Egalitarianism in Online Coalition Formation

Extended Abstract

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ABSTRACT

We investigate the *online coalition formation* problem, where agents arrive one by one and must be assigned to coalitions, with their utilities for others revealed upon arrival. Our focus lies on *additively separable hedonic games*, where agents assign cardinal utilities to others, assumed to be controlled by an adversary in our online context. This paper introduces the evaluation of partitions based on their *egalitarian social welfare*, with the goal of maximizing the minimum utility of any agent. This objective strikes balance between fairness and efficiency by prioritizing the satisfaction of the least well-off agents. For various real-life scenarios, we establish tight or nearly tight upper bounds on the competitive ratio and complement these findings with ***optimal*** or ***near-optimal*** algorithms. However, we also demonstrate that in some cases, no competitive algorithm is feasible. In particular, under the classic *worst-case adversarial model*, where agents arrive in an *arbitrary* order, we show that no algorithm has a non-trivial competitive ratio, if at all.

KEYWORDS

Online Coalition Formation, Hedonic Games, Egalitarianism

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1 BACKGROUND

Coalition formation is a vibrant topic in multi-agent systems, whose goal is to partition a set of agents into disjoint coalitions (henceforth, *partition*). *Hedonic games* are a standard framework for the investigation of coalition formation (see, e.g., [5, 10]), where agents have preferences for the coalitions they are part of by disregarding externalities. This research considers *additively separable hedonic games* (ASHGs) with *symmetric* preferences [2], where agents have cardinal utilities for one another, and an agent's utility for a coalition is the *sum* of her utilities from other members of that coalition.

The traditional hedonic games literature often regards an *offline* setting, making the possibly unrealistic assumption that the game is entirely known upfront. On the contrary, in this work we study numerous practical scenarios where agents arrive *online*, one at a time [3, 4, 6–8, 12]. We consider realistic situations where the number of coalitions and their sizes are constrained due to limited *physical* space (see, e.g., [7, 12] for examples). Unlike previous research on online coalition formation [3, 4, 7–9], in this paper we initiate the study on evaluating partitions in terms of their *egalitarian social welfare*, with the objective of maximizing the minimum utility of any agent in our *online* setting. The egalitarian social welfare captures various realistic scenarios, seeking to balance fairness and efficiency by ensuring that the least satisfied agents achieve the highest possible level of satisfaction.

Egalitarian welfare maximization in ASHGs has been previously studied in *offline* settings [1, 13–15]. In particular, even in *offline* ASHGs with *symmetric* preferences, Hanaka et al. [13] proved that computing a maximum egalitarian partition is strongly NP-hard. However, Peters [14] showed that this problem becomes polynomial time tractable when the game's underlying graph has bounded treewidth. Unlike prior works on that topic, which focus exclusively on *offline* ASHGs, we are the first to examine egalitarian welfare maximization within the *online* variant of coalition formation. Specifically, whereas existing studies on online coalition formation predominantly address *utilitarian* welfare maximization [3, 7, 8] or stability [4, 9], our work introduces a novel perspective by formulating and analyzing online coalition formation through the lens of egalitarian welfare maximization.

2 PRELIMINARIES

We focus on *additively separable hedonic games* (ASHGs) with *symmetric* preferences, where every pair of agents assigns the same cardinal utility toward one another, and an agent's utility for a given coalition is the *sum* of her utilities from other members of that coalition. The outcome of an ASHG is a *partition* of the set of agents into disjoint coalitions. We concentrate on realistic situations, where the number and sizes of coalitions within a partition may be constrained due to *physical* constraints. For positive integers α and k, we seek a partition that contains at most k coalitions, each of size at most α. We assume that $k \geq 2$ and $\alpha \geq 2$ since scenarios where $k = 1$ and/or $\alpha = 1$ are trivial. If $k < \infty$, then we say that the number of coalitions is *bounded*; otherwise, if $k = \infty$, it is said to be *unbounded*. Likewise, coalition sizes are *bounded* when $\alpha < \infty$ and *unbounded* otherwise. In our setting, partitions are assessed via their their *egalitarian social welfare*, i.e., we seek a partition that maximizes the minimum utility obtained by any agent. That is, the egalitarian social welfare of a given partition is given by minimum utility among the utilities of all agents.

