Cooperation and Coordination in Heterogeneous Populations with Interaction Diversity

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ABSTRACT

Cooperation, a prosocial behavior enhancing collective rewards in multi-agent games, intricately intertwines with coordination. This study explores how interaction diversity and zero-sum gifting influence cooperation and coordination in heterogeneous populations, where agents engage in threshold public goods games with multiple equilibria. Our model accommodates two sources of inequality: variations in agents' capabilities to provide public goods and differences in the rewards they receive upon successful public good provision. In the absence of gifting, we demonstrate the inevitability of intermediate interaction intensity in fostering global cooperation, elucidating conditions for co-dominance, coexistence, and the polarized state of cooperation. While gifting introduces reciprocity opportunities, our findings highlight the importance of maintaining moderate levels of gifting, as excessive gifting can paradoxically undermine global cooperation. This research contributes valuable insights into the emergence of cooperation and coordination dynamics.

KEYWORDS

Cooperation; Coordination; Heterogeneous Populations; Public Goods Game; Evolutionary Game Theory

ACM Reference Format:
Hao Guo, Zhen Wang, Junliang Xing, Pin Tao, and Yuanchun Shi. 2024. Cooperation and Coordination in Heterogeneous Populations with Interaction Diversity. In *Proc. of the 23rd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2024), Auckland, New Zealand, May 6 – 10, 2024*, IFAAMAS, 9 pages.

1 INTRODUCTION

Cooperation, a behavior that entails incurring costs to benefit others [49], is the backbone of maintaining large collective populations (e.g., human society). Interaction among unrelated individuals always creates social dilemmas [18, 37], where each individual aims to maximize their own benefit, leading to a sub-optimal solution for the collective. At its core lies the prisoner's dilemma (PD) and public goods game (PGG) [13, 46]. These games describe the situations in which natural selection favors defection over cooperation and makes the mean population payoff lower than if everybody had cooperated. Understanding the maintenance and evolution of cooperation among selfish individuals prompted academic interest during the past decades. The typical mechanism to explain emergent cooperation includes kin and group selections [14, 42], direct, indirect, and spatial reciprocities [2, 19, 28, 35]. Most research on cooperation, however, has traditionally concentrated on agents with homogeneous identities. Actually, agents often exhibit asymmetry in many scenarios [4, 6]. Factors such as their position in social networks, inherent wealth inequality, and their history of interactions can lead to differences in their abilities and behaviors, then create situations of asymmetric interactions and games [16, 26].

Coordination plays a crucial role in fostering prosocial behaviors and tackling collective issues [29, 44], like climate change. It involves overcoming multifaceted challenges due to various interconnected factors [23, 36]. Tackling a collective task necessitates not only cooperation but also cooperation of a minimum number of agents, constituting a coordination challenge [45]. For instance, a village's failure to gather enough resources or manpower to build a dam demonstrates coordination challenges. Therefore, coordination is often accompanied by the risk of failure [34], resulting in a waste of resources (e.g., an unfinished dam would waste labor, funds, and resources). The threshold public good game (TPGG) model [9, 22], which integrates individual efforts towards a collective goal, highlights this aspect. Along this line, one can naturally consider the heterogeneous composition of the population [12]. Different from the conventional symmetry assumption, these participating individuals may differ: they can differ in what extent they are able to contribute to the collective task (e.g., wealth and productivity) and what consequences their actions have (e.g., success or failure in achieving the target). Our research is dedicated to explaining the emergence of cooperation and coordination in heterogeneous multi-agent systems.

Extensive research has been conducted on wealth inequality and productivity disparities in PGGs and TPGGs, utilizing evolutionary game theory [43], learning theory [25, 27], and behavioral experiments [17, 45]. These studies, however, have often overlooked

Proc. of the 23rd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2024), N. Alechina, V. Dignum, M. Dastani, J.S. Sichman (eds.), May 6 – 10, 2024, Auckland, New Zealand. © 2024 International Foundation for Autonomous Agents and Multiagent Systems (www.ifaamas.org).

specific aspects: i) the ability of individuals or a small group to independently resolve collective issues; ii) *interaction diversity*, a term defined to quantify the fraction of intrapopulation and interpopulation interactions [5, 51]. In community structures, where diverse interactions are common, individuals often form stronger connections within their own communities while having fewer connections with other communities [51]. Given the presence of inequality and diverse interactions, it is natural to contemplate the consequences of reducing inequality in asymmetric interactions. To this end, we introduce the concept of *zero-sum gifting*, frequently associated with peer rewards in multi-agent reinforcement learning [21, 44]. However, this setup has been relatively unexplored in heterogeneous populations. In light of these considerations, our motivation questions are:

What are the evolutionary dynamics in a system where an individual or a certain proportion of individuals can address public goods provision? How does the interaction diversity influence the equilibrium selection? Can introducing gifting stimulate global cooperation and coordination in these contexts?

To address these questions, we propose a novel model based on evolutionary game theory (EGT), focusing on one-shot TPGG where agents lack historical data or past interaction memory [11]. This model considers individual attributes like wealth and productivity as measures of capability, leading to the development of a capability threshold public goods game (CTPGG). We primarily focus on two-player games within heterogeneous populations to simplify our analysis. The system encompasses two distinct populations: one identified as the weak population, and the other as the strong population. Two forms of inequality manifest between the weak and the strong populations: variations in agents' capabilities to provide public goods and the rewards they receive upon completing public good provision. Within this system, we aim to explore the evolutionary dynamics of cooperation and coordination under the interplay between interaction diversity and gifting.

Employing replicator dynamics [32], we provide definitive responses to our motivation questions. Specifically, in the scenario of exclusive interpopulation interaction, the system converges to a state wherein strong cooperators provide public goods while weak agents free ride on it. However, introducing gifting at a moderate level fosters global cooperation and coordination (Theorem 1). Our study also elucidates the possibility of cyclic dynamics (Theorem 3). In scenarios incorporating intrapopulation and interpopulation interactions, we showcase that global cooperation and coordination can be achieved even in the absence of gifting, contingent upon an intermediate level of interaction intensity (Theorem 4). Interestingly, our results underscore the significance of maintaining moderate gifting, as excessive gifting can paradoxically undermine global cooperation. Furthermore, we derive the conditions for codominance, coexistence, and the polarized state of cooperation.

