Agreement Games in Multi-Agent Systems

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ABSTRACT

Multi-Agent Systems are composed of distributed intelligent components, known as agents. In real-world scenarios, these agents often lack sufficient resources or access to global system information to achieve their objectives. In such cases, intelligent information sharing can enable agents to collaborate effectively and reach their goals. This work introduces the concept of *Agreement Game* as a framework to address these challenges.

KEYWORDS

Strategic Reasoning, Agreement Games, Multi-Agent Systems

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1 INTRODUCTION

Multi-Agent Systems (MAS) are extensively used in domains such as robotics, sensor networks, e-commerce, and smart grids [5, 7, 9]. These systems comprise autonomous agents that operate independently, pursuing goals based on local knowledge and capabilities. However, in many real-world scenarios, agents face constraints in resources and information, limiting their ability to achieve objectives effectively. While the distributed nature of MAS provides scalability and fault tolerance, it also introduces significant challenges in coordination and cooperation, particularly in environments characterized by uncertainty, incomplete information, and dynamic changes. Addressing these challenges requires robust mechanisms for collaboration and intelligent information sharing. In this paper, we present the *Agreement Game* (AG), a framework rooted in the formal verification of MAS [1, 6, 8], with a particular focus on settings involving imperfect information [4]. The AG models agent interactions as a strategic process, enabling agents to share information and collaborate effectively.

2 BACKGROUND

If $\rho = x_1, x_2, \ldots$ is a (finite or infinite) sequence, we denote its length as $|\rho|$, and its (j-th) element x_j as ρ_j. For $j \leq |\rho|$, let $\rho_{\geq j}$ be the suffix $\rho_j, \rho_{j+1} \ldots$ of ρ starting at ρ_j and $\rho_{\leq j}$ the prefix $\rho_1, \ldots, \rho_j$ of ρ. We denote by $\#(X)$ the cardinality of a given set X. If X is a set of sets $X = \{Y_1, Y_2, \ldots\}$, we write $\bigcup X$ for $Y_1 \cup Y_2 \cup \cdots$. We use $\mathbf{a}, \mathbf{b}, \mathbf{c}, \ldots$ as variables for tuples of elements of a given set and $\mathbf{a}[i]$ to denote the (i-th) element of such a tuple.

Thanks to [3], we have a class of models that we can use as basis to define agreements in the multi-agent setting.

Definition 2.1. A Resource **Action** Based CGS (RAB-CGS) is a tuple $M = \langle Ap, Ag, S, s_I, \{act_i\}_{i \in Ag}, P, t, L, r, \$ \rangle$ such that:

- Ap is a non-empty set of atomic propositions;
- $Ag = \{1, \ldots, n\}$ is a finite set of agents;
- S is a non-empty set of states and $s_I \in S$ is the initial state;
- for any $i \in Ag$, act_i is a set of actions, $ACT = \Pi_{i \in Ag} act_i$, and $act = \bigcup_{i \in Ag} act_i$;
- $P : Ag \times S \longrightarrow (2^{act} \setminus \emptyset)$ is the protocol function that associates to any agent i and state s a non-empty subset of act_i representing the actions that are available for i at s. We impose that the idle action $\star$ always belong to $P(i, s)$ for any i;
- $t : S \times ACT \longrightarrow S$ is the transition function, that is given a state s and a tuple of actions $\mathbf{a}$ (where $\forall i$, $\mathbf{a}[i] \in P(i, s)$) such function outputs a state s';
- $L : S \longrightarrow 2^{Ap}$ is the labelling function;
- $r \geq 1$ is a natural number (the number of resources types);
- $\$: S \times act \times ACT \longrightarrow \mathbb{N}^r$ is a function mapping a state s, an action a, and a tuple of actions $\mathbf{a} = \langle a_1, \ldots, a_n \rangle$ to a natural number vector of length r. We impose that a is one of the a_i composing $\mathbf{a}$, and impose that $\$(s, \star, \mathbf{a}) = 0$.

A **path** ρ is an infinite alternated sequence $s_1, \mathbf{a}_1, s_2, \ldots$ of states and tuples in ACT such that for all $i \geq 1$, $t(s_i, \mathbf{a}_i) = s_{i+1}$. If ρ is a path, we denote by ρ^S the sub-sequence of ρ only containing states. If $h \in S^+$ is a finite sequence of states, we say that h is a **history** iff there is a path ρ such that $h = \rho^S_{\leq i}$ for some $i \in \mathbb{N}$. We use H to denote the set of all histories. A (memoryful) strategy for an agent j is a function $\sigma_j : H \longrightarrow act_j$, that maps a history to an action $a_j \in act_j$. Let C be a subset of Ag, s a state, and $\mathbf{c}$ be a tuple of actions one for each agent in C. We denote by $Post(s, \mathbf{c})$ the set of states $\{s' \in S \mid t(s, \mathbf{a}) = s' \wedge \mathbf{c}$ is a sub-sequence of $\mathbf{a}\}$. We denote with $P_C(s)$ the set of tuples of actions that are available at s for the coalition C. A path $\rho = s_1, \mathbf{a}_1, s_2 \ldots$ is **compatible** with a strategy σ_j if for every $i \geq 1$ it holds that $\sigma_j(\rho^S_{\leq i}) = \mathbf{a}_i[j]$. A joint strategy σ_C for C is a tuple of strategies: one for each agent in C. A path ρ is compatible with a joint strategy σ_C if it is compatible with any strategy σ_i composing the joint strategy. We denote with $out(s, \sigma_C)$ the set of all σ_C-compatible paths whose first element is s.

