Truthful and Welfare-maximizing Resource Scheduling with Application to Electric Vehicles

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ABSTRACT

We consider the problem of scheduling resources with monetary transfers among agents in a setting where multiple outlets can dispense these resources at different rates within fixed time-slots. This problem is motivated by applications such as electric vehicle (EV) charging where energy is the resource and EVs are available within a convenient time window of its owners. The agents' valuations depend on the contiguous time slots at a given outlet that dispense the resource to them. We show that for *monotone* and its sub-class of *dichotomous* valuations, computing the *social welfare-maximizing allocation* is NP-hard, even if there is only one outlet. For monotone and dichotomous valuations, we provide a randomized 2-approximation mechanism that is *truthful in dominant strategies* and *individually rational* for a single outlet and a randomized $O(\sqrt{|S|})$-approximation algorithm with the same properties for multiple outlets (S is the set of time-slots). However, for *single-minded* valuations, the welfare maximization problem for multiple outlets is in $\mathbb{P}$. This allows us to use standard mechanisms like VCG to ensure truthfulness and individual rationality.

KEYWORDS

Electric Vehicle Charging; Truthfulness; Social Welfare.

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1 INTRODUCTION

Allocating resources efficiently among time-constrained consumers is a critical challenge across industries. For instance, power grids distribute electricity to many organizations operating heavy electrical equipments, while food delivery apps allocate incoming food orders to delivery agents. Similarly, a *charging point operator* (CPO) of electric vehicles (EVs) manages multiple charging stations, with EVs coming as consumers to charge themselves. These diverse scenarios share some common features: (a) resources take significant time to serve each consumer, (b) consumers have specific preferences over the schedules (e.g., delivery agents preferring certain geographical areas and times or EVs needing charging at specific times and locations), and (c) payments are allowed, with the option of price discrimination among consumers. This creates a generalized framework of mechanism design for *resource scheduling with monetary transfers*, where the planner (e.g., a grid manager, delivery app, or CPO) must adhere to certain key principles. The first goal is *truthfulness*, that ensures consumers to reveal true electricity demand or delivery agents to disclose real preferences. The second goal is *social welfare maximization*, that aims to maximize the collective consumer satisfaction. Finally, *individual rationality* ensures that the consumers are not penalized for participation.

In this paper, we consider the problem of truthful, individually rational, and welfare-maximizing resource scheduling problem with payments where the allocation and payment decisions are made at certain given epochs, e.g., at certain hours of the day depending on the number of consumer requests that arrive at that time epoch. We keep the electric vehicle allocation as our running example and develop the theory and notation accordingly. However, we want to emphasize that the same framework can also be easily adapted to any resource allocation problem discussed above. The special structure of this setting allows us to show that maximizing welfare is computationally hard and therefore needs to be approximated. However, for such approximated welfare mechanisms, non-trivial allocation and payment rules need to be designed to ensure truthfulness. In this paper, we consider a static setup where the consumers report their values and the mechanism decides the allocation and payments in one go. Making the decision epochs sufficiently fine, a close approximation of the dynamic decision problem can be obtained. Even in this static setup, we find the problem to be quite challenging and therefore a general analysis of an online resource scheduling problem is left as a future exercise.

1.1 Related Work

The literature on truthful resource scheduling is diverse primarily because of the history and application domains of such problems.

The first strand of this literature comes from the classical domain of machine scheduling. In this domain, the primary objective is to minimize *makespan* [2, 10, 11, 15, 17, e.g.]. The question of social welfare has been addressed sporadically, e.g., Koutsoupias [24] defined it as the negation of the sum of executing times of all machines and provided approximation to the optimal.

The second strand comprises of discrete interval scheduling problems, where a set of *weighted* jobs can be executed over multiple machines and the the goal is to *maximize the weighted sum* of executed jobs [5, 6, 8, 32]. While this literature focuses on providing approximation schemes, it does not consider the objective of truthfulness or capture the rich valuation structure of agents.

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Table 1: Summary of results.

Valuation	*# outlets*	*Complexity*	*Mechanism*	*Guarantee*	*DSIC*	*IR*
Monotone/dichotomous	Multiple	NP-Hard	RAE (Algorithm 1 + Algorithm 2 in separation oracle)	$2\sqrt{\lvert S\rvert}$-approx	✓	✓
Monotone/dichotomous	Single	NP-Hard	RAE (Algorithm 1 + Algorithm 3 in separation oracle)	2-approx	✓	✓
Single-minded	Multiple	$\mathbb{P}$	DAE (Algorithm 4)	Optimal	✓	✓

The third strand of literature addresses the axiomatic questions of properties such as truthfulness, budget balance, independence of irrelevant alternatives [20], risk aversion [25], and provide characterization results. These works do not consider the computational complexity of the mechanisms that yield these properties. Kress et al. [26] and Kolen et al. [22] provide nice surveys of these three strands.

The fourth strand is from the algorithmic mechanism design viewpoint, where the computational questions in welfare maximization and truthfulness are considered together. If the time is discrete and each agent desires a set of contiguous time-slots of a resource, then this problem reduces to a special combinatorial allocation problem. We consider this setting in our paper and, therefore, this literature is the most relevant one. For mechanisms with payments, VCG [12, 19, 33] is the most widely used one for guaranteeing truthfulness and welfare maximization. However, it requires the *optimal social welfare* (OSW) allocation to be computed in order to ensure truthfulness. The OSW problem in combinatorial auctions is known to be NP-Hard [13, 29], even in the case where agents are single-minded. In addition, approximating the social welfare to a factor within $k^{1/2-\epsilon}$ (where k is the number of objects or goods) is also NP-Hard [13, 29]. In the case of multi-unit combinatorial auction under the constraint that no object is allocated more than y times (hence considered as the number of units of every object) and every agent gets at most one bundle, approximating social welfare within a factor of $O(k^{1-\epsilon/y+1})$ is NP-Hard [7]. Thus, the approach taken in the literature is to approximately maximize social welfare, while ensuring truthfulness and individual rationality (IR).

There are several algorithms that provide $O(k^{1/y+1})$ approximation guarantee [9, 23, 30] to the social welfare maximization problem. However, for general monotone valuations, only [7, 27] are known to be truthful. Note that the VCG mechanism with approximate social welfare does not generally guarantee truthfulness [28]. Bartal et al. [7] give a deterministic $O(yk^{1/y-2})$ approximation algorithm that ensures truthfulness and IR. However, this approach only works for $y \geqslant 3$. In contrast, Lavi and Swamy [27] provide a randomized mechanism that uses VCG in a computationally tractable manner and achieves $O(k^{1/y+1})$ approximation guarantee for the social welfare $\forall y \geqslant 1$, ensuring truthfulness and IR. Several other works address the single minded buyers [3, 9, 28, e.g.], single-valued buyers [4], and subadditive valuations [16].

