite their neighbors to join the auction [5, 6, 8, 11, 15, 16, 16, 17], which is called *diffusion auction design* [7]. A recent review of these studies can be found in [3]. The challenge is that buyers would not invite each other by default as they are competitors [4]. The model of diffusion auction design was initiated in [10] and it demonstrated that the classic VCG mechanism [1, 2, 13] will give a deficit. Then, they proposed the first incentive compatible mechanism: Information Diffusion Mechanism. Later, [9] further proposed a class of mechanisms. However, they are not designed to optimize the revenue. Therefore, we focus on what maximal revenue can be achieved, more precisely, given prior distributions of the buyers' valuations, an incentive compatible (IC) and individually rational (IR) mechanism to maximize the expected revenue.

The well-known optimal mechanism for traditional single-item auction, Myerson's mechanism [12], is not IC here as it does not consider the connections between buyers. The main difficulty is that the revenue of a diffusion auction may highly depend on the network structure. In this work, we define optimality over structures, and propose a class of IC and IR diffusion auctions called the k-Partial Winner of Myerson's (k-PWM), where k is a parameter associated with the structure. This class can give an optimal mechanism for any given structures, but a k-PWM may get zero revenue for the other structures. Unfortunately, we show that the existence of k-PWM implies that there is no mechanism that can have higher expected revenue than others in all structures. Therefore, our next goal is to find a mechanism to approximate the optimal revenue under all structures. We propose a mechanism called the *Closest Winner of Myerson's* (CWM) and a general class called the *CWM with Shifted Reserve Prices* (CWM-SRP). We show that the CWM has a bounded approximation of the optimal revenue.

2 PRELIMINARIES

We consider an auction where a seller s sells one item in a social network that contains herself and a buyer set $N = \{1, 2, \ldots, n\}$. Each buyer $i \in N$ has a private type of $t_i = (v_i, r_i)$, where v_i is her valuation of the item satisfying $v_i \in [\underline{v}, \overline{v}]$ ($\underline{v}$ and $\overline{v}$ are public), and $r_i \subseteq N \backslash \{i\}$ is the set of all her direct neighbors. Let $r_s \subseteq N$ represent the direct neighbors of the seller. Let T_i be the type space of buyer i. In an auction mechanism, each buyer i is asked to report her type, and her report is denoted by $t'_i = (v'_i, r'_i)$, where $v'_i \in [\underline{v}, \overline{v}]$ and $r'_i \subseteq r_i$. Denote the report profile of all buyers by t'. Let t'_{-i} be the report profile of all buyers except for i, and then we can represent t' as (t'_i, t'_{-i}). Initially, the seller only knows her neighbors r_s, and can only notify them of the auction. Buyers who are aware of the sale can further invite their neighbors to join. Eventually, the seller can sell the item to those who are finally informed of the auction. In practice, buyers who are not informed of the sale cannot participate in the sale. We call buyer i a **valid buyer** if there exists a path from the seller to i with their reports. Let $V(t')$ be the set of all valid buyers. An **diffusion auction mechanism** M is composed of an allocation policy $\pi = \{\pi_i\}_{i \in N}$ and a payment policy $p = \{p_i\}_{i \in N}$, that only run among $V(t')$, and are independent of $N \setminus V(t')$. Then, a buyer i has utility $u_i(t_i, t', (\pi, p)) = \pi_i(t')v_i - p_i(t')$.

Definition 2.1. A mechanism (π, p) is individually rational (IR) if for all $i \in N$, $t_i \in T_i$ and any t'_{-i}, $u_i(t_i, ((t_i, t'_{-i}), (\pi, p))) \geq 0$.

Definition 2.2. A mechanism (π, p) is incentive compatible (IC) if for all $i \in N$, $t_i, t'_i \in T_i$ and any t'_{-i}, $u_i(t_i, (t_i, t'_{-i}), (\pi, p)) \geq u_i(t_i, (t'_i, t'_{-i}), (\pi, p))$.

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The expected revenue of a mechanism may be affected by the **structure profile** of the network $r = (r_s, r_1, \ldots, r_n)$. Let R_k be the space of all connected structure profiles of k buyers, and $R = \bigcup_{k \in \mathbb{N}^*} R_k$ be the space of all connected structure profiles. The revenue of an IC and IR M is $rev_M^r(v) = \sum_{i \in N} p_i(t)$. Then, an IC and IR M is **optimal over** S if for any $r \in S$, and any other IC and IR M', $\mathbb{E}_{\{v_i\} \sim \{F_i\}}[rev_M^r(v)] \geq \mathbb{E}_{\{v_i\} \sim \{F_i\}}[rev_{M'}^r(v)]$, where the buyers' valuations are drawn independently from c.d.f. $\{F_i\}_{i \in N}$. A virtual bid of any buyer i is $\tilde{v}_i = \phi_i(v_i') = v_i' - (1 - F_i(v_i'))/f_i(v_i')$, where f_i is the p.d.f. of F_i with monotone non-decreasing $f_i(z)/(1 - F_i(z))$.