In the following, we provide the related work in section 2. Section 3 presents the details about the capability threshold public good game in the two-player scenario and introduces evolutionary dynamics. Our findings, in both the presence and absence of gifting, are presented in Section 4. Finally, we conclude our study and outline avenues for future research in section 5. We present the agent-based simulation results and provide proof for our theoretical findings in the supplementary information (SI)[1].

2 RELATED WORK

This study investigates cooperation and coordination within heterogeneous multi-agent systems, drawing inspiration from established game models like the threshold public goods game and the collective risk dilemma game. We propose a novel model, the capability threshold public goods game, aimed at exploring the evolutionary dynamics of cooperation and coordination. Leveraging the framework of evolutionary game theory, this paper sheds light on the emergence of global cooperation and coordination under the interplay between inequalities and interaction intensity, combined with the effect of gifting. Evolutionary game theory has provided a powerful framework for studying human behavior and strategy evolution and has become of interest to economists, statisticians, sociologists, and computer scientists. It has been used to solve the conundrum in opinion dynamics [50], the evolution of social conventions [3], and social norms [31, 33], particularly in the realm of cooperation evolution [30, 40, 46]. Over the past few decades, EGT on cooperation has extended from symmetric scenarios to asymmetric scenarios [24, 38].

Inequality or asymmetry, a pervasive phenomenon, has been extensively studied in social dilemma games, including prisoner's dilemma games, as well as linear and threshold public good games. Previous investigations on the impact of inequality in public goods games have yielded mixed results [7, 8]. A comprehensive understanding of inequality in the evolution of cooperation and coordination remains elusive [1, 20]. In the context of one-shot games, EGT provides a framework for exploring the role of heterogeneity in cooperation through ecological and genotypic forms [24]. Ecological asymmetry emerges mainly from the location of the agents (e.g. the location in a structured population), whereas genotypic asymmetry is derived from the players themselves (e.g. the productivity in the provision of public goods). Heterogeneity introduced by resources suggests that cooperation can be sustained when some agents receive more resources than others [20]. Additionally, studies have demonstrated that asymmetric relationships within social networks can enhance cooperation, particularly in the scenario involving a moderate proportion of one-way interactions [39].

As the game is repeated, the historical information individuals memorize makes it a complicated scenario. When considering endowment and productivity inequality in games, empirical findings have revealed that heterogeneous endowment exert a more negative influence on human cooperation [45]. However, the negative effect of endowment heterogeneity can be partially mitigated by peer punishment [48]. Although strong inequality in wealth and productivity has been proven to inhibit cooperation, slightly unequal wealth may benefit cooperation when other inequalities exist [16]. Another typical behavior that triggers prosocial behavior includes helping and gifting. Deliberate reward passing or gifting has been identified as a powerful way of altering the learning progression of multi-agent systems [21]. Without inequality, zero-sum gifting with reinforcement learning algorithms has demonstrated its efficacy in promoting the convergence of high-risk, general-sum coordination

[1] https://github.com/DC-Freezing/AAMAS2024SI

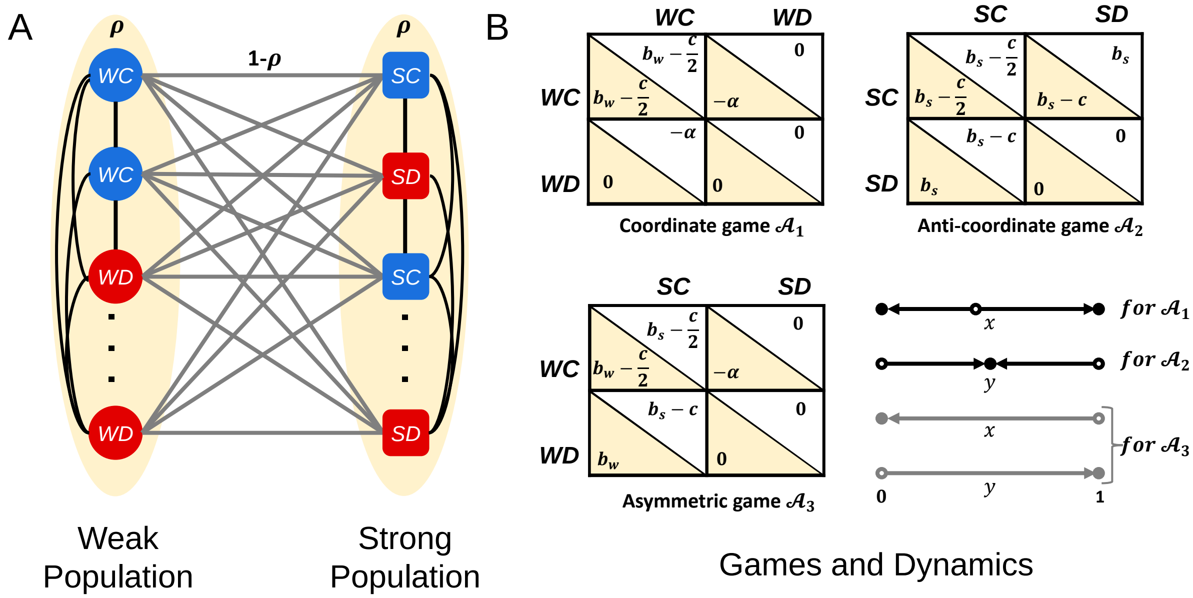

Figure 1: Schematic representation of population games. (A) The multi-agent system consists of two heterogeneous populations, with agents well-mixed within each population. Connections between agents from the two populations are complete. Each agent interacts with its counterpart from the same or a different population with a probability of ρ or $1-\rho$. (B) The weak population participates in a coordination game ($\mathcal{A}_1$), while the strong population engages in an anti-coordinate game ($\mathcal{A}_2$). Games involving agents from different populations ($\mathcal{A}_3$) are asymmetric. The lines illustrate the evolutionary dynamics of the three games.

games to the prosocial equilibrium [44]. With inequality, it becomes conceivable for the rich to sustain cooperation by assisting the poor within a certain degree of uncertainty [1].