Our resource scheduling problem and the results are distinct from that in the literature. In our setup, every outlet-timeslot pair is a good, and exactly one unit of this is available. Hence, this naturally falls in the setup of [27]. However, applying their method directly in our setting where the number of goods is $|S||M|$ (S and M are the set of time slots and outlets respectively) achieves an approximation guarantee of $O(\sqrt{|S||M|})$. Using the structure of the allocation space of our problem, we provide an improved $O(\sqrt{|S|})$-approximation for multiple outlets and a 2-factor approximation for the single outlet (see Section 1.2 and Table 1 for more details).

1.2 Our Contributions

In this paper, we consider the consumers (agents) who are looking for contiguous time-slots to consume resource from an outlet at a rate that is fixed for that consumer-outlet pair. They have different valuations for different such contiguous slots, e.g., infeasible slots have zero values. The planner wants to allocate the resources to maximize the sum of the valuations of all the agents (i.e., welfare-maximizing) while ensuring that agents are truthful. Monetary transfers can be used to achieve this goal. In this setting, our contributions can be summarized as follows.

- For *monotone* and *dichotomous* valuations, computing the welfare-maximizing allocation is NP-Hard even for a single outlet (Theorems 1 and 4).
- When considering a single outlet for the above valuations, we provide a 2-approximate welfare-maximizing mechanism that satisfies *truthfulness in dominant strategies* and *individual rationality* (Theorem 3).
- For the case with multiple outlets, we provide a $O(\sqrt{|S|})$-approximate welfare-maximizing mechanism (S is the set of time-slots) that is also *truthful in dominant strategies* and *individually rational* (Theorem 2).
- For *single minded* agents (agents who get a fixed positive valuation only when a specific set of contiguous slots at a particular outlet is allocated to them) with multiple outlets, we show that the welfare-maximization problem can be reduced to a linear program and hence is efficiently solvable (Theorem 5). Therefore, truthfulness and IR can be ensured via the VCG mechanism.

Our results are summarized in Table 1. The different cases are motivated by the practical limitations of resource scheduling problems. For instance, if the resource outlets are not interconnected and a simultaneous decision over all the requests coming at all outlets cannot be made or if each agent prefers to get resource at any single outlet but has valuations for several set of contiguous time slots at that outlet, then the planner can run the algorithm individually at every outlet and guarantee a constant factor approximation. For the relatively difficult problem of a single CPO (in the context of EV charging) jointly allocating the consumers having preference over multiple outlets, the approximation guarantee becomes worse because the underlying optimization problem gets harder. For certain special settings, e.g., every EV has a desired (outlet, interval) and does not consider any other (outlet, interval), the problem becomes computationally easy.

2 PRELIMINARIES

In this section, we formally describe the resource scheduling problem setup and the mechanism design goals using electric vehicle charging as the motivation.

2.1 Model

Let $N = \{1, 2, 3, \ldots, n\}$ denote the set of electric vehicles (EVs) requesting to charge themselves (e.g., via a mobile application) from a single *charging point operator* (CPO) who owns charging *stations* in a region. Each station has several charging *outlets* and every outlet has a fixed maximum charging rate at which it can charge an EV. We collect together all the outlets in the region (irrespective of whether they are at the same station) and denote $M = \{1, 2, 3, \ldots, m\}$ to be the set of all outlets that the CPO owns in that region. EVs have preferences over different outlets based on their location and charging rates. For instance, an EV would prefer to charge at a charging outlet based on its proximity, the rate of charging (fast/slow), pricing, and various similar factors. We consider CPO as the planner whose goal is to allocate EVs (agents) to outlets and decide an appropriate pricing scheme for the allocation. Since charging an EV requires time, the planner also needs to factor in the time allocated while assigning agents to the outlets. Consider a time horizon (e.g. the working hours of a day) which is discretized into s slots of equal duration denoted by $S = \{1, 2, 3, \ldots, s\}$. Each slot $j \in S$ is an indivisible unit representing the minimum amount of time an agent must charge once plugged in at an outlet $k \in M$. The planner solves the problem of allocating agents to slots at the outlets given a set of charging requests by EVs. Hence, the resource that each EV can be allocated is a pair of 'time slot and outlet'.

Each EV $i \in N$ is allocated a collection of (slot, outlet) pairs which we will be calling a *bundle*. Since no EV can charge at two different outlets at the same time slot, we denote a bundle by $b \in (M \cup \{0\})^S$, which implies that a bundle is a vector of length $|S|$ where the coordinates correspond to the time slots and the value at each coordinate represents the assigned outlets $\{1, 2, \ldots, m\} \cup \{0\}$ at the corresponding time-slot. The special outlet 0 denotes 'unassigned' at that slot. We use b_j to denote the j^{th} coordinate of b, where b_j is the charging outlet assigned at time slot $j \in S$. We assume that each EV wants to be assigned contiguous time-slots exactly at one outlet. This assumption captures the practical problem of repeatedly switching between outlets or stop-start charging, which are infeasible in practice. This implies that we are only considering the types of bundles that satisfy the following: (i) $\forall i, j \in S$, if $b_i, b_j \neq 0$, then $b_i = b_j$ and (ii) there exists i^*, j^*, s.t. $b_i = 0, \forall i < i^*, i > j^*$ and $b_i \neq 0, \forall i^* \leqslant i \leqslant j^*$. The first condition ensures that the bundle consists of time slots at exactly one outlet, while the second condition imposes the contiguity requirement. Denote the set of all such *feasible* bundles by B.

Each agent $i \in N$ comes with a type $\theta_i : B \to \mathbb{R}$, where $\theta_i(b)$ represents the satisfaction of agent i for bundle $b \in B$. We assume that the types satisfy monotonicity unless stated otherwise, i.e., for all $b, b' \in B$ where b' is a sub-bundle of b (b' is a *sub-bundle* of b if b contains all the allocated time-slots of b' at the same outlet, and is represented as $b' \sqsubseteq b$)

$$\theta_i(b') \leqslant \theta_i(b), \forall i \in N, \text{ and } \theta_i(\{0\}^{|S|}) = 0, \forall i \in N. \tag{1}$$

Note that, in this definition, θ_is account for the agent i's preference over outlets (fast/slow chargers), time slots (e.g., their preferred arrival and departure), and their charge demand, in a consolidated manner. Since θ_i is agent i's private information, we need mechanisms to truthfully elicit this information to take an *efficient* decision. We use θ_{-i} to represent the types of agents other than i and $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$ to denote the type profile. Set Θ_i denotes the *monotone* type set of agent i and $\Theta = \prod_{i \in N} \Theta_i$ denotes the set of type profiles. When requesting for charging services, each agent reports $\hat{\theta}_i$ which could be different from their true type θ_i. The planner needs to design a mechanism (the allocation and payment schemes) using the reported type vector $\hat{\theta}$. In a real-world setting, time slots can be categorized into different time-periods of the day, e.g., morning, afternoon, evening, night, and these mechanisms can allocate slots to all the EVs that placed a request before the time-period started.

