Extended Abstract of Diffusion Auction Design with Transaction Costs

JAAMAS Track

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ABSTRACT

We study multi-unit diffusion auctions powered by intermediated markets, where all transactions are processed by intermediaries and incur certain costs. The classic Vickrey-Clarke-Groves (VCG) mechanism within the scenario can obtain the maximum social welfare, but it can lead to a deficit for the seller. To address the revenue issue, we develop two deficit reduction strategies and further propose a family of diffusion auctions, called Critical Neighborhood Auctions (CNA). The CNA not only maximizes the social welfare, but also achieves a (non-negative) revenue that is no less than the revenue given by the VCG mechanism with/without intermediaries. This is the first set of diffusion auctions with welfare and revenue advantages that can handle multiple items and transaction costs.

KEYWORDS

Diffusion auction design; Transaction cost; Intermediated market

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1 INTRODUCTION

Intermediaries, which have become an integral part of modern business, take advantage of information technologies to reduce the search costs between suppliers and consumers, promote commodity circulation, and increase trade efficiency [2]. Instead of taking the ownership of the products sold, the intermediaries bring the buyers and sellers together to make a deal and collect commissions from successful transactions. This work considers auction markets powered by intermediaries, where each intermediary owns a private set of buyers and all intermediaries are networked with each other. Due to information asymmetry, intermediaries are initially partially accessible to the seller, which would result in a local transaction without promotion. To address this problem, we build a diffusion-based auction framework to incentivize the intermediaries to share the auction information to individuals they can reach, including their private buyers and neighboring intermediaries, so that more potential buyers are able to participate in the auction.

We contribute to the literature on diffusion mechanism design [4, 7] by exploring multi-unit diffusion auctions with intermediaries and identifying a set of diffusion auctions which is proved to maximize the social welfare and increase the seller's revenue comparing to the VCG with/without using intermediaries. It is known that social welfare maximization and (weak) budget balance are two conflicting objectives for diffusion auctions on social networks [5], i.e., the Myerson-Satterthwaite theorem [8] can extend to social-network-based diffusion auctions. Nevertheless, our results suggest that the two properties can be obtained simultaneously in the intermediated auction markets.

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2 PRELIMINARIES

Let s denote a seller endowed with a set of D identical commodities. Besides the seller, the intermediated market consists of a set of agents, denoted by N, which are divided into two disjoint categories: a set of intermediaries I and a set of buyers B. Each intermediary $i \in I$ owns a private set of buyers, and all intermediaries in the market are networked with each other. Let $r_i \subseteq N \setminus \{i\}$ denote the agents with whom intermediary i can communicate in the market. Each buyer $j \in B$ is unit-demand, that is her value, denoted by v_j, for consuming one or more commodities is unchanged. A *transaction* is defined by an agent path $\{a_k\}_{k=0}^m$, where a_0 denotes the seller, a_m is the winning buyer, and $a_k \in r_{a_{k-1}}$. Let $c_{i,k} = w_{i,k} \cdot n_{i,k}$ denote the *transaction cost* between agents i and k, where $w_{i,k}$ is the cost per transaction and $n_{i,k}$ is the number of transactions involving (i, k). We make two assumptions on the market model: 1) for every $i \in I$ and $j \in B$, r_i and v_j are *private information*, and 2) for any two agents i and k participating in the sale, $w_{i,k}$ is *fixed and known*. For convenience's sake, we will slightly abuse notations and refer to i as an intermediary, j as a buyer, and k as an arbitrary agent.

We next present a formal model for auctions in intermediated markets. Let θ_k be the private type of agent k and Θ_k be k's type space. If k is an intermediary, then $\theta_k = r_k$ and $\Theta_k = \mathcal{P}(N)$ where $\mathcal{P}(N)$ is the power set of N; otherwise, $\theta_k = v_k$ and $\Theta_k = \mathbb{R}_+$ for a buyer k. We use $\Theta = \times\Theta_{k \in N}$ to denote the type profile space of all agents. Accordingly, let θ'_k be k's reported type. Since every intermediary in the market is only aware of her neighbors, the misreport space of r'_k is limited to $\mathcal{P}(r_k)$. In addition, let θ' be the reported type profile of all agents, θ'_{-k} be the reported type profile of all agents except k, i.e., $\theta' = (\theta'_k, \theta'_{-k})$. Given a reported type profile θ', we say agent $k \in N$ is a valid agent if there is a "transaction path" from the seller s to k, and k is invalid if such a "transaction path" does not exist. Let $F(\theta')$ denote all valid agents for θ'. We use T and $T(\theta')$ to denote the space of all possible transactions regarding to $N \cup \{s\}$ and all valid agents $F(\theta')$, respectively.

