Optimal Mechanism Design for Crowdfunding of Public Goods

Extended Abstract

Yukun Cheng
School of Business, Jiangnan University
Wuxi, China
ykcheng@amss.ac.cn

Xiaotie Deng
CFCS, School of Computer Science, Peking University
Beijing, China
xiaotie@pku.edu.cn

Baqiao Quan
School of Mathematical Sciences, Peking University
Beijing, China
2200010895@stu.pku.edu.cn

ABSTRACT

Mechanisms for crowdfunding public goods are essential for ensuring that societies can collectively benefit from public goods. Unlike previous researches on crowdfunding for public goods, which focused on binary outcomes—either full provision or none at all, this paper proposes an auction framework to examine the partial provision of public goods, based on the funds raised, with the goal of maximizing the final investment amount. We develop truthful investment mechanisms that achieve the (approximate) optimal expected investment amount across different models, taking into account the number of agents.

KEYWORDS

Public goods; Truthful; Crowdfunding; Mechanism design; Optimal investment

ACM Reference Format:
Yukun Cheng, Xiaotie Deng, and Baqiao Quan. 2025. Optimal Mechanism Design for Crowdfunding of Public Goods: Extended Abstract. In *Proc. of the 24th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2025), Detroit, Michigan, USA, May 19 – 23, 2025*, IFAAMAS, 3 pages.

1 INTRODUCTION

Public goods, characterized by their non-excludability and non-rivalrous nature, offer universal benefits to all members of society. Although many public goods receive governments' support, a significant portion still relies on funding through public contributions. However, public goods frequently encounter the problem of under-provision, leading to inefficient social services. One primary cause is the free-rider problem [9]. Generally, the greater the number of individuals sharing the benefits, the more pronounced the issue of proportional under-provision becomes. Additionally, problems can also arise from poorly designed mechanisms that either permit agents to secure additional benefits through dishonest bidding or lead to the inefficient production of public goods. In many scenarios, the provider organizes an auction to crowd-fund the construction of public goods from a set of agents. During this process, each agent is required to submit a sealed bid b_i, which represents the maximum amount that they are willing to pay. The total expenditure required to fully realize the public good is denoted by c. Once the public good is fully produced, each participant can derive a private benefit, v_i, from it.

Several studies on public goods [2, 7] have explored auctions that yield binary outcomes: either full provision of the public good if the sum of all bids meets or exceeds the specified cost c, or no provision if it falls short. In instances where the public good is fully provided, each agent is obligated to pay their respective bid b_i. Conversely, if the provision threshold is not met, no payments are expected from any of the agents. However, in other scenarios [1], [11], [3], [4] and [6], even if the crowdfunded investment falls short of the total expenditure c, the funds raised can still be utilized to partially provide the public good. This partial provision allows for some level of benefit to be derived by the participants, albeit not the full extent initially intended.

Therefore, based on those practical situations, this paper discusses a continuum auction framework for crowdfunding public goods. Within this framework, although agents' valuations from the totally constructed public good are private, the joint probability distribution π of these valuations is known. Consequently, the provider applies the agents bidding profile b to determine an investment level $f_i(b)$ and assigns charges $g_i(b)$ to each agent. In our proposed public goods auction, the outcomes are no longer binary; instead, the public good can be partially constructed, and correspondingly, agents can derive partial valuation that is proportional to their total value v. Unlike studies that aim to maximize social welfare [5, 10], we endeavor to develop a truthful mechanism that also targets an (approximate) optimal expected investment amount within our continuum auction framework, which allows for the partial provision of public goods.

2 PRELIMINARIES

Consider a public good that necessitates crowdfunding from a set of agents, represented as $N = \{1, 2, \cdots, n\}$. The full construction of the good requires an investment of $c \in \mathbb{R}_+$. If the total amount f is less than c, then this good can be partially constructed, with the completion level being directly proportional to the investment, i.e. f/c. Upon full completion, each agent gains a private value $v_i \in [0, +\infty)$ from this good. For partial completion with investment f, the value is $(f/c)v_i$. Despite the private nature of individual values, the joint probability distribution π, which determines the value profile $\mathbf{v} = (v_1, \ldots, v_n)$ with a non-increasing density function $p(\mathbf{v})$, is public. The provider organizes n agents to participate in crowdfunding. The procedure of the auction is as follows.

• Stage 1. Setting Bid Bounds. The provider establishes an upper bid bound $\bar{b}_i \in [0, c]$ for each agent i. This step is designed to encourage greater investment participation.

Proc. of the 24th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2025), Y. Vorobeychik, S. Das, A. Nowé (eds.), May 19 – 23, 2025, Detroit, Michigan, USA.