An allocation is represented as $x = [x(i, b), \forall i \in N, b \in B]$, where $x(i, b) = 1$ when agent i is allocated $b \in B$, and $x(i, b) = 0$ otherwise. We call an allocation *feasible* if it satisfies the following: (a) every agent is allocated at most one bundle, i.e., $\sum_{b \in B} x(i, b) \leqslant 1, \forall i \in N$, and, (b) no more than a single unit of any "time slot, outlet" pair is allocated. Let $B_{jk} = \{b \in B : b_j = k\}$ denote the set of bundles in which the pair $(j, k), j \in S, k \in M$, exists. Then $\sum_{b \in B_{jk}} \sum_{i \in N} x(i, b) \leqslant 1, \quad \forall j \in S, k \in M$. The set of all such feasible allocations is denoted by X.

An allocation function $f : \Theta \to X$ is a mapping that yields a feasible allocation $f(\theta) \in X$ for every type profile $\theta \in \Theta$. The *valuation* of agent $i \in N$ is described by $v_i : X \times \Theta_i \to \mathbb{R}$, which for a given $\theta_i \in \theta$ and a feasible allocation $x \in X$ gives a value $v_i(x, \theta_i) = \sum_{b \in B} \theta_i(b)\, x(i, b)$. Note that if θ_i satisfies monotonicity then so does v_i. Every EV is also asked a payment for a given allocation. A payment function for agent i is given by $\pi_i : \Theta \to \mathbb{R}$ which maps the reported type profile $\theta \in \Theta$ to a real number.

Given the above formulation, the utilities of the agents take a quasi-linear form. Formally, given the reported type profile of agents $\hat{\theta}$, an allocation function f and payment functions $\pi_i, \forall i \in N$, the utility of agent i when its true type is θ_i is given by: $u_i((f(\hat{\theta}), \pi(\hat{\theta})), \theta_i) = v_i(f(\hat{\theta}), \theta_i) - \pi_i(\hat{\theta})$.

Note that in the definitions above, we defined the allocation and the payments to be deterministic. But more generally, the planner can also output randomized allocation and payments. A randomized allocation can be seen as a probability distribution over all deterministic allocations in X. Denote the set of all randomized allocations by $\Delta_X = \{\lambda \in [0, 1]^{|X|} : \sum_{x \in X} \lambda_x = 1 \text{ and } \lambda_x \geqslant 0, \forall x \in X\}$, where λ represents a randomized allocation and λ_x denotes the probability of choosing the deterministic allocation $x \in X$. Note, Δ_X is the convex hull of the set X. Given this, we extend the allocation function $f : \Theta \to \Delta_X$ to be a mapping which yields a randomized allocation $f(\theta) \in \Delta_X$ for a given type profile $\theta \in \Theta$. We also extend the the valuation function of agent i, $v_i : \Delta_X \times \Theta_i \to \mathbb{R}$ to have all randomized allocations Δ_x in the domain. Thus, with a slight abuse of notation we denote $v_i(\lambda, \theta_i) = \sum_{x \in X} \lambda_x\, v_i(x, \theta_i) = \sum_{x \in X} \lambda_x \sum_{b \in B} \theta_i(b)\, x(i, b)$ to be the expected valuation of agent i for the randomized allocation $\lambda \in \Delta_X$ when its type is θ_i. Likewise, the payment $\pi_i(\theta)$ denotes the randomized payment to be made by agent i. For a given the reported type profile $\hat{\theta}$, this gives us the expected utility of an agent i when its true type is θ_i as follows: $u_i((f(\hat{\theta}), \pi(\hat{\theta})), \theta_i) = v_i(f(\hat{\theta}), \theta_i) - \mathbb{E}[\pi_i(\hat{\theta})]$.

In summary, the planner needs to design a social choice function or a mechanism (f, π) such that several desirable properties are satisfied. We define the desirable properties in the following section.

2.2 Design Desiderata

In this paper, our objective is to maximize social welfare through a mechanism that is dominant strategy incentive compatible and individually rational. These properties are defined as follows.

Definition 1 (Efficiency). A deterministic mechanism (f, π) maximizes social welfare and therefore is *efficient* if for every $\theta \in \Theta$, $f(\theta) = \arg\max_{x \in X} \sum_{i \in N} \sum_{b \in B} \theta_i(b)\ x(i, b)$. Correspondingly, a randomized mechanism (f, π) is efficient if for every $\theta \in \Theta$, $f(\theta) = \arg\max_{\lambda \in \Delta_X} \sum_{x \in X} \lambda_x\ \sum_{i \in N} \sum_{b \in B} \theta_i(b)\ x(i, b)$.

The next property incentivizes agents to participate in the game ensuring that their utility is non-negative for every type profile.

Definition 2 (Individual Rationality (IR)). A deterministic mechanism (f, π) is *individually rational (IR)* if for every $\theta \in \Theta$ and for every $i \in N$, $v_i(f(\theta), \theta_i) - \pi_i(\theta) \geqslant 0$. Likewise, a randomized mechanism (f, π) is *ex-post* individually rational if for every $\theta \in \Theta$ and for every $i \in N$, $v_i(x, \theta_i) - p_i \geqslant 0$ for every sample x and p_i drawn from $f(\theta)$ and $\pi_i(\theta)$ respectively.

Finally, since the planner's decision is dependent on the agents' reported types $\hat{\theta}$, we need to incentivize them to report it truthfully.

Definition 3 (Dominant Strategy Incentive Compatible (DSIC)). A deterministic mechanism (f, π) is *dominant strategy incentive compatible (DSIC)* if for every agent $i \in N, \forall\ \theta_i, \tilde{\theta}_i \in \Theta_i$, and $\forall\ \theta_{-i} \in \Theta_{-i}$, $v_i(f(\theta_i, \theta_{-i}), \theta_i) - \pi_i(\theta_i, \theta_{-i}) \geqslant v_i(f(\tilde{\theta}_i, \theta_{-i}), \theta_i) - \pi_i(\tilde{\theta}_i, \theta_{-i})$. Correspondingly, a randomized mechanism (f, π) is DSIC if $\forall i \in N, \forall \theta_i, \tilde{\theta}_i \in \Theta_i$, and $\forall \theta_{-i} \in \Theta_{-i}$, $v_i(f(\theta_i, \theta_{-i}), \theta_i) - \mathbb{E}[\pi_i(\theta_i, \theta_{-i})] \geqslant v_i(f(\tilde{\theta}_i, \theta_{-i}), \theta_i) - \mathbb{E}[\pi_i(\tilde{\theta}_i, \theta_{-i})]$.

In the sections that follow, we focus on mechanisms that achieve the above set of properties in a computationally efficient manner.

3 MONOTONE VALUATIONS

The social welfare maximization problem for monotone valuations can be formulated as the following Integer linear program (ILP).

$$\begin{aligned} \max \quad & \sum_{i \in N} \sum_{b \in B} \theta_i(b)\ x(i, b) \\ \text{s.t.} \quad & \sum_{b \in B_{jk}} \sum_{i \in N} x(i, b) \leqslant 1, \forall j \in S, k \in M, \\ & \sum_{b \in B} x(i, b) \leqslant 1, \forall i \in N, \\ & x(i, b) \in \{0, 1\}, \forall i \in N, b \in B. \end{aligned} \tag{2}$$

We first prove that this problem is NP-Hard even if the true types of the agents θ are known via a polynomial reduction from the *Job Interval Selection Problem* [13, 32] which is known to be NP-Complete. Due to paucity of space, we move the proof of this result and some other proofs to the full version [1].

