On the Distortion of Multi-Winner Elections on the Line Metric

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ABSTRACT

We consider the problem of selecting a committee of k alternatives among m alternatives, based on the ordinal preferences of voters. Our focus is on the case where both voters and alternatives lie on a metric space—specifically, on the line—and the objective is to minimize the social additive cost. Social additive cost is the sum of the costs for all voters, where the cost for each voter is defined as the sum of their distances to each member of the selected committee.

We propose a new voting rule, the *Polar Comparison Rule*, which achieves an upper bound of $1 + \sqrt{2} \approx 2.41$ distortion for $k = 2$, and we show that this bound is tight. Furthermore, we generalize this rule and show that it maintains a distortion of 2.41 for even committee sizes and $2.41 + (2 - \sqrt{2})/k$ for odd committee sizes. We also establish lower bounds on the distortion based on the parity of k and for both small and large committee sizes.

KEYWORDS

Metric Distortion, Voting, Committee Selection

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1 INTRODUCTION

Imagine a city council election where residents must select a committee of three representatives from a pool of nine candidates. Each voter has cardinal cost or utility values for the alternatives, reflecting their preferences. However, voters only submit ordinal rankings derived from these values. The problem is how to use these rankings to form a reasonable committee.

The above scenario raises two key questions. First, what defines a "reasonable" committee? Second, given the limitations of ordinal rankings, can we achieve—or at least approximately achieve—this reasonable objective without access to cardinal values?

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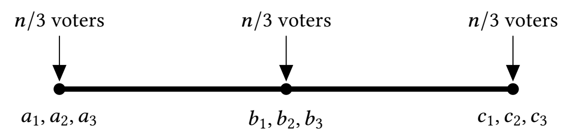

Figure 1: An instance consisting of n voters and 9 alternatives. If we want to select a committee of $k = 3$ alternatives, the best committee varies depending on how the cost of a committee is defined for a voter, based on its members.

Ultimately, what constitutes a reasonable outcome depends on the expectations from the committee and the trade-off between efficiency and fairness. Let us set aside the limitation of ordinal rankings for now and consider the example in Figure 1, where the goal is to select three candidates. If the objective is to minimize the social additive cost for all voters, selecting b_1, b_2, b_3 achieves the lowest cost. However, one might argue that this choice fails to fairly represent voters' preferences. Alternatively, selecting a_1, b_1, c_1 increases the social additive cost but ensures that each voter's top choice is included.

Several well-studied criteria for multi-winner voting rules address these trade-offs. These include minimizing the social additive cost of the elected committee [15], minimizing the cost of the q'th nearest candidate [7–9], and more generally, optimizing social cost based on a scoring function over the committee [7].

The second challenge is the lack of cardinal information. If cardinal values were available, finding the optimal outcome under any of the above criteria would be simple. However, assuming access to such detailed information is often unrealistic. Voters may find it difficult to precisely quantify their preferences, and requiring cardinal inputs may lead to inconsistencies or increased costs in data collection. For these and other reasons, ordinal rankings are preferred, despite offering less detailed information about preferences.

The efficiency gap due to the lack of cardinal information is captured by the term distortion [21]. In single-winner elections, distortion of a voting rule f is defined as the worst-case ratio (across all instances) between the social welfare (or social cost) of the candidate selected by f and that of the optimal candidate, which is based on hidden cardinal values. Distortion is usually studied in two main frameworks: the utilitarian framework and the metric framework. In the utilitarian framework, cardinal values are utilities that often sum to one for each voter, and in the metric framework, cardinal

Table 1: Summary of Lower and Upper Bounds on the Distortion in Multi-Winner Elections.

	$k < m/2$	$k \geq m/2$
Upper Bound	$2.41 + \frac{2-\sqrt{2}}{k}$ (k odd) 2.41 (k even)	
Lower Bound	$2 + \frac{1}{k}$ (k odd) $1 + \sqrt{1 + \frac{2}{k}}$ (k even)	$1 + \frac{m-k}{3k-m}$

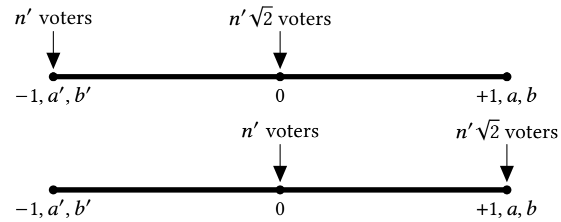

Figure 2: Metrics d_1 and d_2, used for proving lower bounds on the distortion of any 2-winner voting rule. $a, a', b,$ and b' are the alternatives, distributed on locations -1 and $+1$. Note that $n' = n/(1+\sqrt{2})$.

values are costs that satisfy metric properties. There are many arguments that justify why these two scenarios are reasonable and applicable to real-world situations [2, 10, 16, 18].