Unlike the aforementioned studies, which mainly emphasized the impact of endowment and productivity, our study expands its scope to encompass individual capability, particularly scenarios involving individuals capable of independently providing public goods. Given the ubiquitous inequality in modern society, understanding and promoting cooperation and coordination in such contexts constitutes an interesting topic. To this end, we introduce two pivotal elements: interaction diversity and gifting in heterogeneous populations. We investigate the dynamics of cooperation and coordination in games with multiple equilibria, and explore the influence of inequality, interaction diversity, and gifting. Our study paves fertile avenues for studying cooperation and coordination in the future.

3 AN EVOLUTIONARY GAME THEORETIC MODEL

A public good is a common resource from which all individuals may benefit regardless of their contributions [18]. Due to the non-excludability of these resources, there is a temptation to free-ride on the contribution of others. This scenario results in a social dilemma effectively represented through public goods games. In the game, although all individuals would be better off by cooperating, they are all incentivized to choose defection, resulting in a sub-optimal outcome. In PGGs with a threshold [22], the public goods are modeled as a form of protection to ensure individual rewards. Specifically, if and only if the group's contribution reaches a certain threshold, the public goods can be formalized, and each group member can be rewarded. However, if the threshold is not met, contributing individuals may even lose their invested resources, therefore introducing a certain amount of risk. Building upon TPGGs, this study introduces heterogeneous populations and extends the model to incorporate inequality and interaction diversity. We develop a comparably simple set-up: a two-population, two-player capability threshold public goods game. The payoff matrix for a general two-strategy game is as follows:

$$\mathcal{A} = \begin{pmatrix} a_{11}, b_{11} & a_{12}, b_{12} \\ a_{21}, b_{21} & a_{22}, b_{22} \end{pmatrix}, \tag{1}$$

where the first element is the payoff obtained by the row player, and the second element represents the payoff for the column player.

3.1 Two-Player Games with Inequality

To explore the effect of inequality on cooperation and coordination, we develop a model involving two heterogeneous populations. The first population is labeled as weak, composed of agents possessing limited capability to address collective tasks and deriving modest rewards from the completed provisions. In contrast, the second population is labeled as strong, consisting of agents with heightened capabilities in resolving collective tasks and obtaining substantial rewards from their completion. Consequently, two forms of inequality manifest between these populations: unequal capability and reward. We consider a two-player game that encompasses pairs of agents drawn from weak and strong populations. Agents from either the weak or strong population face the same strategy set denoted as $\mathcal{S} = \{C, D\}$, where C and D mean cooperation and defection, respectively. Subsequently, there are four types of agents: weak cooperators (WCs), weak defectors (WDs), strong cooperators (SCs), and strong defectors (SDs). The model investigates three interaction types: within the weak population, within the strong population, and between agents from both populations (see Fig. 1A).

In our proposed two-player CTPGG, these two agents confront a threshold for the provision of public goods, denoted as c ($c > 0$). Each agent has to make a decision from the strategy set. Selecting cooperation means the agent pays the cost or contributes to the public goods. Selecting defection implies the agent free-rides on others' contributions and benefits without sowing. Specifically, asymmetric capability is explained as follows: cooperators within the strong population can independently achieve the required threshold ($\alpha_s \geq c$), whereas those within the weak population can not. To attain the threshold, agents in the weak population must cooperate with their counterparts, denoting their capability as $\alpha = \frac{c}{2}$. This implies that the combined capabilities of the two weak agents are precisely equal to the threshold. In other words, to achieve the threshold, the minimum number of cooperators needed is one in the strong population and two in the weak population.

3.2 Interaction Diversity

Interactions between strong agents. When two strong cooperators interact, their cumulative capability satisfies $2\alpha_s > c$. Therefore, they evenly share the threshold and receive a benefit denoted as b_s, resulting in a payoff of $b_s - \frac{c}{2}$. When a strong cooperator encounters a strong defector, the former bears the cost alone, yielding a payoff of $b_s - c$, while the latter obtains a payoff of b_s as a free rider. If both opt for defection, they fail to meet the threshold and gain nothing. The payoff ranking follows the sequence $a_{21} > a_{11} > a_{12} > a_{22}$, classifying this scenario as a snowdrift game [10], which is also known as an anti-coordination game (see Fig. 1B $\mathcal{A}_2$). The pure Nash Equilibria involve (C, D) and (D, C), implying the optimal choice is to diverge from your counterpart's strategy.

Interactions between weak agents. In interactions within the weak population (see Fig. 1B $\mathcal{A}_1$), a weak cooperator, due to its inability to independently fulfill the collective task, suffers a loss of α when interacting with a defector. Meanwhile, the defector gains nothing as the collective threshold cannot be met. However, agents can fulfill the provision of public goods by choosing cooperation. Subsequently, mutual cooperation results in a payoff of $b_w - \frac{c}{2}$, where b_w ($b_w < b_s$) is the reward for weak agents. On the other hand, mutual defection yields a payoff of 0. With payoff ranking satisfying: (i) $a_{11} > a_{22}, b_{11} > b_{22}$; (ii) $a_{11} = b_{11}, a_{22} = b_{22}, a_{21} = b_{12}$; (iii) $a_{11} - a_{21} < a_{22} - a_{12}$, this game aligns with the stag hunt game, which is also known as a kind of coordination game [15]. The pure Nash Equilibria involve (C, C) and (D, D), signifying the optimal choice is to align your strategy with your counterpart's.