Theorem 1. *For monotone valuations and for a given K, the decision problem of whether the optimal allocation to the EV charging problem has a social welfare of at least K is NP-complete even when the number of outlets $|M| = 1$.*

Given the above result, the VCG mechanism is intractable for our setup. Thus, we focus on providing mechanisms that maximize social welfare approximately. In particular, we provide randomized mechanisms that ensure DSIC and IR, and approximate the social welfare to within a factor of $O(\sqrt{|S|})$ for multiple outlets and to within a factor of 2 for the single outlet case.

3.1 Mechanism for multi-outlet scenario

Our construction of the randomized mechanism proceeds as follows. We first use the classic VCG mechanism in the *fractional* space to obtain an optimal *efficient fractional* allocation and payments that ensure DSIC and IR. A randomized mechanism is then constructed such that the randomized allocation is a convex decomposition of the fractional allocation scaled by a factor $\alpha > 1$. Also, the randomized payment is set such that the expected payment of every agent is equal to the α-scaled VCG payment calculated in the fractional space. To get the desired convex decomposition of the α-scaled fractional allocation, we need an α-approximation algorithm that gives guarantees w.r.t. the fractional optimal solution for every *monotone* valuation. We provide a greedy algorithm with $O(\sqrt{|S|})$-approximation factor for this purpose. This method approximates the social welfare to within a factor of $O(\sqrt{|S|})$ and retains DSIC and IR via VCG in the fractional space. We call this method Randomized Allocatively Efficient (RAE) mechanism, which is detailed out in Algorithm 1. Note that Algorithm 1 takes an α-approximation algorithm $\mathcal{A}$ as input. It internally employs the ellipsoid method with a *separation oracle* [34] that uses the approximation algorithm $\mathcal{A}$. Thus, Algorithm 1 acts as a template, where the variable $\mathcal{A}$ can be set appropriately. We show in the following result how we can achieve all desirable properties.

Theorem 2. *For monotone valuations and multiple outlets, the* RAE *mechanism (Algorithm 1) that uses Algorithm 2 as $\mathcal{A}$ in the separation oracle approximates the social welfare within a factor of $O(\sqrt{|S|})$ and ensures DSIC and IR.*

The general technique of constructing a randomized mechanism using VCG in a tractable manner was originally proposed by Lavi and Swamy [27] in the context of combinatorial auctions. We adapt their technique to ensure DSIC and IR for monotone valuations, but improve on the approximation guarantees for our setup. In particular, the approximation algorithm proposed in [27] translates to a factor of $O(\sqrt{|S||M|})$ in our setting since every (j, k) pair, where $j \in S, k \in M$ can be seen as a good. However, Theorem 2 provides an improved $O(\sqrt{|S|})$-approximation for the multi-outlet case. Later, we improve it to a constant factor for a single outlet.

For given reported types $\hat{\theta}$, Algorithm 1 first solves the LP relaxation of ILP (2) given by LP (3) to obtain an optimal fractional allocation. This can be computed in polynomial time since the number of variables and constraints in LP (3) are polynomial in $|N|$, $|M|$, and $|S|$. Particularly, note that the number of bundles in $B = O(|M||S|^2)$ since EVs are assigned contiguous time slots at any one outlet. Denote the optimal solution of LP (3) by $x^{\text{fr}}(\hat{\theta})$. Wherever clear from

Algorithm 1: RAE Mechanism

Input: Agent reports $\hat{\theta}$ and an α-approximation algorithm $\mathcal{A}$ ($\alpha > 1$) that provides an integer solution with a value of at least $1/\alpha$ times the value of the fractional optimal of LP (3) for any monotone $\hat{\theta}$.

Output: A randomized allocation $f(\hat{\theta}) \in \Delta_X$ and randomized payments $\pi_i(\hat{\theta}), \forall i \in N$.

Solve LP (3) to get an optimal fractional allocation $x^{\text{fr}}(\hat{\theta})$.

Set payments $p_i^{\text{fr}}(\hat{\theta})$ for every agent i using VCG in the fractional space X^{fr} as given by Equation (4).

Scale $x^{\text{fr}}(\hat{\theta})$ and $p_i^{\text{fr}}(\hat{\theta}), \forall i \in N$ by α.

Using `GetConvexDecomposition`$(x^{\text{fr}}(\hat{\theta}), \mathcal{A})$, construct a convex decomposition of $x^{\text{fr}}(\hat{\theta})/\alpha = \sum_{x^I \in X} \lambda^*_{x^I} x^I$ with polynomially many $\lambda^*_{x^I} > 0$.

Set the randomized allocation $f(\hat{\theta})$ and payments $\pi_i(\hat{\theta}), \forall i \in N$ according to Equation (8).

return $f(\hat{\theta}), \pi(\hat{\theta})$

Procedure `GetConvexDecomposition`$(x^{\text{fr}}(\hat{\theta}), \mathcal{A})$:

- Solve the dual LP (6) using ellipsoid method with `SeparationOracle()` that uses $\mathcal{A}$ and $x^{\text{fr}}(\hat{\theta})$. This identifies an LP that is equivalent to LP (6) but with only polynomial no. of constraints from $\sum_{i \in N} \sum_{b \in B} x^I(i,b)\ w(i,b) + z \leqslant 1, \forall x^I \in X$.
- Solve the primal LP (5) by considering polynomially many variables corresponding to the above identified constraints to get the optimal solution λ^*.
- **return** λ^*

Procedure `SeparationOracle()`:

- **Input:** $x^{\text{fr}}(\hat{\theta})$, an α-approximation algorithm $\mathcal{A}$, and any point (w, z), where $w = [w(i,b), \forall i \in N, b \in B]$ is unconstrained.
- **Output:** A separating hyperplane which is used to cut the ellipsoid in a given iteration.
- **if** $z + \frac{1}{\alpha} \sum_{i \in N} \sum_{b \in B} x^{\text{fr}}(i,b)\ w(i,b) > 1$ **then**
  - Using $\mathcal{A}$, get an $x^I \in X$ s.t. $\sum_{i \in N} \sum_{b \in B} x^I(i,b) w(i,b) \geqslant \frac{1}{\alpha} \max_{x \in X^{\text{fr}}} \sum_{i \in N} \sum_{b \in B} x(i,b) w(i,b)$.
  - Using the above inequality and the condition in the if statement, we get a violated constraint $\sum_{i \in N} \sum_{b \in B} x^I(i,b)\ w(i,b) + z > 1$ of the LP (6) for the point (w, z).
  - **return** $\sum_{i \in N} \sum_{b \in B} x^I(i,b)\ w(i,b) + z = 1$
- **else**
  - **return** $z + \frac{1}{\alpha} \sum_{i \in N} \sum_{b \in B} x^{\text{fr}}(i,b)\ w(i,b) = 1$
- **end**

context, we will use x^{fr} instead of $x^{\text{fr}}(\hat{\theta})$ for brevity. Note that, x^{fr} is a fractional allocation, where $x^{\text{fr}}(i,b) \in [0,1], \forall i \in N, b \in B$ denotes the fraction of bundle b allocated to agent i. Denote X^{fr} to be the set of all feasible fractional allocations[1].