In this work, we study the *online* version of ASHGs, where agents arrive one by one. Once an agent arrives, she reveals her preferences toward previously arrived agents, which are assumed to be set by an adversary in our *online* settings. See Figure 1 for a sample instance

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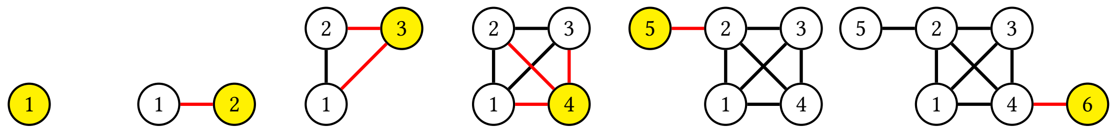

(a) $t = 1$ **(b)** $t = 2$ **(c)** $t = 3$ **(d)** $t = 4$ **(e)** $t = 5$ **(f)** $t = 6$

Figure 1: A visual example of our problem with *binary* utilities. Each subfigure corresponds to the arrival of a single agent. Examples of maximum egalitarian partitions for this instance are $(\{1,3\},\{2,5\},\{4,6\})$, $(\{1,2,3,5\},\{4,6\})$ and $(\{2,5\},\{1,3,4,6\})$, each attaining the maximal egalitarian welfare of 1 since agents 5 and 6 obtain their maximal possible utility of 1.

in such settings. Afterwards, a central authority (i.e., an *online* algorithm) must then *immediately* and *irrevocably* assign the arriving agent to an existing coalition or a new one containing, at this moment, only her. Specifically, at each round, the algorithm constructs a partial partition of the agents who arrived so far, without changing the partition generated in previous rounds and without any knowledge of future agents as well as their corresponding utilities.

That is, our problem can be viewed as a game between an adversary and an online assignment algorithm. We consider the standard *worst-case adaptive adversary* [3, 7, 8, 12], with the objective of severely hurting an algorithm's performance by carefully picking the utilities and the arrival order of all agents. The adversary is *adaptive* in the sense that, at each round, it decides to either release a new agent or stop the arrival sequence based on the partition formed so far. If the adversary introduces an agent at that round, then the adversary selects the mutual utilities that the newly arriving agent and the previously disclosed ones have toward each other.

Since maximizing egalitarian welfare is known to be strongly NP-hard even in *offline* settings [1], we aim to approximate the maximum egalitarian welfare in our *online* context. As common in online algorithms (see, e.g., [11]), we thus measure the performance of an online algorithm in terms of its *competitive ratio*, i.e., the minimum ratio between the egalitarian welfare of the algorithm's produced partition to that of the optimal (offline) partition. The competitive ratio notions that we consider are at most 1, with better performance corresponding to a larger competitive ratio.

The adaptive adversary that we consider can make *randomized* decisions when constructing the game, enabling the adversary to generate complex and unexpected scenarios. This enables the adversary to hinder an algorithm's potential randomness, steering the algorithm toward worst-case performance. Therefore, in our setting, the competitive ratio is defined with respect to the expected maximal egalitarian welfare and the expected egalitarian welfare of the potentially randomized online algorithm. We also remark that, in prior works on online coalition formation (see, e.g., [12]), a notion of competitiveness, termed as the weak competitive ratio, was considered, which is *weaker* than our competitive ratio notions.

3 OUR CONTRIBUTIONS

For numerous real-world situations, we derive tight or nearly tight upper bounds on the competitive ratio and complement these findings with ***optimal*** or ***near-optimal*** algorithms. Nevertheless, we also demonstrate that in some cases, no competitive algorithm is feasible. Particularly, we show that no algorithm has a non-trivial competitive ratio, if at all.

General Utilities. First, we analyze the case of *general* utilities that can take any negative or non-negative real value. When the number and sizes of coalitions are either bounded or unbounded, we illustrate that no competitive algorithm exists in general, even for randomized algorithms and restricted utilities.

Non-Negative Utilities. The strong impossibility result for general utilities prompts us to consider significantly more constrained instances with *non-negative* utilities, for which we obtain:

Bounded Coalition Sizes. Sadly, so long as coalition sizes are *bounded*, we show that a competitive algorithm still does not exist, even for a special subclass of non-negative utilities.

Unbounded Coalition Sizes. We are thus motivated to explore a less constrained family of instances that captures many practical contexts, where coalition sizes are *unbounded* and the number of coalitions is either *bounded* or *unbounded*. Though the grand coalition trivially maximizes egalitarian welfare, its formation is impractical in realistic scenarios. For example, it is unreasonable for a hospital administrator to assign all doctors to a *single* department while leaving others understaffed, as patients in the neglected departments would receive no care. Accordingly, we explore the most general model where a partition must contain at least *two* coalitions. Within this general setup, we show that the competitive ratio of an algorithm may be positive, and that a naive randomized algorithm is ***optimal***. Particularly, we illustrate the optimality of our algorithm by deriving tight or nearly tight upper bounds on the competitive ratio of any randomized algorithm. Surprisingly, we also show that no *deterministic* algorithm can surpass our *randomized* ones, opposed to *utilitarian* welfare maximization where the *deterministic* greedy algorithm by Flammini et al. [12] is optimal.

4 FUTURE WORK

Our work opens the way for many future works, like the study of other classes of hedonic games, other solution concepts and other adversarial models. Future works should also study more general classes of social welfare functions. Finally, future studies should further explore settings where partitions can be modified with a penalty by migrating agents between coalitions, assignments may be postponed with a cost for making late decisions or both.

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