Interactions between agents from different populations. Shifting attention to interactions spanning both populations (see Fig. 1B $\mathcal{A}_3$), notable differences arise in the $WC - SD$ and $WD - SC$ pairwise scenarios. In the case of a weak cooperator encountering a strong defector, the former receives $-\alpha$, while the latter obtains 0. When a weak defector interacts with a strong cooperator, the former gains b_w, whereas the latter receives $b_s - c$. In this game, cooperation is a dominant strategy for strong agents as both $b_{11} > b_{12}$ and $b_{21} > b_{22}$ are met. Consequently, the optimal choice for a weak agent is defection, given that $a_{21} > a_{11}$ always holds.

3.3 Population Setup

We consider two well-mixed populations denoted as $\mathcal{P}_i = \{1, 2, \cdots, N\}$, encompassing both weak and strong populations. This study mainly focuses on populations with infinitely many agents, *i.e.*, $N \rightarrow +\infty$. Each agent has an equal opportunity to interact with any other agent within the population. Notably, our analysis encompasses not only intrapopulation interaction but also interpopulation interaction between the weak and strong populations. Agents between two populations are fully connected as well. We denote the interaction intensity as ρ, whereby the fraction of the intrapopulation and interpopulation interactions are ρ and $1 - \rho$, respectively (see Fig. 1A).

Denote the fraction of cooperators in the weak and strong populations as x and y, respectively. Correspondingly, $1 - x$ and $1 - y$ represent the fraction of defectors in weak and strong populations. With the predefined payoff matrices, the expected payoff of cooperation and defection in the weak population can be calculated as follows:

$$\begin{aligned}\pi_{WC} &= \rho(x(b_w - \frac{c}{2}) + (1-x)(-\alpha)) \\ &\quad + (1-\rho)(y(b_w - \frac{c}{2}) + (1-y)(-\alpha)), \\ \pi_{wD} &= (1-\rho)yb_w,\end{aligned} \tag{2}$$

where the first term of the right-hand side of π_{WC} represents the payoff derived from intrapopulation interaction, while the second term means the payoff obtained from interactions with strong agents. Similarly, we can calculate the expected payoff of cooperation and defection in the strong population as follows:

$$\begin{aligned}\pi_{SC} &= \rho(y(b_s - \frac{c}{2}) + (1-y)(b_s - c)) \\ &\quad + (1-\rho)(x(b_s - \frac{c}{2}) + (1-x)(b_s - c)), \\ \pi_{SD} &= \rho y b_s,\end{aligned} \tag{3}$$

where the first term of the right-hand side of π_{SC} represents the payoff acquired from intrapopulation interaction, and the second term means the payoff obtained from interacting with weak agents.

3.4 Replicator Dynamics

In this study, we assume that both weak and strong populations evolve in accordance with the replicator dynamics. The replicator equation is a differential equation commonly used to depict evolutionary dynamics in infinitely large populations [32]. This equation describes how the growth of a specific strategy is proportional to the difference in payoffs. Therefore, the evolutionary dynamics of our game model can be represented as follows:

$$\begin{aligned}\dot{x} &= x(1-x)(\pi_{WC}-\pi_{WD}),\\ \dot{y} &= y(1-y)(\pi_{SC}-\pi_{SD}),\end{aligned} \tag{4}$$

where $\dot{x}$ and $\dot{y}$ represent the derivative of weak and strong cooperation with respect to time, respectively. We denote the difference in expected payoffs as:

$$\begin{aligned}h_1(x,y) &= \pi_{WC}-\pi_{WD}\\ &= b_w\rho x-\alpha,\\ h_2(x,y) &= \pi_{SC}-\pi_{SD}\\ &= (1-\rho)\alpha x-\rho(b_s-\alpha)y+b_s-2\alpha.\end{aligned} \tag{5}$$

According to the game model described above, the parameters consistently follow the rank: $b_s > 2\alpha$, $b_s > b_w$, and $b_w > \alpha$.

3.5 Asymmetric Games with Gifting

Due to asymmetric rewards, strong agents consistently receive higher payoffs from the completing provision of public goods ($b_s > b_w$). To mitigate this inequality while fostering prosocial behavior [44], we introduce a zero-sum gifting mechanism during interpopulation interaction when mutual cooperation occurs. In this setup, strong cooperators incur a cost η while providing a bonus of the same magnitude (η) to weak cooperators. This form of gifting can be interpreted as a reward for the cooperative behavior of weak agents or as an exogenous incentive to encourage such behavior. Consequently, the first element of $\mathcal{A}_3$ is modified to $a_{11} = b_w - \frac{c}{2} + \eta$ and $b_{11} = b_s - \frac{c}{2} - \eta$. In this scenario, the difference in expected payoffs becomes:

$$\begin{aligned}g_1(x,y) &= \pi_{WC}-\pi_{WD}\\ &= b_w\rho x+(1-\rho)\eta y-\alpha,\\ g_2(x,y) &= \pi_{SC}-\pi_{SD}\\ &= (1-\rho)(\alpha-\eta)x-\rho(b_s-\alpha)y+b_s-2\alpha.\end{aligned} \tag{6}$$

By substituting Eq. 6 into Eq. 4, we can derive the evolutionary dynamics of the system when gifting is included.

4 THEORETIC RESULTS

We organize our theoretical results from simpler to more complex scenarios. When examining evolutionary dynamics with solely intrapopulation interaction (see SI for details), we identify two asymptotically stable equilibrium points in the stag hunt game (matrix $\mathcal{A}_1$): a cooperation state ($x = 1$) and a defection state ($x = 0$). Additionally, an unstable equilibrium point exists at $x^* = \frac{\alpha}{b_w}$. The population converges to a defection state if the initial density of WC is smaller than x^*, and to a cooperation state otherwise. The sketch of evolutionary dynamics for $\mathcal{A}_1$ is shown in Fig. 1 B. Concerning dynamics in the strong population (see SI for details), a unique asymptotically stable equilibrium point exists at $y^* = \frac{b_s-2\alpha}{b_s-\alpha}$ in the anti-coordinated game. In addition, two unstable equilibrium points exist at $y = 0$ and $y = 1$. The sketch of evolutionary dynamics for $\mathcal{A}_2$ is shown in Fig. 1B. In contrast, evolutionary dynamics with solely interpopulation interaction converge to the state $x = 0$ and $y = 1$. The sketch of evolutionary dynamics for $\mathcal{A}_3$ is shown in Fig. 1B.