Definition 2.1. An auction mechanism consists of an allocation policy $\pi : \Theta \rightarrow \mathcal{P}(T)$ and a payment policy $x = \{x_k : \Theta \rightarrow \mathbb{R}\}_{k \in N}$, where for all reported type profile θ', π and x satisfy

1. $\pi(\theta')$ and $x(\theta')$ are independent of the reports of invalid agents $N \setminus F(\theta')$;
2. $\pi(\theta') \subseteq T(\theta')$ and $|\pi(\theta')| \leq D$;
3. $x_k(\theta') = 0, \forall k \notin F(\theta')$.

Given an auction mechanism $M = (\pi, x)$ and a reported type profile θ', the seller's revenue (or utility) can be formulated as $\text{Rev}(M, \theta') = \sum_{k \in N} x_k(\theta') - C(\theta')$, where $C(\theta') = \sum_{i \in I, k \in r_i} w_{i,k} \cdot n_{i,k}$ denotes the transaction costs. In addition, each agent k's utility is defined as $u_k(\theta_k, \theta', M) = z_k(\theta')\theta_k - x_k(\theta')$, where $z_k(\theta') = 1$ for a winning buyer and $z_k(\theta') = 0$ otherwise.

An auction M is *incentive-compatible* (IC) if acting truthfully is a dominant strategy, i.e., $u_k(\theta_k, (\theta_k, \theta'_{-k}), M) \geq u_k(\theta_k, (\theta'_k, \theta'_{-k}), M)$ for all $k \in N$, all θ'_k, and all θ'_{-k}, and it is *individually rational* (IR) if $u_k(\theta_k, (\theta_k, \theta'_{-k}), M) \geq 0$ for all $k \in N$, and all (θ_k, θ'_{-k}). In addition, M is called *non-degenerated* (ND) if for all $i \in I$, $u_i(r_i, \theta, M) > u_i(r_i, \theta', M) \geq 0$ for some type profile $\theta = (r_i, \theta_{-i})$ and $\theta' = (r'_i, \theta_{-i})$ with $r'_i \subset r_i$. The social welfare is defined as the total utilities of all agents (including the seller), which can be expressed as $W(\theta') = \sum_{j \in B} z_j(\theta')v_j - C(\theta')$. We say M is *efficient* (EF) if it maximizes the social welfare for all θ'. Lastly, we say M is *weakly budget balanced* (WBB) if $\text{Rev}(M, \theta') \geq 0$ for all θ'.

3 SUMMARY OF THE MAIN RESULTS

The VCG mechanism [1, 3, 9] is a classic mechanism that implements the efficient allocation policy. In the VCG mechanism, the commodities are allocated to maximize the social welfare and each agent is charged the social welfare decrease of others caused by her participation. Let $T^*_j(\theta')$ denote the transaction with the least costs between s and buyer j, and $W^*_j(\theta')$ the associated social welfare. For convenience, we relabel all buyers such that $T^*_j(\theta')$ represents the individual transaction with the j-th highest social welfare. Then, for each agent $k \in N$, her VCG payment can be expressed as

$$x_k^{vcg}(\theta') = W^*(\theta'_{-k}) - (W^*(\theta') - z_k(\theta')v'_k), \tag{1}$$

where $W^*(\theta') = \sum_{h=1}^{D} W^*_{h^+}(\theta')$ denotes the maximum social welfare obtained in θ' and $W^*_{h^+}(\theta') = \max(0, W^*_h(\theta'))$. The VCG mechanism can incentivize information diffusion and achieve the optimal social welfare within the intermediated markets. However, the seller may have a large deficit by using such a mechanism [6, 7].