$$\begin{aligned} \max \quad & \sum_{i \in N} \sum_{b \in B} \hat{\theta}_i(b)\ x(i,b) \\ \text{s.t.} \quad & \sum_{b \in B_{jk}} \sum_{i \in N} x(i,b) \leqslant 1, \forall j \in S, k \in M \\ & \sum_{b \in B} x(i,b) \leqslant 1, \forall i \in N; \quad x(i,b) \geqslant 0, \forall i \in N, b \in B \end{aligned} \tag{3}$$

The payment of every agent i is then given by the VCG payment in the fractional space X^{fr} as follows.

$$p_i^{\text{fr}}(\hat{\theta}) = \max_{x \in X^{\text{fr}}} \sum_{i' \in N \setminus \{i\}} \sum_{b \in B} \hat{\theta}_{i'}(b) x(i',b) - \sum_{i' \in N \setminus \{i\}} \sum_{b \in B} \hat{\theta}_{i'}(b) x^{\text{fr}}(i',b) \tag{4}$$

The *fractional mechanism* $(x^{\text{fr}}, p^{\text{fr}})$ guarantees DSIC and IR since it is the VCG mechanism in the fractional space X^{fr}. Note that, even if the allocation and the payments are scaled by some $\alpha > 1$, i.e., $x^{\text{fr}}(i,b)/\alpha, \forall i \in N, b \in B$ and $p_i^{\text{fr}}(\hat{\theta})/\alpha, \forall i \in N$, DSIC and IR still hold. This is due to v_i's linearity in x^{fr} i.e., $v_i(x^{\text{fr}}(\hat{\theta}), \theta_i) = \sum_{b \in B} \theta_i(b)\ x^{\text{fr}}(i,b), \forall x^{\text{fr}}(\hat{\theta}) \in X^{\text{fr}}$. Note that we also overload v_i for a fractional allocation. From the above discussion, we get the following lemma.

Lemma 1. *For every $\theta \in \Theta$, a mechanism that outputs the fractional allocation $x^{\text{fr}}(\theta)/\alpha$ and the VCG payments $p^{\text{fr}}(\theta)/\alpha$, for any $\alpha > 1$ is DSIC and IR in X^{fr}.*

However, note that the mechanism $(x^{\text{fr}}(\hat{\theta})/\alpha, p^{\text{fr}}(\hat{\theta})/\alpha)$ cannot be implemented since it gives a fractional allocation. For this reason, we construct a convex decomposition of $x^{\text{fr}}(\hat{\theta})/\alpha = \sum_{x^I \in X} \lambda_{x^I} x^I$ to obtain a randomized allocation $\lambda \in \Delta_X$ that has only polynomially many $\lambda_{x^I} > 0$. Note that $x^I \in X$ denotes a deterministic/integer allocation. The problem of finding such a decomposition can be formulated as the following linear program.

$$\begin{aligned} \min \quad & \sum_{x^I \in X} \lambda_{x^I} \\ \text{s.t.} \quad & \sum_{x^I \in X} \lambda_{x^I} x^I(i,b) = x^{\text{fr}}(\hat{\theta})(i,b)/\alpha, \forall i \in N, b \in B \\ & \sum_{x^I \in X} \lambda_{x^I} \geqslant 1; \quad \lambda_{x^I} \geqslant 0, \forall x^I \in X. \end{aligned} \tag{5}$$

If we can show that the optimal value of LP (5) is 1 (for some fixed $\alpha > 1$), then the solution of the LP is a convex decomposition of the fractional allocation. This gives a randomized allocation that approximates the social welfare to within a factor of α since we have $\sum_{x^I \in X} \lambda_{x^I}\ \sum_{i \in N} \sum_{b \in B} \theta_i(b)\ x^I(i,b) = \sum_{i \in N} \sum_{b \in B} \theta_i(b) \sum_{x^I \in X} \lambda_{x^I}\ x^I(i,b) = \sum_{i \in N} \sum_{b \in B} \theta_i(b)\ x^{\text{fr}}(i,b)/\alpha$. In addition, using the properties of *fractional mechanism* (Lemma 1) we can also ensure DSIC and IR. We next show that for a particular choice of α we can guarantee an optimal value of 1 for LP (5) for every monotone $\hat{\theta}$. Moreover, LP (5) can be solved in polynomial time to give a $\lambda \in \Delta_X$ having only polynomially many $\lambda_{x^I} > 0$ ($x^I \in X$).

Observe that LP (5) can have exponentially many variables, since the number of deterministic allocations for a given instance of our

[1]Note that, X^{fr} is the feasible region of LP (3). This may be different from the convex hull of X, since the corner points of the feasible region X^{fr} may not be deterministic allocations. If that happens to be the case, then ILP (2) is solvable in polynomial time.

EV charging problem can be exponential in the number of agents, outlets and time slots. For this reason, we consider its dual LP (6).

$$\begin{aligned} \max \quad & z + \frac{1}{\alpha} \sum_{i\in N}\sum_{b\in B} x^{\text{fr}}(\hat{\theta})(i,b)\; w(i,b) \\ \text{s.t.} \quad & z \geqslant 0 \\ & \sum_{i\in N}\sum_{b\in B} x^{I}(i,b)\; w(i,b) \; + z \leqslant 1, \forall x^I \in X \end{aligned} \tag{6}$$

The dual program has a polynomial number of variables and an exponential number of constraints. But an LP with exponentially many constraints can be solved in polynomial time using the ellipsoid method if one can construct an efficient *separation oracle* [18, 34]. This is because the ellipsoid method solves an LP without the explicit description of the program itself. For our dual LP (6), an α-approximation algorithm that provides an integer solution with a value of at least $1/\alpha$ times the value of the optimal fractional solution of LP (3) for every *monotone* $\theta \in \Theta$ can be used to construct such an efficient separation oracle (see Algorithm 1). This has two implications for the choice of α: (1) We require α to be at least the *integrality gap (IG)*; (2) To obtain the convex decomposition we need an accompanying α-approximation algorithm which provides an integer solution having guarantees w.r.t. to the fractional optimal for every *monotone* $\theta \in \Theta$. The *integrality gap* is the maximal ratio between the optimal fractional solution and optimal integer solution of the social welfare maximization problem across all *monotone* valuations as defined below.

$$\text{IG} := \sup_{\theta\in\Theta^{\text{MONO}}} \frac{\max_{x\in X^{\text{fr}}} \sum_{i\in N}\sum_{b\in B} \theta_i(b)\; x(i,b)}{\max_{x^I\in X} \sum_{i\in N}\sum_{b\in B} \theta_i(b)\; x^I(i,b)} \tag{7}$$

If α was less than the integrality gap, then by the above definition there exists a $\theta \in \Theta$ for which no integer solution $x^I \in X$ gives $\sum_{i\in N}\sum_{b\in B} x^I(i,b)\;\theta_i(b) \geqslant \frac{1}{\alpha}\max_{x\in X^{\text{fr}}}\sum_{i\in N}\sum_{b\in B} x(i,b)\;\theta_i(b)$.