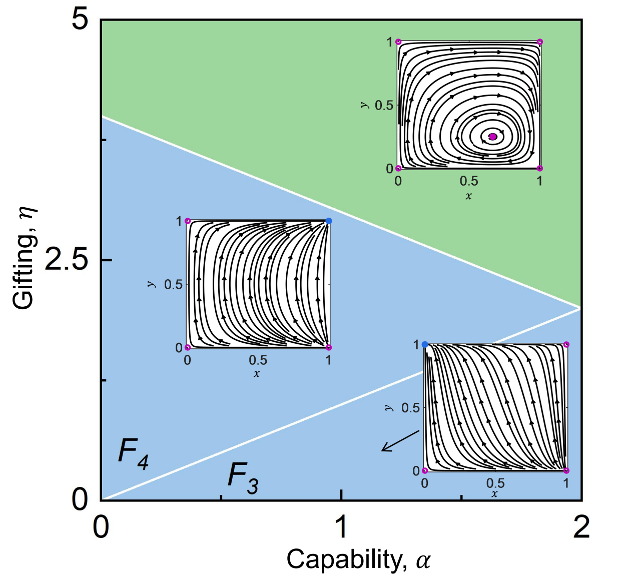

Figure 2: Phase diagram of equilibrium in the scenario involving only interpopulation interaction with gifting. The blue regions show the conditions under which the unique asymptotically stable state is achieved. In the green region, the interior equilibrium point is neutrally stable. The closed and open circles represent stable and unstable equilibria, respectively. Parameters are fixed at $b_w = 2$, $\rho = 0$, and $b_s = 4$. The insets, arranged from bottom to top, correspond to fixed gifting values of 0.5, 2, and 4, given $\alpha = 1$.

4.1 The Role of Gifting under Interpopulation Interaction

In the scenario with solely interpopulation interaction, *i.e.*, $\rho = 0$, the evolutionary dynamics in the presence of gifting are given as follows:

$$\begin{aligned}\dot{x} &= x(1-x)(\eta y-\alpha),\\ \dot{y} &= y(1-y)((\alpha-\eta)x+b_s-2\alpha).\end{aligned} \tag{7}$$

By solving the replicator equations, we can derive five fixed (or equilibrium) points, including:

- $F_1 = (0,0)$: $x = 0$ and $y = 0$, signifying the *co-extinction* of *WC* and *SC*.
- $F_2 = (1,0)$: $x = 1$ and $y = 0$, signifying a *polarized* state with the dominance of *WC* and the extinction of *SC*.
- $F_3 = (0,1)$: $x = 0$ and $y = 1$, signifying a *polarized* state with the extinction of *WC* and the dominance of *SC*.
- $F_4 = (1,1)$: $x = 1$ and $y = 1$, signifying a global cooperation and coordination state with the *co-dominance* of *WC* and *SC*.
- $F_5 = (\frac{2\alpha-b_s}{\alpha-\eta}, \frac{\alpha}{\eta})$: $x^* = \frac{2\alpha-b_s}{\alpha-\eta}$ and $y^* = \frac{\alpha}{\eta}$, signifying the coexistence of *WC*, *WD*, *SC*, and *SD*. Note that this interior equilibrium point exists if and only if $0 < x^* < 1$ and $0 < y^* < 1$.

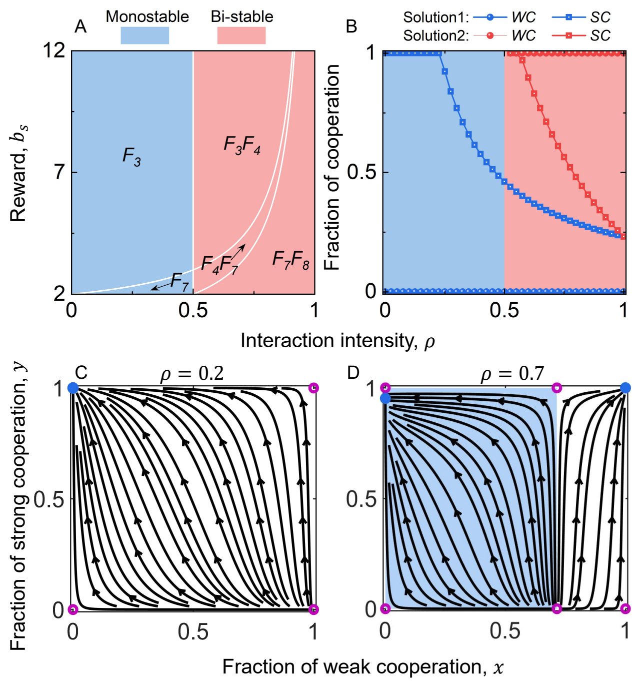

Figure 3: Equilibrium as a function of the (ρ, b_s) pair in the absence of gifting. (A) The phase diagram illustrates how the asymptotically stable equilibria vary. The blue regions show the monostable state, where the equilibrium is insensitive to the initial values of (x, y) pair. The red region indicates the bi-stable state, where the equilibrium to which the system converges relies on the domain of attraction. (B) The fraction of cooperation as a function of ρ, given $b_s = 2.3$. (C) and (D) Phase portrait with different values of ρ, given $b_s = 4$.

To analyze the stability conditions of these equilibria, we employ Lyapunov's indirect method (see SI for details). It's essential to note that the conditions $b_s > b_w$, $b_s > 2\alpha$, and $b_w > \alpha$ consistently hold. We begin by addressing the question, "Can introducing gifting stimulate global cooperation and coordination?" in solely interpopulation interaction.