3 MECHANISMS AND MAIN CONCLUSIONS

First, we will design a class of optimal diffusion mechanisms over different structures. We say a buyer i who is the Myerson's winner without her invitations is a **potential winner**. The corresponding payment p_i^* is **potential payment**. $W(t')$ is the set of all potential winners. Among $W(t')$, we call a buyer i is a k**-partial potential winner** if there exists $r_i'' \subseteq r_i'$ such that $|V(((v_i', r_i''), t_{-i}'))| = k$ and she is the winner of Myerson's mechanism among $V(((v_i', r_i''), t_{-i}'))$. The corresponding payment is k**-partial potential payment**. We can observe that there exists at most one k-partial potential winner.

The k-Partial Winner of Myerson's (k-PWM)

INPUT: a set of buyers N and their type report profile t'.

(1) Let $m = |V(t')|$. If $m < k$, let $\pi_i = 0$, $p_i = 0$ for all i.
(2) If $m = k$, then run Myerson's Mechanism among $V(t')$.
(3) If $m > k$, if there exists a k-partial potential winner, then let her be the winner, and her payment is her **minimal** k-partial potential payment; otherwise, let $\pi_i = 0$, $p_i = 0$ for all i.

OUTPUT: the allocation π and the payment p.

THEOREM 3.1. *k-PWM is IR, IC and optimal over R_k.*

Since the optimal expected revenue in any structure is the same as Myerson's mechanism with the same number of buyers without diffusion, it unfortunately leads a negative result that **an optimal mechanism over R does not exist**. We then consider the approximation mechanisms. We say an IC and IR M is α-optimal if

$$\inf_{r \in R} \mathbb{E}_{\{v_i\} \sim \{F_i\}}[rev_M^r] / \mathbb{E}_{\{v_i\} \sim \{F_i\}}[rev^r_{|V((\cdot, r))|\text{-PWM}}] \geq \alpha$$

We then propose a new mechanism as follows.

The Closest Winner of Myerson's (CWM)

INPUT: a set of buyers N and their report profile t'.

(1) Let the potential winner set $W(t') = \{w_1, w_2, \ldots, w_m\}$, sorted by the distance to s. If $W(t') = \emptyset$, then set $\pi_i(t') = 0$, $p_i(t') = 0$ for all $i \in N$ and goto OUTPUT.
(2) Set $\pi_{w_1}(t') = 1$ and $p_{w_1}(t') = p_{w_1}^*(t')$.

OUTPUT: the allocation π and the payment p.

THEOREM 3.2. *CWM is IR, IC and $(\phi^{-1}(0)/\overline{v})$-optimal (tight) if all buyers' valuations are drawn independently from an identical F.*

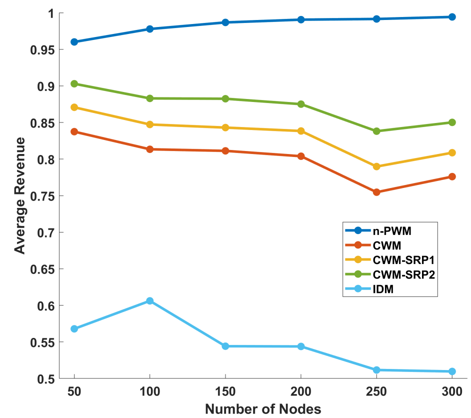

Figure 1: The average of the estimated expected revenue over all sampled structures with $50 \leq n \leq 300$.

4 NUMERICAL RESULTS

As CWM always chooses the first potential winner, buyers away from the seller have fewer opportunities. To avoid this, a direct idea is increasing the reserve prices of buyers that are close to the seller. Let d_i be the length of the shortest path from s to i. We can define a monotone non-increasing *shifting function* σ s.t. $0 \leq \sigma(d_i) \leq |\overline{v} - \underline{v}|$.

CWM with Shifted Reserve Prices (CWM-SRP)

INPUT: a set of buyers N and their report profile t'.

(1) Let the potential winner set $W(t') = \{w_1, w_2, \ldots, w_m\}$, sorted by the distance to s. If $W(t') = \emptyset$, then set $\pi_i(t') = 0$, $p_i(t') = 0$ for all $i \in N$ and goto OUTPUT.
(2) $k = 1$ to m: if $v'_{w_k} \geq \phi_{w_k}^{-1}(0) + \sigma(d_{w_k})$, set $\pi_{w_k}(t') = 1$, $p_{w_k}(t') = \max\{p_{w_k}^*(t'), \phi_{w_k}^{-1}(0) + \sigma(d_{w_k})\}$, and BREAK.
(3) Set $\pi_i(t') = 0$, $p_i(t') = 0$ for all i except for winner.

OUTPUT: the allocation π and the payment p.

To provide ideas on how to set shifting functions, we evaluate these methods through experiments. In all experiments, the valuations of buyers are drawn independently from U[0, 1]. Mechanisms that are evaluated and compared include (i) **IDM** [10]: a representative of existing mechanisms, (ii) **CWM**, (iii) **CWM-SRP1**: with $\sigma_1(d_i) = 0.1 \cdot \mathbb{I}(d_i = 1)$, and (iv) **CWM-SRP2**: with $\sigma_2(d_i) = 0.1(3 - d_i) \cdot \mathbb{I}(d_i \leq 2)$. We sample 100 structures of small-world networks [14] for each n. For each structure, we average 100 samples of valuations to approximate the expected revenue. Figure 1 summarizes the results, where the CWM-SRPs can greatly improve the expected revenue, and **when the graph is larger, a more aggressive shifting function may perform better**.

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