An important point to highlight is that in the separation oracle the α-approximation algorithm is used to provide an integer solution with a value of at least $1/\alpha$ times the value of the fractional optimal for any unconstrained w (not monotone w). However, for our EV charging problem an α-approximation algorithm that works for *monotone* w can also be used to provide the required integer solution for any unconstrained w. This is stated as the following lemma. We note that an α-approximation algorithm that provides an integer solution with a value of at least $1/\alpha$ times the value of the fractional optimal for every *positive* $\theta \in \Theta$ can also be used for the separation oracle[2]. For details, we refer the reader to the proof.

Lemma 2. *For any unconstrained* $w = [w(i,b), \forall i \in N, b \in B]$, *an* α*-approximation algorithm that provides an integer solution with a value of at least* $1/\alpha$ *times the value of the fractional optimal of Equation* (3) *for every monotone* $\theta \in \Theta$, *can be used to construct an* $x^I \in X$ *in polynomial time such that* $\sum_{i\in N}\sum_{b\in B} x^I(i,b)\; w(i,b) \geqslant 1/\alpha \cdot \max_{x\in X^{\text{fr}}} \sum_{i\in N}\sum_{b\in B} x(i,b)\; w(i,b)$.

The ellipsoid method with this efficient separation oracle identifies an LP that is equivalent[3] to the dual LP (6), but with only polynomially many constraints from $\sum_{i\in N}\sum_{b\in B} x^I(i,b)\; w(i,b) + z \leqslant 1, \forall x^I \in X$. These constitute the set of violated constraints returned by the separation oracle which is used to cut the ellipsoid at some iteration. The primal LP (5) is then solved by considering only polynomially many variables corresponding to these violated constraints to get the optimal solution λ^*. Since the ellipsoid method runs in polynomially many iterations, we get the decomposition in polynomial time. It can also be shown that the optimal value of the dual, and hence, the primal is 1 which yields the desired convex combination in polynomial time.

Lemma 3. *A decomposition of* $x^{\text{fr}}(\hat{\theta})/\alpha = \sum_{x^I\in X} \lambda^*_{x^I} x^I$ *with only polynomially many* $\lambda^*_{x^I} > 0$ *and* $\sum_{x^I\in X}\lambda^*_{x^I} = 1$ *can be obtained in polynomial time.*

Finally, the allocation and payments of the randomized mechanism are given by Equation (8). The randomized allocation $f(\hat{\theta})$ is set to λ^* and the randomized payment $\pi_i(\hat{\theta})$ of agent i is set such that the expected payment is $p_i^{\text{fr}}(\hat{\theta})/\alpha$. This ensures DSIC, IR, and gives an α-approximation to the social welfare. In summary, the above discussion highlights that for our EV charging problem any α-approximate algorithm that gives guarantees w.r.t to the fractional optimal for *monotone* inputs can be used to give a α-approximate mechanism that is DSIC and IR.

$$\begin{aligned} f(\hat{\theta}) &= \{\lambda^*_{x^I}, \forall x^I \in X\} \\ \pi_i(\hat{\theta}) &= \begin{cases} \frac{p_i^{\text{fr}}(\hat{\theta})/\alpha}{v_i(f(\hat{\theta}),\hat{\theta}_i)}\; v_i(x^I,\hat{\theta}_i), & \text{if } v_i(f(\hat{\theta}),\hat{\theta}_i) > 0 \;\&\; x^I \in X \text{ is sampled.} \\ 0, & \text{Otherwise.} \end{cases} \end{aligned} \tag{8}$$

Lemma 4. *The randomized mechanism* (f,π) *given by Equation* (8) *is DSIC, IR, and approximates the social welfare to within a factor of* α.

For the α-approximation algorithm, we leverage the greedy allocation strategy in [28] to provide an $O(\sqrt{|S|})$-approximation guarantee. Algorithm 2 describes the procedure. Note that Lehmann et al. [28] propose a truthful greedy mechanism for the combinatorial auction problem with *single-minded* valuations. This approximates the social welfare within a factor of $O(\sqrt{\kappa})$ (κ is the number of goods). While their results do not extend for monotone valuations, their greedy allocation scheme can be combined with Algorithm 1 to ensure DSIC and IR since it provides guarantees w.r.t. the optimal fractional solution for every monotone valuation. Moreover, we demonstrate that by exploiting the structure of the allocation space of our EV charging model, we can get an improved $O(\sqrt{|S|})$-approximation factor instead of $O(\sqrt{|S||M|})$ dependency on the number of goods. The key idea utilizes the fact that: (1) A bundle $b \in B$ is a collection of contiguous time slots at any single outlet and (2) For any pair (i,b) that is removed when $x^I(i',b')$ is set to 1, if the outlets corresponding to the bundles b and b' are not the same, then $i = i'$. For more details, we refer the reader to the complete proof.

Lemma 5. *For monotone valuations and multiple outlets, the greedy Algorithm 2 approximates the social welfare of the EV charging problem within a factor of* $O(\sqrt{|S|})$.

From Lemmas 1 to 5, we conclude Theorem 2.

[2] This follows from the packing property [27] since our EV problem (can be represented as a combinatorial auction problem) is an instance of the set packing problem [14].

[3] It has the same the same optimal value as LP (6), but has only polynomially many constraints from $\sum_{i\in N}\sum_{b\in B} x^I(i,b)\, w(i,b) \; + z \leqslant 1, \forall x^I \in X$.

Algorithm 2: Greedy $O(\sqrt{|S|})$-approximation algorithm

Input: Monotone $\hat{\theta} = [\hat{\theta}_i(b), \forall i \in N, b \in B]$.
Output: $x^I \in X$ such that $\sum_{i\in N}\sum_{b\in B} x^I(i,b)\,\hat{\theta}_i(b) \geqslant \frac{1}{2\sqrt{|S|}} \max_{x\in X^{\text{fr}}} \sum_{i\in N}\sum_{b\in B} x(i,b)\,\hat{\theta}_i(b)$.