Theorem 1. *In the scenario of solely interpopulation interaction with gifting, i.e., $\eta > 0$, the equilibrium point $(1, 1)$ is asymptotically stable if $\alpha < \eta < b_s - \alpha$.*

This theorem demonstrates that gifting can indeed stimulate global cooperation and coordination when $\alpha < \eta < b_s - \alpha$. These conditions significantly restrict the acceptable range of gifting. The rationale behind these conditions is straightforward: (i) $\eta > \alpha$ ensures that $a_{11} > a_{21}$, thereby incentivizing weak agents to choose cooperation; (ii) $\eta < b_s - \alpha$ guarantees the condition $b_{11} > b_{21}$, preserving the advantage of cooperation for strong agents. Therefore, when gifting occurs at a moderate level, it promotes global cooperation and coordination.

Theorem 2. *In the scenario of solely interpopulation interaction with gifting, i.e., $\eta > 0$, the equilibrium point $(0, 1)$ is asymptotically stable if $\eta < \alpha$, given $b_s > 2\alpha$.*

This theorem clearly reveals that when $0 < \eta < \alpha$, the asymptotically stable equilibrium point remains unchanged from the scenario without gifting. Consequently, the system converges to a polarized state in which all agents in the weak population defect while all agents in the strong population cooperate. It's evident that gifting can not make sense if the values are not substantial enough. Having established these results for $\eta < b_s - \alpha$, it is interesting to see how the equilibrium changes if gifting exceeds $b_s - \alpha$.

Theorem 3. *In the scenario of solely interpopulation interaction with gifting, i.e., $\eta > 0$, the equilibrium point $(\frac{2\alpha - b_s}{\alpha - \eta}, \frac{\alpha}{\eta})$ is neutrally stable with all the boundary equilibrium points are unstable if $\eta > b_s - \alpha$.*

This theorem reveals that the system enters an oscillatory state when gifting is sufficiently large, satisfying $\eta > b_s - \alpha$. In the weak population, cyclic dominance unfolds between WC and WD, while in the strong population, a cyclic dominance of $SC \rightarrow SD \rightarrow SC$ emerges. Regarding other equilibrium points, they are always unstable. To visually represent the stable states based on the (α, η) pair, we provide a phase diagram in Fig. 2. The blue regions represent monostable states, where only one asymptotically stable equilibrium exists. The insets show the evolutionary dynamics for various initial values of the (x, y) pair. In the monostable state, the equilibrium to which the system converges is irrespective of the initial conditions. In scenarios with a fixed capability (e.g., $\alpha = 1$), as gifting increases, the system transitions from state $(0, 1)$ to state $(1, 1)$. If η continues to increase beyond $b_s - \alpha$, the system enters the cyclic dominance state. In the green area, the inset demonstrates numerous clockwise cycles surrounding the neutrally stable equilibrium F_5.

4.2 The Role of Interaction Diversity and Gifting

This section concentrates on the scenario considering both intrapopulation and interpopulation interactions, *i.e.*, $0 < \rho < 1$ condition. By substituting Eq. 6 into Eq. 4 and solving the replicator equations, we derive nine equilibrium points, including:

- $F_1 = (0, 0)$, $F_2 = (1, 0)$, $F_3 = (0, 1)$, and $F_4 = (1, 1)$.
- $F_5 = (\frac{\alpha}{b_w \rho}, 0)$: $x_1^* = \frac{\alpha}{b_w \rho}$ and $y = 0$, signifying the existence of *WC* in the extinction of *SC*. Note that this equilibrium exists if and only if $0 < x_1^* < 1$.
- $F_6 = (\frac{\eta\rho + \alpha - \eta}{b_w \rho}, 1)$: $x_2^* = \frac{\eta\rho + \alpha - \eta}{b_w \rho}$ and $y = 1$, signifying the existence of *WC* in the dominance of *SC*. Note that this equilibrium exists if and only if $0 < x_2^* < 1$.
- $F_7 = (0, \frac{b_s - 2\alpha}{\rho(b_s - \alpha)})$: $x = 0$ and $y_1^* = \frac{b_s - 2\alpha}{\rho(b_s - \alpha)}$, signifying the existence of *SC* in the extinction of *WC*. Note that this equilibrium exists if and only if $0 < y_1^* < 1$.
- $F_8 = (1, \frac{\rho(\eta - \alpha) + b_s - \eta - \alpha}{\rho(b_s - \alpha)})$: $x = 1$ and $y_2^* = \frac{\rho(\eta - \alpha) + b_s - \eta - \alpha}{\rho(b_s - \alpha)}$, signifying the existence of *SC* in the dominance of *WC*. Note that this equilibrium exists if and only if $0 < y_2^* < 1$.
- $F_9 = (x^*, y^*)$: $x^* = \frac{\rho\alpha(b_s - \alpha) + \eta(2\alpha - b_s)(1 - \rho)}{(1 - \rho)^2 \eta(\alpha - \eta) + \rho^2 b_w (b_s - \alpha)}$ and $y^* = \frac{\alpha(1 - \rho)(\alpha - \eta) + b_w \rho(b_s - 2\alpha)}{(1 - \rho)^2 \eta(\alpha - \eta) + \rho^2 b_w (b_s - \alpha)}$, signifying the coexistence of *WC*,