Distortion in both utilitarian and metric frameworks is well-studied for single-winner elections. In the utilitarian case, any voting rule has a distortion of at least $\Omega(m^2)$, with the plurality rule reaching this bound [6]. In the metric framework, the lower bound for any deterministic rule is 3 [1], and rules like Plurality Veto [17] and Plurality Matching [14] achieve this distortion. Additionally, tight bounds have been established for many voting rules in both frameworks [1].

In recent years, several studies have focused on the distortion for multi-winner elections. For the additive cost objective, it has been shown by Goel *et al.* [15] that repeatedly applying any single-winner rule with distortion δ results in a distortion of at most δ. Therefore, within the metric framework, applying the Plurality Veto rule [17] k times yields a committee of size k with distortion at most 3. Additionally, for the q-cost objective in the metric framework, Caragiannis *et al.* [7] show that for $q \leq k/3$, distortion is unbounded; for $k/3 < q \leq k/2$, it is tightly bounded by n; and for $q \geq k/2$, it is tightly bounded by 3. However, for the case $q \geq k/2$, they provided a voting rule with distortion 3 that has exponential running time, and another voting rule with distortion 9 that runs in polynomial time. Kizilkaya and Kempe [17] resolved this gap and provided a polynomial voting rule with distortion 3 for $q \geq k/2$ using Plurality veto. In utilitarian framework, Caragiannis *et al.* [5] and Borodin *et al.* [4] establish a tight distortion of $\Theta(\min(m/k, \sqrt{m}))$ for q-cost objective when $q = 1$.

In this paper, we investigate the distortion associated with selecting a committee of k alternatives under the social additive cost objective. We focus on the setting where both voters and alternatives are positioned on a line metric. Line metric is an important special case of metric space, which is widely studied [3, 11–13, 19, 20, 22]. For example, the line metric can model political preferences along a political spectrum, ranging from liberal to conservative.

The additive cost objective is particularly relevant when all committee members influence the outcome of the committee equally. For example, in a conference, if the reviewers' combined ratings determine the final score of a paper, selecting reviewers whose expertise aligns closely with the paper minimizes the total distance between the paper's field and the reviewers' expertise. Note that committee selection with an additive cost objective is equivalent to a randomized voting rule that applies a uniform distribution over the selected candidates with a fixed support size. Thus, the outcome of this rule can also be interpreted as a randomized voting rule.

2 OUR CONTRIBUTIONS AND TECHNIQUES

The problem of selecting a committee of k alternatives, based on voters' ordinal preferences, is both classical and practically significant. However, choosing more than one alternative introduces new challenges in minimizing the social additive cost. Since we study social additive cost, choosing consecutive alternatives leads to a committee with lower distortion compared to other configurations. Although the median voter appears to be a strong representative–indeed, when selecting a single alternative, the closest one to its left or right is optimal–forming a committee based solely on the median voter's preferences cannot achieve a distortion better than 3. Consider an instance in which the median voter prefers all alternatives on one side of it to all alternatives on the other side. Thus, it is important to choose alternatives from both sides of the median voter.

To leverage this insight, we design a new voting rule, the *Polar Comparison Rule*, which prioritizes committees that include alternatives from both sides of the median voter, thereby increasing the likelihood of selecting better alternatives. We first introduce this voting rule for selecting a committee of size two. Specifically, the rule selects the top-ranked alternative of the median voter and then compares the two closest alternatives to the median voter on opposite sides of each other that have not yet been picked. Based on the ratio of voters who prefer one alternative to the other, the rule selects the alternative that is more preferred, while introducing a bias toward the alternative on the opposite side of the previously chosen alternative. We then generalize it to any committee size by iteratively applying the rule for $k = 2$.

The *Polar Comparison Rule* achieves an upper bound of $1 + \sqrt{2} \approx 2.41$ distortion for $k = 2$, and we show that this bound is tight. We further extend these results by showing how different voting rules can be effectively combined to improve the distortion bounds for various committee sizes. Table 1 contains the resulting upper bounds on the distortion for different committee sizes.

We next complement these results with lower bounds. At a high level, we construct two instances where voters share the same preference profile and show that, regardless of the voting rule outcome, at least one of these instances results in a high distortion. See Figure 2 for an example. A summary of our results is provided in Table 1.

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