1. Initialize allocation $x^I = [0, \ \forall i \in N, b \in B]$.
2. Initialize set $Y = \{(i,b), \forall i \in N, b \in B\}$.
3. **while** $Y \neq \emptyset$ **do**
4. Determine $(i', b') = \arg\max_{(i,b)\in Y} \hat{\theta}_i(b)/\sqrt{\sum_{j\in S} \mathbb{I}\{b_j \neq 0\}}$
5. Set $x^I(i', b') = 1$.
6. For every (i,b) such that $i = i'$ or $b'_j = b_j \neq 0$ (for some $j \in S$), $Y = Y \setminus \{(i,b)\}$.
7. **end**
8. **return** x^I

3.2 Mechanism for single outlet scenario

As shown in Theorem 1, the social welfare maximization problem under monotone valuations is NP-Hard even for the case of single outlet. However, we show that we can achieve a constant-factor approximation for this scenario. We leverage [6] to give an LP based 2-approximation algorithm that gives guarantees w.r.t. the optimal fractional solution. In particular, their ideas of rounding a fractional solution and using graph coloring to get the desired integer solution can be extended for our single outlet and monotone valuations setup. Since the algorithm works for all monotone inputs, this gives DSIC and IR via the RAE mechanism.

Firstly, LP (9) is solved to obtain the optimal fractional solution $(x^*, \texttt{OPT})$ for any monotone w[4]. Since $|M| = 1$, a bundle $b \in \{0,1\}^{|S|}$, such that : (i) $\forall i, j \in S$, if $b_i, b_j \neq 0$, then $b_i = b_j = 1$ and (ii) there exists i^*, j^*, s.t. $b_i = 0, \forall i < i^*, i > j^*$ and $b_i = 1, \forall i^* \leqslant i \leqslant j^*$.

$$\begin{aligned} \max \quad & \sum_{i\in N}\sum_{b\in B} w(i,b)x(i,b) \\ \text{s.t.} \quad & \sum_{b\in B} x(i,b) \leqslant 1, \forall i \in N \\ & \sum_{b\in B_j}\sum_{i\in N} x(i,b) \leqslant 1, \forall j \in S \\ & x(i,b) \geqslant 0, \forall i \in N, b \in B \end{aligned} \tag{9}$$

Each $x^*(i,b)$ is then rounded down to the nearest fraction of the form p/Q for some $p \in \{1,2,\ldots,Q\}$, where $Q = |N|^2(|S|(|S|+1)/2)^2$. Denote the rounded solution by x^{rou}. Observe that every $x^{\text{rou}}(i,b)$ is at most $1/Q$ smaller than $x^*(i,b)$. This implies that the value of the objective function for x^{rou} decreases by at most $\max_{i,b} w(i,b)/\sqrt{Q}$. This is because the summation is taken over all agents $|N|$ and bundles $(|S|(|S|+1)/2)$. Moreover, since $\texttt{OPT} \geqslant \max_{i,b} w(i,b)$, we get

$$\sum_{i\in N}\sum_{b\in B} w(i,b)\, x^{\text{rou}}(i,b) \geqslant (1 - 1/\sqrt{Q})\ \texttt{OPT}. \tag{10}$$

Denote $\{x^\ell, \forall \ell \in L\}$ to be a set of feasible integral solutions of LP (9), where $L = \{1,2,\ldots,l\}$. Let $\texttt{val}(x^\ell) = \sum_{i\in N}\sum_{b\in B} w(i,b)\, x^\ell(i,b)$. It is easy to see that for non-negative values of $\{\beta_\ell, \ell \in L\}$, if $\sum_{\ell\in L} \texttt{val}(x^\ell)\beta_\ell \geqslant (1 - 1/\sqrt{Q})\ \texttt{OPT}$ and $\sum_{\ell\in L}\beta_\ell \leqslant 2$, then by convexity there exists an $\ell' \in L$ such that $\texttt{val}(x^{\ell'}) \geqslant (1 - 1/\sqrt{Q})\texttt{OPT}/2$. Thus, if one can find such a set of integral solutions $\{x^\ell, \forall \ell \in L\}$ with polynomial size of L, then we get a 2-factor approximation (with a negligible rounding loss) for the single outlet case. Using the rounded solution x^{rou}, we next construct a graph and color it appropriately to get the desired set of integral solutions.

Construct a graph G with $x^{\text{rou}}(i,b)\cdot Q$ vertices corresponding to each $i \in N, b \in B$. Any two vertices y, z corresponding to (i^y, b^y) and (i^z, b^z) respectively have an edge between them if $i^y = i^z$ or $b^y_j = b^z_j = 1$ for some $j \in S$. This implies that two vertices have an edge if either they correspond to the same agent or if their corresponding bundles overlap (i.e., have a common slot allotted). The vertices of G are colored such that no two vertices y, z having an edge between them get the same color. We will call such a coloring of vertices of G as a *feasible coloring*. Observe that the set of vertices that get the same color is an independent set in G and form a feasible integral solution for LP (9). It can be shown that a *feasible coloring* can be achieved with at most $(2Q-1)$ colors using a greedy strategy. For details, we refer the reader to the proof.

Lemma 6. *For graph G, there exists a feasible coloring for vertices that requires at most $(2Q-1)$ colors.*

Let L be the set of colors and x^ℓ for $\ell \in L$ denote an integral solution where $x^\ell(i,b) = 1$ if a vertex corresponding to (i,b) has color ℓ, and $x^\ell(i,b) = 0$ otherwise. As before, denote $\texttt{val}(x^\ell) = \sum_{i\in N}\sum_{b\in B} w(i,b)\, x^\ell(i,b)$. Let $\beta_\ell = 1/Q, \forall \ell \in L$. This gives $\sum_{\ell\in L}\beta_\ell \leqslant (2Q-1)/Q \leqslant 2$, since the size of L is at most $2Q-1$. Moreover, we have

$$\sum_{\ell\in L} \texttt{val}(x^\ell) = \sum_{i\in N}\sum_{b\in B} w(i,b)(Q\, x^{\text{rou}}(i,b)) \geqslant Q((1 - 1/\sqrt{Q})\texttt{OPT}).$$

The equality holds because graph G contains $x^{\text{rou}}(i,b)\cdot Q$ vertices for each $i \in N, b \in B$. The inequality holds due to Equation (10). This implies $\sum_{\ell\in L}\texttt{val}(x^\ell)\beta_\ell = \sum_{\ell\in L}\texttt{val}(x^\ell)1/Q \geqslant (1 - 1/\sqrt{Q})\texttt{OPT}$. Since $\sum_{\ell\in L}\beta_\ell \leqslant 2$, there exists a color $\ell' \in L$ for which $\texttt{val}(x^{\ell'}) \geqslant (1 - 1/\sqrt{Q})\frac{\texttt{OPT}}{2}$. We can obtain $x^{\ell'}$ by choosing the color having the maximum value. From the above discussion, we conclude the following.

Lemma 7. *For the monotone valuations and single outlet case, Algorithm 3 that rounds the optimal fractional solution approximates the social welfare to within a factor of 2 in polynomial time.*

From Lemmas 1 to 4 and 7, we conclude the following result.

Theorem 3. *For monotone valuations and a single outlet, the RAE mechanism (Algorithm 1) that uses approximation Algorithm 3 as $\mathcal{A}$ in the separation oracle, approximates the social welfare within a factor of 2 and ensures DSIC and IR.*

4 DICHOTOMOUS VALUATIONS

Although our assumption that agents have monotone valuations is fairly general, it becomes quite demanding in the EV charging setup. A more restricted, yet practical scenario arises when agents have dichotomous valuations. Consider a setting where each agent i requires c_i units of charge and derives a value of $q^*_i \in \mathbb{R}^+$ if it

[4] We omit the set of outlets M from the linear program since $|M| = 1$.