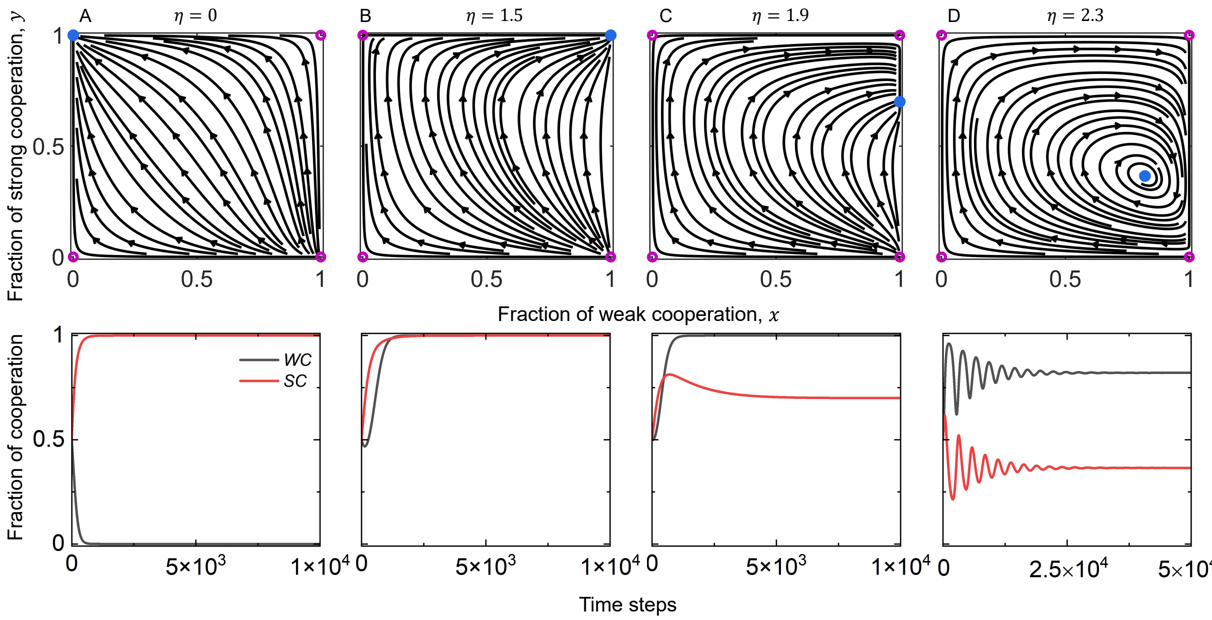

Figure 4: Phase portrait and time series of cooperation under various gifting values η, with parameters set at $\rho = 0.2$, $\alpha = 1$, $b_w = 2$, and $b_s = 3$.

WD, *SC*, and *SD*. Note that this equilibrium exists if and only if $0 < x^* < 1$ and $0 < y^* < 1$.

We continue to employ Lyapunov's indirect method to analyze the stability conditions for the aforementioned equilibria. We emphasize discerning the interplay between inequality and interaction diversity in these equilibria in the absence and presence of gifting. In this context, we present the subsequent theorems:

Theorem 4. *In the absence of gifting, i.e., $\eta = 0$, the equilibrium point $(1, 1)$ is asymptotically stable if $b_s > \frac{\alpha}{1-\rho}$ and $b_w > \frac{\alpha}{\rho}$. In the presence of gifting, i.e., $\eta > 0$, the equilibrium point $(1, 1)$ is asymptotically stable if $b_s > \frac{\alpha+\eta(1-\rho)}{1-\rho}$ and $b_w > \frac{\eta(\rho-1)+\alpha}{\rho}$.*

This theorem demonstrates that introducing interaction intensity ρ can stimulate the system converging to global cooperation and coordination even without gifting. Notably, the critical values of b_s and b_w for stable equilibrium are closely related to α and ρ. When gifting is considered, we observe that global cooperation and coordination can still be achieved. The difference is that the inclusion of gifting modifies these critical conditions by increasing the threshold for b_s while reducing the threshold for b_w.

Theorem 5. *In the absence of gifting, i.e., $\eta = 0$, the equilibrium point $(0, 1)$ is asymptotically stable if $b_s > \frac{(2-\rho)\alpha}{1-\rho}$. In the presence of gifting, i.e., $\eta > 0$, the equilibrium point $(0, 1)$ is asymptotically stable if $\eta < \frac{\alpha}{1-\rho}$ and $b_s > \frac{(2-\rho)\alpha}{1-\rho}$.*

We've revealed that equilibrium point $(0, 1)$ is asymptotically stable when there is only interpopulation interaction. However, when both intrapopulation and interpopulation interactions are considered, this theorem indicates that the critical value for b_s is enlarged. The reason behind this finding is that incorporating the intrapopulation interaction opposes the dominance of cooperation in strong population. Moreover, even though the presence of gifting can further reduce the payoff of *SC*, the polarized state with the dominance of *SC* and the extinction of *WC* remains asymptotically stable as long as $\eta < \frac{\alpha}{1-\rho}$ can be satisfied.

Theorem 6. *In the absence of gifting, i.e., $\eta = 0$, the equilibrium point $(1, 0)$ is unstable. In the presence of gifting, i.e., $\eta > 0$, the equilibrium point $(1, 0)$ is asymptotically stable if $b_w > \frac{\alpha}{\rho}$ and $\eta > \frac{b_s-(1+\rho)\alpha}{1-\rho}$.*

Interaction diversity, on its own, cannot promote the polarized state where cooperation dominates in the weak population while vanishing in the strong population. However, when gifting is considered, the equilibrium $(1, 0)$ can become stable under the interplay between inequality and interaction intensity.

Theorem 7. *In the absence of gifting, i.e., $\eta = 0$, the equilibrium point $(0, \frac{b_s-2\alpha}{\rho(b_s-\alpha)})$ is asymptotically stable, given $0 < y_1^* < 1$. In the presence of gifting, i.e., $\eta > 0$, the equilibrium point $(0, \frac{b_s-2\alpha}{\rho(b_s-\alpha)})$ is asymptotically stable if $\eta < \frac{\rho\alpha(b_s-\alpha)}{(1-\rho)(b_s-2\alpha)}$, given $0 < y_1^* < 1$.*

Recall that the equilibrium in a strong population with only intrapopulation interaction is $\frac{b_s-2\alpha}{b_s-\alpha}$. In comparison, this theorem demonstrates that incorporating interpopulation interaction can significantly enhance the prevalence of *SC*. However, if gifting is introduced, the equilibrium remains stable only if the gifting amount is below the threshold $\frac{\rho\alpha(b_s-\alpha)}{(1-\rho)(b_s-2\alpha)}$. This condition is crucial for maintaining the desired equilibrium in the presence of gifting.

