Parameterized Complexity of Hedonic Games with Enemy-Oriented Preferences

Extended Abstract

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ABSTRACT

Hedonic games model settings in which a set of agents have to be partitioned into groups which we call coalitions. In the enemy aversion model, each agent has friends and enemies, and an agent prefers to be in a coalition with as few enemies as possible and, subject to that, as many friends as possible. A partition should be stable, i.e., no subset of agents prefer to be together rather than being in their assigned coalitions under the partition. We look at two stability concepts: core stability and strict core stability. This yields several algorithmic problems: determining whether a (strictly) core stable partition exists, finding such a partition, and checking whether a given partition is (strictly) core stable. Several of these problems have been shown to be NP-complete, or even beyond NP. This motivates the study of parameterized complexity. We conduct a thorough computational study using several parameters: treewidth, number of friends, number of enemies, partition size, and coalition size. We conclude this paper with results in the setting in which agents can have neutral relations with each other, including hardness-results for very restricted instances.

KEYWORDS

Hedonic Games; Parameterized Complexity; Coalition Formation

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1 INTRODUCTION

In this paper we study *hedonic games with enemy aversion*. Hedonic games describe coalition formation, where agents preferences only depend on the agents in the coalition. This setting was introduced by Drèze and Greenberg [8]. Dimitrov et al. [6] introduce *friend appreciation* and *enemy aversion*: Each agent divides other agents into *friends* and *enemies*. Ota et al. [12] add *neutrals* to the model. When preferences are derived from enemy aversion, an agent prefers a coalition containing fewer enemies over one containing more enemies. Subject to that, the agent prefers a coalition containing more friends. An example of hedonic games with enemy aversion would be a setting where politicians form political coalitions: The more members a group has, the larger their weight. However, ideological conflicts can stop members from working together.

When selecting a partition, we desire that the partition is in some sense *stable*, i.e., a group of agents will not deviate and form a coalition together. In this paper we focus on the stability concepts called *core stability* and *strict core stability*.

Under enemy aversion, Dimitrov et al. [6] show that a core stable partition always exists when there are no neutral relations, although finding one is NP-hard. Finding a strictly core stable partition is harder: Rey et al. [14] show it is beyond NP. Sung and Dimitrov [15] show that verifying whether a partition is core stable is NP-complete under the enemy aversion, which also holds for strict core stability [1]. This computational hardness motivates us to look into parameterized complexity.

Our Contribution. We study the parameterized complexity of hedonic games under enemy aversion with and without neutrals.

In this paper, we represent the friendship and enemy relations with graphs. Each agent corresponds to a vertex and there is an edge between two vertices in the friendship graph G^g (resp. enemy graph G^b) if their corresponding agents are friends (resp. enemies). Coalitions can then be interpreted as subsets of vertices.

In the setting without neutrals, we study the following problems: CF – finding a core stable partition, (S)CV– verifying that a given partition is (strictly) core stable, and (S)CE – determining whether a (strictly) core stable partition exists.

Given a problem A, we use A^N to denote the problem A in which we allow neutral relations. We focus on the following parameters: The maximum degree of the friendship graph Δ^g (resp. the enemy graph Δ^b, the union of the friendship and enemy graph $\Delta^{g,b}$) the

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treewidth of the friendship graph tw^g (resp. the enemy graph tw^b), the maximum coalition size $|C|$, and the maximum number of coalitions $|\Pi|$. For verification problems, the last two parameters correspond to the size of the largest coalition and the number of coalitions in the given partition, respectively. For finding and existence problems they are additional constraints on the solution.

The proofs are in the full version of the paper [9].

Table 1: Overview of parameterized results. Rows correspond to instance restrictions, columns correspond to problems. The letter "B" indicates bipartite and "I" interval graphs. The known results are in italics and the reference can be found in their cell. The symbol ($^\diamond$) indicates that the result additionally holds even when $G^g \cup G^b$ is bipartite.

	CF	CV	SCE	SCV
B G^g/ G^b	P	P	P	P
I G^g/ G^b	P	P	P	P
tw^g/ tw^b	FPT	FPT	FPT	FPT
Δ^g	FPT	FPT	NPh (≥ 4) P (≤ 3)	FPT
Δ^b	NPh (≥ 3) P (≤ 2)	NPc (≥ 8) P (≤ 2)	NPh (≥ 6) P (≤ 2)	NPc (≥ 16) P (≤ 2)
$\|C\|$	W[1]h XP	*W[1]h* [10] *XP* [10]	NPh (≥ 3) P (≤ 2)	*W[1]h* [10] *XP* [10]
$\|\Pi\|$	NPh (≥ 3) P (≤ 2)	NPc (≥ 3) P (≤ 2)	NPh (≥ 3) P (≤ 2)	NPc (≥ 3) P (≤ 2)
	CE^N	CV^N	SCE^N	SCV^N
B $G^g \cup G^b$	?	NPc	NPh	NPc
$\Delta^{g,b} + \|C\|$	NPh	NPc ($^\diamond$)	NPh	NPc ($^\diamond$)
$\Delta^{g,b} + \|\Pi\|$	NPh	NPc ($^\diamond$)	NPh	NPc ($^\diamond$)

Related Work. Brandt et al. [2] show that under enemy aversion individually stable partition always exists and can be found in polynomial time, whereas determining the existence of a Nash stable partition is NP-complete even when the coalition size is bounded; this reduction can be modified for constant degree [4].