Algorithm 3: 2-approximation algorithm

Input: Montone $w = [w(i,b), \forall i \in N, b \in B]$.
Output: An $x^I \in X$ s.t. $\sum_{i\in N}\sum_{b\in B} x^I(i,b)\ w(i,b) \geqslant \frac{1}{2}\max_{x\in X^{\text{fr}}}\sum_{i\in N}\sum_{b\in B} x(i,b)\ w(i,b)$.

Solve LP (9) to get the optimal fractional solution x^*.
Set $Q = |N|^2(|S|(|S|+1)/2)^2$.
Round down every $x^*(i,b)$ to the nearest fraction of the form p/Q for some $p \in \{1,2,\dots,Q\}$ to get x^{rou}.
Construct a graph G with $x^{\text{rou}}(i,b)\cdot Q$ vertices corresponding to the each $i \in N, b \in B$. Add an edge between any two vertices y,z if either $i^y = i^z$ or $b^y_j = b^z_j = 1$ for some $j \in S$.
For every vertex y denote $b^y_{\min} = \min_{j\in S:b^y_j=1} j$.
Sort the vertices in ascending order of $b_{\min}$ and color them using at most $2Q-1$ colors (Lemma 6) from left to right such that we get a feasible coloring of vertices.
Let L be the set of colors and let $x^\ell, \forall \ell \in L$ be an integer solution where $x^\ell(i,b) = 1$ if a vertex corresponding to (i,b) has color ℓ, and $x^\ell(i,b) = 0$ otherwise.
$x^I = \arg\max_{x^\ell:\ell\in L}\sum_{i\in N}\sum_{b\in B} w(i,b)\ x^\ell(i,b)$.
return x^I

receives c_i units, and 0 if it receives less than c_i. To receive c_i units of charge, agent i must be allocated some ℓ_{ik} contiguous time slots at outlet $k \in M$. Define the set of all bundles that provide c_i units to agent i by $B_i = \{b \in B : \sum_{j\in S}\mathbb{I}\{b_j \neq 0\} = \ell_{ik_b}\}$, where $k_b \in M$ denotes the outlet corresponding to bundle b i.e., $b_j = k_b, \forall j \in S$ with $b_j \neq 0$. Let $B^*_i \subseteq B_i$ denote the set of *acceptable* bundles for agent i. Type θ_i is said to be dichotomous if $\theta_i(b) = q^*_i, \forall b \in B^*_i$, else $\theta_i(b) = 0$. It can be shown that the welfare maximization problem for dichotomous valuations is also NP-Hard via a reduction from the *Job Interval Selection Problem (JISPk)* [32] where all intervals have equal length, which is known to be NP-Complete.

Theorem 4. *For dichotomous valuations, the social welfare maximization problem is NP-Hard even if the number of outlets $|M| = 1$.*

Since dichotomous valuations are a subset of monotone valuations and our approximation guarantees give a lower bound on $ALG(\theta)/OPT(\theta)$ for all monotone θ, our results for monotone valuations naturally extend to dichotomous valuations.

5 SINGLE-MINDED VALUATIONS

The type θ_i for every agent $i \in N$ is said to be single-minded if there exists a bundle $b^i \in B$ and $q^i \in \mathbb{R}^+$ such that $\theta_i(b) = q^i, \forall b \supseteq b^i$ and $\theta_i(b) = 0$ otherwise. In other words, each agent prefers to charge at a single outlet for a specific set of contiguous time slots or not charge at all. For single-minded reports $\hat{\theta}$, we can drop the constraint $\sum_{b\in B} x(i,b) \leqslant 1, \forall i \in N$ from ILP (2) since each agent is interested in exactly one bundle (or any super-set of that bundle). Hence, the LP-relaxation reduces to LP (11). It can be shown that this LP always has an optimal integer solution (since the constraint matrix is totally unimodular [21]).

$$\begin{aligned} \max \quad & \sum_{i\in N}\sum_{b\in B}\hat{\theta}_i(b)\ x(i,b) \\ \text{s.t.} \quad & \sum_{b\in B_{jk}}\sum_{i\in N} x(i,b) \leqslant 1, \forall j \in S, k \in M, \\ & x(i,b) \geqslant 0, \forall i \in N, b \in B. \end{aligned} \tag{11}$$

Algorithm 4: DAE mechanism

Input: Agents report type profile $\hat{\theta}$.
Output: $f(\hat{\theta}) \in X$ and $\pi_i(\hat{\theta}) \in \mathbb{R}, \forall i \in N$.

Solve LP (11) with parameters given by $\hat{\theta}$ to get an optimal deterministic allocation $f(\hat{\theta}) = x^*$.
For every agent i, set payment using VCG $\pi_i(\hat{\theta}) = \max_{x\in X}\sum_{i'\in N\setminus\{i\}}\sum_{b\in B}\hat{\theta}_{i'}(b)x(i',b) - \sum_{i'\in N\setminus\{i\}}\sum_{b\in B}\hat{\theta}_{i'}(b)x^*(i',b)$.
return $f(\hat{\theta}), \pi(\hat{\theta})$

A polytope $(Ax \leqslant b,\ x \geqslant 0)$ is said to be integral if and only if all its corners have integer coordinates. It is well known that a linear program with an integral polytope always has an optimal integer solution [31]. A sufficient condition to identify integral polytopes is by total unimodularity. A polytope $Ax \leqslant b, x \geqslant 0$ is integral if A is totally unimodular (TU)[5] and b is integral [31]. Observe from the constraints $(Ax \leqslant b,\ x \geqslant 0)$ of LP (11) that b is integral. Additionally, the constraint matrix A is totally unimodular because it is a 0-1 matrix with consecutive ones in each column. This implies that the LP (11) is integral and can be solved to obtain an optimal deterministic allocation in polynomial time. Since computing the *efficient* allocation is tractable, the VCG mechanism can be used to ensure DSIC and IR which concludes the following result.

Theorem 5. *For single-minded valuations, the* Deterministic Allocatively Efficient (DAE) *mechanism (Algorithm 4) ensures DSIC and IR, and gives an efficient allocation in polynomial time.*

6 CONCLUSION

We studied resource scheduling with monetary transfers across multiple outlets with varying dispensing rates. We proved NP-hardness of social welfare maximization for monotone and dichotomous valuations in a single outlet. We proposed a randomized 2-approximation mechanism for a single outlet and an $O(\sqrt{|S|})$-approximation mechanism for multiple outlets, ensuring DSIC and IR. For single-minded agents, the problem is in $\mathbb{P}$, allowing VCG to be used. Future work includes improving approximation ratios, establishing matching lower bounds, and extending the model to dynamic agent arrivals and departures.

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[5] A matrix A is TU if every square sub-matrix of A has a determinant of 0, 1, or -1.

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