Theorem 8. *In the absence of gifting, i.e., $\eta = 0$, the equilibrium point $(1, \frac{b_s-\alpha-\alpha\rho}{\rho(b_s-\alpha)})$ is asymptotically stable if $\frac{\alpha}{b_w} < \rho < 1$, given $0 < y_2^* < 1$. In the presence of gifting, i.e., $\eta > 0$, the equilibrium point $(1, \frac{\rho(\eta-\alpha)+b_s-\eta-\alpha}{\rho(b_s-\alpha)})$ is asymptotically stable if $b_w > \frac{\alpha-(1-\rho)\eta y_2^*}{\rho}$, given $0 < y_2^* < 1$.*

In scenarios involving interaction diversity, it becomes attainable to achieve a state of complete cooperation in the weak population along with the presence of *SC*. The prevalence of *SC* in these settings is notably higher than in purely intrapopulation interaction. Although distinct from global cooperation and coordination, adjusting interaction intensity proves to be an effective method for boosting cooperation in both weak and strong populations. Furthermore, this equilibrium remains stable when gifting is considered, though it requires a stricter critical threshold. Unsurprisingly, the critical threshold for rewards of weak agent b_w lowers as gifting advantages to the weak cooperators.

Theorem 9. *In the absence of gifting, i.e., $\eta = 0$, the equilibrium point $(\frac{\alpha}{b_w\rho}, \frac{b_s b_w\rho-2\alpha b_w\rho-\alpha^2\rho+\alpha^2}{b_w\rho^2(b_s-\alpha)})$ is unstable. In the presence of gifting, i.e., $\eta > 0$, the equilibrium point (x^*, y^*) is asymptotically stable if $b_w\rho x^*(1-x^*) - \rho(b_s-\alpha)y^*(1-y^*) < 0$ and $\rho^2 b_w(b_s-\alpha) + (1-\rho)^2\eta(\alpha-\eta) < 0$, given $0 < x^* < 1$ and $0 < y^* < 1$.*

The interior equilibrium point remains unstable even though intrapopulation and interpopulation interactions exist simultaneously. However, introducing gifting stabilizes this point, as specified by the theorem, though identifying the precise parameter values meeting these conditions requires numerical methods due to their complexity. It's important to note that the interaction diversity and gifting may not significantly affect the stability of certain equilibrium points, such as $(0,0)$, $(\frac{\alpha}{b_w\rho}, 0)$, and $(\frac{\eta\rho-\eta+\alpha}{b_w\rho}, 1)$, which stay consistently unstable.

We visualize how equilibria change with varying pairs of (ρ, b_s) in the absence of gifting, as depicted in Fig. 3. Fig. 3A shows that when $\rho < \frac{1}{2}$, the system falls into a monostable state where only one stable equilibrium exists. In such cases, the weak population typically shifts towards complete defection, falling into either the F_3 or F_7 region. Meanwhile, the strong population can achieve complete cooperation if $b_s > \frac{2-\rho}{1-\rho}$ (see Fig. 3C), or the coexistence of *SC* and *SD* (F_7) if $b_s < \frac{2-\rho}{1-\rho}$. For $\rho > 0.5$, the system enters a bi-stable state where two stable equilibria exist for specific (ρ, b_s) pairs. In this region, the system's final state is influenced by the initial values of x and y. These starting points thus determine the system's trajectory towards equilibrium. Fig.3D demonstrates that the system's trajectory leads to the left closed circle if starting in the blue area, and to the right circle otherwise. Fig. 3A and B reveal that with a fixed b_s, intermediate values of ρ stimulate global cooperation and coordination. Also, a rise in b_s consistently enhances *SC* (see Fig. 3A). The impact of gifting on equilibrium selection, as shown in Fig.4, reveals that moderate levels of gifting promote global cooperation (see Fig. 4B). Excessive gifting can undermine cooperation in the strong population (see Fig. 4C) and even diminish cooperation in the weak population (see Fig. 4D), thereby destroying global coordination.

5 CONCLUSION

In this study, we introduce a novel evolutionary game theoretic model to explore the puzzle of cooperation and coordination. Different from traditional symmetric TPGGs, we consider two heterogeneous populations with diverse interactions. Agents can differ in their capabilities to provide public goods and the rewards they receive upon completing public goods provision. The study delves into both intrapopulation and interpopulation interactions, highlighting how these populations impact each other. It introduces gifting as a means of peer rewards within interpopulation interactions, acknowledging the inherent asymmetries between populations and exploring global cooperation and coordination.

Our analysis of CTPGGs reveals that when one agent can independently finish public goods provision, the agent with less capability opts for free-riding. Specifically, in the scenario exclusively involving interpopulation interaction, two populations converge to divergent behaviors, with the weak one defecting entirely and the strong one fully cooperating. This state can be improved when considering gifting: global cooperation and coordination are realized at an intermediate gifting level, while excessive gifting elucidates the possibility of cyclic dynamics. In scenarios with both intrapopulation and interpopulation interactions, global cooperation becomes attainable by intermediate interaction intensity, even in the absence of gifting. Moreover, we derive and analyze the conditions governing coexistence, co-dominance, and the polarized state of cooperation in the two populations. The introduction of gifting also exhibits an effective setup to promote global cooperation and coordination. However, it is necessary to maintain gifting at moderate levels, as excessive gifting can paradoxically undermine global cooperation. These findings are further validated through agent-based simulations.

In future work, there exist several intriguing and fertile avenues to explore. This paper has examined a two-player game in heterogeneous populations with diverse interactions. This forms a basis that can be expanded to N-player scenarios. Essential factors such as threshold (e.g., the minimum number of cooperators) and group composition (e.g., the number of weak players in the N-player group) have not been well studied. Addressing the challenges regarding cooperation and coordination in N-player games can offer valuable insight into global issues such as climate change and disease transmission. On the other hand, this study has considered only evolutionary dynamics with the same time scale. Exploring dynamics with fast-and-slow systems could be a compelling avenue [41, 47]. Last but not least, another promising way forward is to build connections between theoretical models and human behavior experiments in the future.

ACKNOWLEDGMENTS

This work was supported in part by the Natural Science Foundation of China (Nos. 62025602, 62222606, 62076238 and 11931015) and China Postdoctoral Science Foundation (No. 2023M741852).

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