In what follows, a bound $\mathbf{b}$ will be any element of $\mathbb{N}^r$. Let s be a state, $\mathbf{a} = \langle a_1, \ldots, a_{\#(Ag)} \rangle \in ACT$ be a tuple of actions, such that for all i, $\mathbf{a}[i] \in P(i, s)$ and $\mathbf{c}$ a sub-sequence of $\mathbf{a}$. The cost of $\mathbf{c}$ in s with respect to $\mathbf{a}$ is given by: $cost(s, \mathbf{c}, \mathbf{a}) = \sum_{i=1}^{|\mathbf{c}|} \$(s, \mathbf{c}[i], \mathbf{a})$. Let σ_C be a strategy for the coalition C, $\rho = s_1, \mathbf{a}_1, s_2, \ldots$ be a path in $out(s, \sigma_C)$,

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and $\mathbf{b} \in \mathbb{N}^r$. We say that ρ is $\mathbf{b}$-consistent when for each natural number $n \geq 1$, we have that: $\sum_{k=1}^{n} cost(s_k, \sigma_C(\rho^S_{\leq k}), \mathbf{a}_k) \leq \mathbf{b}$.

A strategy σ_C for a coalition C is $\mathbf{b}$-consistent whenever, for every state s, given any $\rho \in out(s, \sigma_C)$, ρ is $\mathbf{b}$-consistent.

3 AGREEMENT GAMES

We introduce a simple model of information exchange. Suppose that there are n agents $1, 2, \ldots n$, and that each agent can *privately observe* a set of atomic propositions $K_1, K_2, \ldots, K_n$, respectively. Agents can share with each other, via a *communication channel*, propositions from such sets and eventually find an **agreement**. For instance, if agent 1 possesses the proposition p, she can offer it to agent 2 and an exchange will take place if 2 offers her another proposition q that 1 does not possess. The model keeps track of the exchanges that have occurred: a state of the model will be identified with a set of mutually disjoint agreements, where each agreement is a two-element set containing a proposition to which one agent has private access and another proposition to which another agent has private access. Consider an agent i and the set of atomic propositions K_i to which the agent has access. In order to distinguish to which of the agents in $Ag \setminus \{i\}$ the agent i offers a certain proposition p, we index the propositions in K_i with elements of $Ag \setminus \{i\}$, obtaining a set Σ_i. The intuitive meaning of a proposition $p_j \in \Sigma_i$ is "the agent i offers to the agent j the proposition $p \in K_i$".

Definition 3.1. Let $Ag = \{1, \ldots, n\}$ be a finite set of agents and K_i be a non-empty finite set of atomic propositions for any $i \in Ag$. We impose that for all $i, j \in Ag$, $K_i \cap K_j = \emptyset$ whenever $i \neq j$. For any $i \in Ag$ we define: $\Sigma_i = \bigcup_{j \in (Ag \setminus \{i\})} \{p_j \mid p \in K_i\}$ and we denote with Σ the set $\bigcup_{i \in Ag} \Sigma_i$. We say that a two-elements set $\{x, y\} \subseteq \Sigma$ is a Σ-agreement iff for some j and i in Ag we have that $x \in \Sigma_j$, x is of the form p_i, $y \in \Sigma_i$, y is of the form q_j and $j \neq i$. A set X of Σ-agreements is Σ-balanced iff $X = \emptyset$ or it holds that $Y \cap Z = \emptyset$ for all distinct $Y, Z \in X$. We denote by $\mathcal{A}_\Sigma$ the set of Σ-agreements and by $\mathcal{B}_\Sigma$ the set of balanced sets of Σ-agreements. If X is a set of Σ-agreements, $Z \subseteq \Sigma$, and for all $\{x, y\} \in X$, $x \in Z$, and $y \in Z$, we say that X is a set of Σ-agreements over Z.

The following two technical propositions will be used to ensure that the transition function of our AGs is well-defined.

