Group Fairness in Multi-period Mobile Facility Location Problems
Extended Abstract

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ABSTRACT

We study the group-fair multi-period mobile facility location problems, where agents from different groups are located on a real line and arrive in different periods. Our goal is to locate k mobile facilities at each period to serve the arriving agents in order to minimize the maximum total group-fair cost and the maximum average group-fair cost objectives that measure the costs or distances of groups of agents to their corresponding facilities across all periods. We first consider the problems from the algorithmic perspective for both group-fair cost objectives. We then consider the problems from the mechanism design perspective, where the agents' locations and arrival periods are private. For both objectives, we design deterministic strategyproof mechanisms to elicit the agents' locations and arrival periods truthfully while optimizing the group-fair cost objectives and show that our mechanisms have almost tight bounds on the approximation ratios for certain periods and settings. Finally, we discuss the extensions of our results to the online setting where agent arrival information is only known at each period.

KEYWORDS

Facility Location; Group Fairness; Mechanism Design

1 INTRODUCTION

In recent decades, there has been a notable surge of applicational and theoretical interests in facility location problems within the fields of mechanism design and social choice. For instance, the increased interest in these problems is due to their potential ability to capture and model various real-world preference aggregation scenarios (e.g., voting [4, 5, 15] and site locations [6, 16]). In the

most general setting of facility location problems in these fields, a social planner aims to locate facilities to serve agents that have preferences on the facility locations within a given region in order to optimize the total or maximum cost objective, which measures the total or maximum distances of the agents to their allocated facilities, respectively.

While most existing studies have focused on the static aspects of facility location problems, recent studies [7–9, 11, 17–19] have started to consider the multi-period aspects of the problems that often deal with locating or reallocating some forms of mobile facilities (e.g., mobile health clinic services or blood donation centers) within some regions over multiple periods. For example, consider a mobile vaccination vehicle that provides certain medical services for residents, who may choose different time periods to be vaccinated at preferred locations. Therefore, the health providers need to decide where to locate several vehicles over different time periods. Beyond health services, the problems can also be applied to determining the locations of polling centers at different periods by taking into consideration agent location preferences and availabilities [10, 12]. Finally, the multi-period mobile settings can also model problems that are not geographical in nature (e.g., general preference aggregation [1–3, 15]). We refer the readers to other related work [17] for more relevant motivations and examples of mobile facilities.

However, there has been a lack of consideration of group fairness in multi-period mobile studies. As a result, existing models and approaches can lead to certain groups of agents incurring significantly higher costs than other groups of agents. Such a lack of consideration is often not ideal in modern society, especially when providing medical, polling, and education services to groups of agents from different social and economic backgrounds. Therefore, we examine group fairness in multi-period facility location problems, focusing on locating facilities to serve groups of agents fairly over multiple periods subject to group-fair cost objectives over multiple periods.

2 PRELIMINARIES

In a group-fair multi-period mobile facility location problem (GF-MPMFLP), we have a set $N = \{1, \ldots, n\}$ of n agents, where each agent will be served once over $T \geq 1$ periods. The agents are partitioned into $m \geq 1$ disjoint groups based on their associations (e.g., ages, ethnicities, and needs). Let $G = \{G_1, \ldots, G_m\}$ be the set of disjoint groups of the agents. We use $|G_g|$ to denote the

total number of agents in group G_g. Without loss of generality, we assume $|G_g| > 0$ for each $g = 1, \ldots, m$.

Each agent $i \in N$ has a profile $r_i = (x_i, a_i, g_i)$ where the agent location preference is $x_i \in \mathbb{R}$, the arrival period is $a_i \in \{1, \ldots, T\}$, and the group membership is $g_i \in \{1, \ldots, m\}$. A tuple $r = (r_1, \ldots, r_n)$ is a collection of locations, arrival periods, and group memberships.

k facilities are located to serve all the agents arriving at each period. We denote the facility j's location in period t as $f_j^t \in \mathbb{R}$, and let $f_t = (f_1^t, \ldots, f_k^t) \in \mathbb{R}^k$ be the locations of k facilities in period t. A deterministic algorithm (resp. mechanism) is a function F that maps each tuple r to the facility locations of all periods, i.e., $F(r) = (f_1, \ldots, f_T) \in \mathbb{R}^{T \times k}$. Given F and a tuple r, the cost of an agent i is the distance between their preferred location and the closest facility in their arrival period, i.e., $cost(F(r), x_i, a_i) = \min_{1 \le j \le k} |f_j^{a_i} - x_i|$.