• Stage 2. Bidding. Concurrently, each agent i submits a bid $b_i \in [0, \bar{b}_i]$ according her valuation v_i. The bid b_i represents the maximum amount that agent i is willing to invest. The space of bids of all buyers is denoted as $\mathcal{B}$. Note that the true value v_i may be larger than $\bar{b}_i$. So in the setting with bid bounds, truthful bidding dictates that $b_i = v_i$ if $v_i \leq \bar{b}_i$ and $b_i = \bar{b}_i$ otherwise.
• Stage 3. Investment and Charge. Given a bid profile $\mathbf{b} = (b_1, \ldots, b_n)$, an investment mechanism specifies the investment for the good and the corresponding charge assessed for each agent.

Definition 2.1. An investment mechanism $\mathbf{P} = (f, \mathbf{g})$ is characterized by the investment function f and charging functions $\mathbf{g} = (g_i)_{i \in N}$, subject to (1) $g_i(\mathbf{b}) \leq b_i$ and (2) $\sum_{i=1}^n g_i(\mathbf{b}) \geq f(\mathbf{b})$. Here, $f : \mathcal{B} \to [0, c]$ is differentiable and non-decreasing, representing the total amount to be invested in the public good and $g_i : \mathcal{B} \to [0, b_i]$ indicates the charge imposed on each agent i.

Since the investment in public goods depends on the sum of all agents' bids, the investment function $f(\mathbf{b})$ here can be written as $f(\mathbf{b}) = f(\sum_i b_i)$. Based on the bid profile $\mathbf{b}$, the provider receives the investment $f(\mathbf{b})$ and charges each agent $g_i(\mathbf{b})$. So given the private value v_i of agent, each agent obtains a utility $U_i(\mathbf{b}) = \frac{f(\mathbf{b})}{c} v_i - g_i(\mathbf{b})$.

A mechanism is Individually Rational (IR) if, for each buyer, their utility is non-negative when they truthfully report v_i. Due to the bidding bounds, the truthful bidding effectively maps the value v_i into interval $[0, \bar{b}_i]$, such that $\Pi_{[0,\bar{b}_i]}(v_i) = \min(v_i, \bar{b}_i)$. A mechanism is truthful if, for each agent, reporting their true valuations is always a dominant strategy.

Definition 2.2. An investment mechanism $\mathbf{P} = (f, \mathbf{g})$ is truthful if $U_i(\Pi_{[0,\bar{b}_i]}(v_i), \mathbf{b}_{-i}) \geq U_i(b_i, \mathbf{b}_{-i})$, for all $i \in N$, every possible bid $b_i \in [0, \bar{b}_i]$, and $\mathbf{b}_{-i} \in \mathcal{B}_{-i}$.

In this work, our objective is to design an investment mechanism that not only ensures individual rationality and maintains truthfulness, but also maximizes the expected investment $\mathbb{E}_{\mathbf{v} \sim \pi}(f)$ based on the joint distribution π on the value profile $\mathbf{v}$.

3 OPTIMAL INVESTMENT MECHANISMS

Inspired by Myerson's Lemma [8], we propose the following characterization, which is crucial for our mechanism design.

THEOREM 3.1. *An investment mechanism $\mathbf{P} = (f, \mathbf{g})$ is truthful, if and only if for any differential and non-decreasing investment function f, there exists a unique set of charge functions $\{g_i\}_{i \in N}$ satisfying $g_i(b_i, \mathbf{b}_{-i}) = \frac{b_i}{c} f(b_i, \mathbf{b}_{-i}) - \int_0^{b_i} \frac{1}{c} f(t, \mathbf{b}_{-i}) \mathrm{d}t$.*

The mechanisms given by this theorem also satisfy IR property and the requirement $g_i(\mathbf{b}) \leq b_i$. For convenience we denote $s = \sum_{i=1}^n \bar{b}_i - c$ and $q(\sum_{i=1}^n b_i - c) = f(\mathbf{b}) = f(\sum_{i=1}^n b_i)$, then the requirement $\sum_{i=1}^n g_i(\mathbf{b}) \geq f(\mathbf{b})$ can be transformed into

$$(\sum_{i=1}^n b_i - c) q(\sum_{i=1}^n b_i - c) \geq \sum_{i=1}^n \int_{\max(0, \sum b_{-i} - c)}^{\sum_{i=1}^n b_i - c} q(t) \, \mathrm{d}t. \quad (1)$$

Especially, for the two-bidder case, by defining $x = b_1 + b_2 - c$, (1) is equivalent to $x q(x) \geq 2 \int_0^x q(t) \, \mathrm{d}t$.

For $n = 2$, the optimal mechanism is given by Mechanism 1.

Mechanism 1. Optimal Investment Mechanism for $n = 2$

Input: parameter c, bid bounds $\{\bar{b}_1, \bar{b}_2\}$ and bid profile $\mathbf{b}$.