Proposition 3.2. *If X and Y are two Σ-balanced sets and either (i) $X \subseteq Y$ or (ii) $Y \subseteq X$ or (iii) for every $U \in X$ and $W \in Y$, $U \cap W = \emptyset$; then $X \cup Y$ is a Σ-balanced set.*

Proof. The case (i) (resp., (ii)) is immediate because $X \cup Y$ is equal to Y (resp., X) and such a set is Σ-balanced by hypothesis. By definition, a Σ-balanced set is a (eventually empty) set of mutually disjoint Σ-agreements. Thus, if the hypothesis of (iii) is true, then given any P and Q in $X \cup Y$, P and Q are Σ-agreements. Moreover if $P \neq Q$, then they either both belong to X (resp., Y), or one belongs to X and the other to Y. In any case $P \cap Q = \emptyset$ and we can conclude. □

In our AGs, a state of the game is identified with a set of mutually disjoint agreement, representing the information exchange occurred so far. Agents move from one state to another by offering to each other propositions: they can move to a state where new pairs of information exchange took place, i.e., the actions of an agent i will be modeled as propositions belonging to the set Σ_i. To assure that by making actions the agents always moves from a set of mutually disjoints agreements to another, we need the following.

Proposition 3.3. *Let $X \in \mathcal{B}_\Sigma$ and $Y \subseteq \Sigma$. Suppose that $Y = \{y_1, \ldots, y_n\}$ and for each $i \leq n$, (i) $y_i \in \Sigma_i$ and, (ii) there is no Σ-agreement $Z \in X$ such that $y_i \in Z$; then there is a maximal (with respect to inclusion) Σ-balanced set W whose members are Σ-agreements over Y such that $W \cup X \in \mathcal{B}_\Sigma$.*

Proof. By Definition 3.1, and by (i) given any $x \in Y$ there is at most one $y \in Y$ such that $\{x, y\}$ is Σ-agreement. In fact, if $x = q_i \in \Sigma_j$ we can have that $\{x, y\}$ is a Σ-agreement if and only if y is of the form q_j and belongs to Σ_i. Thus, let W be the set of all Σ-agreements over Y. If W is empty, then we are done. Otherwise, W is a Σ-balanced set, since it contains only mutually disjoints Σ-agreements. Moreover, by hypothesis (ii), given any P and Z, such that $P \in W$ and $Z \in X$, $P \cap Z = \emptyset$. Thus, by Proposition 3.2, we can conclude that $W \cup X$ is Σ-balanced. □

Given a set of proposition $P \subseteq \Sigma$, we denote by $P|_i = \{q \mid q \in \Sigma_i\}$. We now have all the ingredients to define Agreement Games.

Definition 3.4. Given a RAB-CGS $M = \langle Ap, Ag, S, s_I, \{act_i\}_{i \in AG}, P, t, L, r, \$\rangle$, we say that M is an Agreement Game whenever:

1. $Ap = \Sigma$; $Ag = \{1, \ldots, n\}$; $S = \mathcal{B}_\Sigma$ and $s_I \in S$; $act_i = \Sigma_i \cup \{\star\}$;
2. $P(i, s) = (\Sigma_i \setminus (\bigcup s)|_i) \cup \{\star\}$, given a state s, an action a_i is either an atomic proposition in Σ_i that does not appear in one of the agreements composing s, or it is the idle action $\star$;
3. if $\mathbf{a} = \langle a_1, \ldots, a_n \rangle$ and for all $i \leq n$, $\mathbf{a}[i] \in act_i$, then the transition function is defined by: $t(s, \mathbf{a}) = (s \cup X)$ where X is the maximal (w.r.t. inclusion) balanced set over the set of $\mathbf{a}[i]$ that are propositions such that $s \cup X$ is balanced. Such a set always exists by the fact that the empty-set is Σ-balanced and by Proposition 3.3. Note that, if $\mathbf{a}$ only contains idle actions, then the definition implies $t(s, \mathbf{a}) = s$;
4. $L(s)$ is the identity function on $\bigcup s$;
5. given a state s, an action a, and a tuple of actions $\mathbf{a}$, such that $a = \mathbf{a}[j]$ for some j, we have that $\$(s, a, \mathbf{a}) > \mathbf{0}$ iff there is a $k \neq j$ such that $\{a, \mathbf{a}[k]\}$ is a Σ-agreement.

Condition 5 says that the cost of an action for an agent i in a state s depends upon what other agents do at s. The intuition is that an agent has to pay some amount of resources only if a communication channel with another agent is open. Opening this channel depends upon the concomitant action of two agents.

Remark that, thanks to our framework, we can ensure sharing agreements between agents, which can be established during a preprocessing phase before the MAS starts its execution.

4 CONCLUSIONS AND FUTURE WORKS

In this paper, we introduced the *Agreement Game* (AG) framework to enable advanced information sharing among distributed agents with imperfect knowledge of the MAS. We formalised AG using concurrent game principles and defined how agents can reach agreements within such structures. Future work will explore applying AG to address information-sharing challenges in MAS. A key application is Runtime Verification [2] in distributed systems, where monitors with limited resources can use AG to collaborate and verify formal properties at runtime.

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