The classical complexity of the problem with friend appreciation has been studied [2, 3, 6, 12]. Chen et al. [3] study the parameterized complexity of hedonic games with friend appreciation. Hedonic games with enemy aversion are a special case of hedonic games with additively separable preferences (ASHG). Their parameterized complexity has attracted prior study [10, 11, 13].

2 PRELIMINARIES

Let $[n] = \{1, 2, \dots, n\}$. We call the *agent set* $A = [n]$. A *partition* of the agent set $\Pi = \{C_1, \dots, C_{|\Pi|}\}$ is a set of disjoint subsets of agents such that $\bigcup_{i=1}^{|\Pi|} C_i = A$. We call each subset in Π a *coalition*. We denote by $\Pi(i)$ the coalition that contains agent i in partition Π. Each agent i partitions other agents in three sets: Her friends $\mathrm{Fr}(i)$, her enemies $\mathrm{En}(i)$, and neutrals $\mathrm{Ne}(i)$.

We define the preference relation, $\succ_i$, for every agent i. We use a definition equivalent to the one of Woeginger [16]. Given two subsets $S_1, S_2 \subseteq A$, with $i \in S_1 \cap S_2$, agent i *prefers* S_1 over S_2, denoted $S_1 \succ_i S_2$, if and only if: (1) $|\mathrm{En}(i) \cap S_1| < |\mathrm{En}(i) \cap S_2|$ or (2) $|\mathrm{En}(i) \cap S_1| = |\mathrm{En}(i) \cap S_2|$ and $|\mathrm{Fr}(i) \cap S_1| > |\mathrm{Fr}(i) \cap S_2|$.

We consider the following two stability notions:

Definition 1 (Core stability and strict core stability). A partition Π of A is *core stable* if there is no $C \subseteq A$ such that all agents in A prefer C over their coalition under Π.

The partition Π is *strictly core stable* if there is no $C \subseteq A$ such that no agent in C prefers her coalition under Π to C, and at least one agent i in C prefers C over $\Pi(i)$.

We assume that the reader is familiar with parameterized complexity, including treewidth. Otherwise, one can refer to the books of Downey and Fellows [7] and Cygan et al. [5].

3 RESULTS

Our results are summarized in Table 1. In the case without neutrals we can assume the preferences to be symmetric; with neutrals our hardess-results hold even when the relations are symmetric.

Most of our algorithms rely on the following lemma:

Lemma 1. *Let $\hat{f}(n)$ denotes the complexity of solving k-Clique or k-Independent Set for an arbitrary $k \in [n]$ and $\hat{g}(n)$ is the complexity of solving Partition Into k' Cliques or k'-Coloring for an arbitrary $k' \in [n]$.*

CF, CV, and SCV can be solved in $n^{O(1)} \hat{f}(n)$. SCE can be solved in $n^{O(1)}(\hat{g}(n) + \hat{f}(n))$.

Lemma 1 implies that if the k-Clique and Partition Into k' Cliques (resp. k-Independent Set and k'-Coloring) problems are tractable on a specific graph class, then CF, CV, SCE, and SCV are tractable as well if the friendship graph (resp. enemy graph) belongs to this class. This also implies that SCE is contained in the complexity class Δ_2^P.

We also show the existence result for core stable partitions [6] in the friends and enemies case does not hold in the presence of neutrals when the relations are symmetric. Ota et al. [12] have already shown this when symmetricity is not required.

Theorem 2. *An enemy oriented hedonic game with neutrals and symmetric relations (A, G^g, G^b) may not admit a core stable partition.*

4 OPEN QUESTIONS

In the case with neutrals, a few open questions remain. We know that all the studied problems remain NP-hard when $G^g \cup G^b$ is an interval graph since a clique is an interval graph and the problems are NP-hard without neutrals. How about the case where only G^g or G^b is an interval graph? We also do not know the complexity of CE^N when $G^g \cup G^b$ is bipartite. The parameterized complexity regarding treewidth is also unknown, although Peters [13] shows that the problems are in FPT with respect to the treewidth and degree combined.

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