The underlying reason that the VCG mechanism is not weakly budget balanced is that it treats the intermediaries the same as the buyers. Hence, it not only pays the intermediaries to diffuse the auction information, but also pays them to bid truthfully, which is a waste as the intermediaries do not bid. To mitigate the revenue issue, a natural approach is reducing the amounts paid to the intermediaries. Following this idea, we propose two strategies to eliminate the seller's deficits in the VCG mechanism.

The first strategy, referred to as "payment scaling", involves increasing each intermediary's VCG payment using a scaling vector $L = (l_1, l_2, \cdots, l_D) \in [0, 1]^D$. In particular, let $W^*(\theta' \mid L) = \sum_{h=1}^{D} l_h \cdot W^*_{h^+}(\theta')$ be the scaled social welfare, then each intermediary's VCG payment with a scaling vector L is defined as

$$x_i^{vcg}(\theta' \mid L) = W^*(\theta'_{-i} \mid L) - W^*(\theta' \mid L). \tag{2}$$

The second strategy, called "payment pruning", involves excluding certain connections when calculating each intermediary's VCG payment. Let $\theta'_{-\tau_i} = (r'_i \setminus \tau_i(\theta'), \theta'_{-i})$ denote the reported type profile pruned by $\tau_i : \Theta \rightarrow \mathbb{P}(N)$, a pruning function designed for i. Then, intermediary i's VCG payment with τ_i is defined as

$$x_i^{vcg}(\theta' \mid \tau_i) = W^*(\theta'_{-i}) - W^*(\theta'_{-\tau_i}). \tag{3}$$

Given an intermediary i with a payment policy $x_i^{vcg}(\cdot \mid \tau_i)$, i will act truthfully *if and only if* τ_i is monotonic. (The formal definition of monotonicity is available in the full paper [6].)

Given an $L \in [0, 1]^D$ and a pruning function profile $\tau = \{\tau_i\}_{i \in I}$, the scaled and pruned VCG payment of intermediary i is defined as

$$x_i^{vcg}(\theta' \mid L, \tau_i) = W^*(\theta'_{-i} \mid L) - W^*(\theta'_{-\tau_i} \mid L). \tag{4}$$

We refer to the VCG mechanism with each intermediary paying $x_i^{vcg}(\theta' \mid L, \tau_i)$ as the VCG mechanism with Payment Scaling and Pruning (abbreviated as VCG-PSP), and have the following result.

THEOREM 3.1. *Given any $L \in [0, 1]^D$ and monotonic τ, the VCG-PSP is IC, IR, EF and $Rev(VCG\text{-}PSP, \theta') \geq Rev(VCG, \theta')$ for all θ'.*

The seller has a wealth of choice for (L, τ). We next present a subset of VCG-PSPs possessing all the expected properties. Given an allocation $\pi(\theta')$, we use $\text{AG}(\theta')$ to denote the allocation graph which is defined as the union of the transactions in $\pi(\theta')$, i.e., $(i, k) \in \text{AG}(\theta') \Leftrightarrow \exists (i, k) \in t \in \pi(\theta')$.

Definition 3.2 (Critical Neighborhood). Given an allocation graph $\text{AG}(\theta')$, we call $\delta_i(\theta') = r'_i \cap \text{AG}(\theta') \cup I_i(\theta')$ the critical neighborhood of i, where $I_i(\theta')$ denotes the intermediaries in r'_i.

Let L_1 be a D-dimensional vector with the form of $(l_1, 0, 0, \cdots, 0)$, where $l_1 \in (0, 1]$. In addition, let $\tau^* = \{\delta_i\}_{i \in I}$ be a pruning function profile. We refer to the VCG-PSP with (L_1, τ^*) as Critical Neighborhood Auction (abbreviated as CNA).

THEOREM 3.3. *The CNA is IC, IR, EF, ND and $Rev(CNA, \theta') \geq \max\{Rev(VCG, \theta'), Rev(VCG\text{-}WI, \theta')\} \geq 0$ for all θ', where the VCG-WI represents the VCG Without Intermediaries.*

4 CONCLUSIONS

In this paper, we put forward the study of diffusion mechanisms in the intermediated markets–an application scenario commonly seen in reality. We showed that the classic VCG mechanism can lead to a larger deficit for the seller, and proposed a set of novel solutions to eliminate all the seller's deficits.

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