Following the existing studies of facility location problems with group fairness [13, 14, 20], we aim to minimize two group-fair cost objectives: the maximum total group cost ($mtgc$) and maximum average group cost ($magc$). Given a tuple r and a function F, $mtgc$ is defined as

$$mtgc(r, F) = \max_{1 \le g \le m} \left\{ \sum_{i \in G_g} cost(F(r), x_i, a_i) \right\}. \tag{1}$$

Moreover, $magc$ is defined as

$$magc(r, F) = \max_{1 \le g \le m} \left\{ \frac{\sum_{i \in G_g} cost(F(r), x_i, a_i)}{|G_g|} \right\}. \tag{2}$$

From the mechanism design perspective, because the agents' locations and arrival periods are private, our goal is to design strategyproof mechanisms to elicit their true information while simultaneously determining facility locations to optimize $mtgc$ and $magc$ objectives. Denote (r_i', r_{-i}) as the tuple r with r_i' in place of r_i.

DEFINITION 1. *A deterministic mechanism F is strategyproof if for any tuple r, any agent i can never benefit by misreporting their location or arrival period regardless of the profiles of other agents. That is, for all agents, r and $r_i' = (x_i', a_i', g_i)$, we have,*

$$cost(F(r), x_i, a_i) \le cost(F((r_i', r_{-i})), x_i, a_i).$$

An algorithm (resp. mechanism) F achieves an approximation ratio of α for minimizing the $mtgc$ (resp. $magc$) in the offline setting, if for any profile r, $mtgc(r, F) \le \alpha \cdot mtgc(r, OPT)$ (resp. $magc(r, F) \le \alpha \cdot magc(r, OPT)$) where OPT is the optimal algorithm that minimizes the $mtgc$ (resp. $magc$) in the offline setting, i.e., with complete knowledge of the agents' information at all periods. Similarly, an algorithm (resp. mechanism) F achieves a competitive ratio of α for minimizing the $mtgc$ (resp. $magc$) in the online setting, if for any profile r, $mtgc(r, F) \le \alpha \cdot mtgc(r, OPT)$ (resp. $magc(r, F) \le \alpha \cdot magc(r, OPT)$) where OPT is the optimal algorithm that minimizes the $mtgc$ (resp. $magc$) in the offline setting.

3 RESULTS

We study the group-fair multi-period mobile facility location problems (GF-MPMFLPs) from both algorithmic and mechanism design perspectives. We summarize our contributions below for different settings from both perspectives.

Single Facility	MTGC	MAGC
Single Period	UB: 2	UB: 2
	LB: 2	LB: 2
Multiple Periods	UB: $\min(T+1, m)$	UB: $T+1$
	LB: 2	LB: 2
Online Setting	UB: $\min(T+1, m)$	UB: $2(n-m+1)$
	LB: $\min(T, \log m) + 1$	LB: $n-m+1$

Multiple Facilities Offline & Online	MTGC	MAGC
$k = 2$	UB: $n-m+2$	UB: $n-m+2$
	LB: $\max(n-m-1, 1)$	LB: $\max(n-m-1, 1)$
$k \ge 3$	∞^*	∞^*

* Anonymous mechanisms.

Table 1: Summary of mechanism approximation results. UB and LB stand for the upper bound and lower bound.

- From the algorithmic perspective, the locations and arrival periods of the agents are publicly known. Our contributions are as follows.
  - For the single facility setting ($k = 1$), we show that GF-MPMFLPs can be solved in polynomial time using linear programming for both $mtgc$ and $magc$ objectives.
  - For the setting with k facilities, we give a slice-wise polynomial (XP) algorithm (i.e., solvable in time $n^{f(k)}$ for some fixed parameter k and computable function f, where n is the input size) to solve the GF-MPMFLPs for both $mtgc$ and $magc$ objectives.
- From the mechanism design perspective, the location and arrival period preferences of the agents are privately known to the agents themselves. We design several mechanisms that elicit the true locations and arrival periods of the agents while simultaneously optimizing the $mtgc$ and $magc$ objectives based on the agents' reported information. Table 1 summarizes our mechanism design results.
- We additionally consider the online GF-MPMFLPs from mechanism design perspective, where the agent arrival information is only known at each period. For the single facility setting ($k = 1$), we establish almost tight bounds for the $mtgc$ objective. We also show that our deterministic strategyproof mechanism has a competitive ratio of $2(n-m+1)$ for the $magc$ objective, and complement this result with a lower bound of $n - m + 1$ for the $magc$ objective. For the setting with more facilities, we show that our previous results can all be extended to the online version.

We refer readers to the full paper for complete results and proofs.

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