(1) If $s = \bar{b}_1 + \bar{b}_2 - c > 0$, then let the investment function be

$$f(b_1, b_2) = \frac{c}{s} \max(0, b_1 + b_2 - c),$$

and define the charge functions of two agents as

$$\begin{cases} g_1(b_1, b_2) = \max(0, \frac{b_1^2 - (c - b_2)^2}{2s}); \\ g_2(b_1, b_2) = \max(0, \frac{b_2^2 - (c - b_1)^2}{2s}); \end{cases}$$

(2) If $s = \bar{b}_1 + \bar{b}_2 - c = 0$, then define the investment function as

$$f(b_1, b_2) = c \mathbf{1}_{b_1 = \bar{b}_1, \, b_2 = \bar{b}_2},$$

and define the charge functions of two agents as

$$\begin{cases} g_1(b_1, b_2) = \bar{b}_1 \mathbf{1}_{b_1 = \bar{b}_1, \, b_2 = \bar{b}_2} \\ g_2(b_1, b_2) = \bar{b}_2 \mathbf{1}_{b_1 = \bar{b}_1, \, b_2 = \bar{b}_2}; \end{cases}$$

(3) If $s = \bar{b}_1 + \bar{b}_2 - c < 0$, then let $f = g_1 = g_2 = 0$.

Output: $f(\mathbf{b})$, $g_1(\mathbf{b})$ and $g_2(\mathbf{b})$.

THEOREM 3.2. *Mechanism 1 is truthful and IR, and is an optimal investment mechanism to maximize the expected investment.*

We also design truthful mechanism for multi-bidder case.

Mechanism 2. The Investment Mechanism for Multi-bidder Scenario

Input: parameter c, bid bounds $\bar{\mathbf{b}} = (\bar{b}_1, \ldots, \bar{b}_n)$, parameter function q and bid profile $\mathbf{b}$.

(1) If $s = \sum_{i=1}^n \bar{b}_i - c > 0$, then define the investment as

$$f(\mathbf{b}) = \begin{cases} q(\sum_{i=1}^n b_i - c) & \text{if } \sum_{i=1}^n b_i \geq c \\ 0 & \text{else,} \end{cases} \quad (2)$$

and let the charges of the agents be

$$g_i(\mathbf{b}) = \frac{b_i}{c} f(\mathbf{b}) - \int_0^{b_i} \frac{1}{c} f(t, \mathbf{b}_{-i}) \mathrm{d}t$$

(2) If $s = \sum_{i=1}^n \bar{b}_i - c = 0$, then the investment and the charge of each agent are

$$f(\mathbf{b}) = c \mathbf{1}_{b_i = \bar{b}_i (i=1,\ldots,n)}, \; g_i(\mathbf{b}) = \bar{b}_i \mathbf{1}_{b_i = \bar{b}_i (i=1,\ldots,n)}$$

(3) If $s = \sum_{i=1}^n \bar{b}_i - c < 0$, then $f = g_i = 0$.

Output: the allocation and payment result $f(\mathbf{b})$, $g_i(\mathbf{b})$.

ACKNOWLEDGMENTS

The first author is supported by NSFCs (No. 12471339); the second author is supported by Wuhan East Lake High-Tech Development Zone (also known as the Optics Valley of China, or OVC) National Comprehensive Experimental Base for Governance of Intelligent Society; and the third author is supported by the Elite Undergraduate Training Program of School of Mathematical Sciences in Peking University. Yukun Cheng and Baqiao Quan are joint first authors. Xiaotie Deng is the corresponding author.

REFERENCES

[1] Robert Agranoff and Michael McGuire. 2001. Big questions in public network management research. *Journal of public administration research and theory* 11, 3 (2001), 295–326.

[2] Dirk Alboth, Anat Lerner, and Jonathan Shalev. 2001. Profit maximizing in auctions of public goods. *Journal of Public Economic Theory* 3, 4 (2001), 501–525.

[3] John M Bryson, Barbara C Crosby, and Melissa Middleton Stone. 2006. The design and implementation of Cross-Sector collaborations: Propositions from the literature. *Public administration review* 66 (2006), 44–55.

[4] John M Bryson, Barbara C Crosby, and Melissa Middleton Stone. 2015. Designing and implementing cross-sector collaborations: Needed and challenging. *Public administration review* 75, 5 (2015), 647–663.

[5] Theodore Groves and John Ledyard. 1977. Optimal allocation of public goods: A solution to the" free rider" problem. *Econometrica: Journal of the Econometric Society* (1977), 783–809.

[6] Chris Huxham and Siv Vangen. 2013. *Managing to collaborate: The theory and practice of collaborative advantage.* Routledge.

[7] Christoph Kuzmics and Jan-Henrik Steg. 2017. On public good provision mechanisms with dominant strategies and balanced budget. *Journal of Economic Theory* 170 (2017), 56–69.

[8] Roger B Myerson. 1981. Optimal auction design. *Mathematics of operations research* 6, 1 (1981), 58–73.

[9] Paul A Samuelson. 1954. The pure theory of public expenditure. *The review of economics and statistics* (1954), 387–389.

[10] Sujay Sanghavi and Bruce Hajek. 2005. A new mechanism for the free-rider problem. In *Proceedings of the 2005 ACM SIGCOMM workshop on Economics of peer-to-peer systems.* 122–127.

[11] Craig W Thomas. 2002. *Bureaucratic landscapes: Interagency cooperation and the preservation of biodiversity.* Mit Press.