Compensating Latent Nonlinear Dynamics for Practical Consensus Control

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ABSTRACT

In this paper, we propose a new kernel-based method for compensating latent nonlinear dynamics for consensus control in multi-agent systems. Although kernel regression is a well-known and thoroughly studied technique, recent research has shown its significant non-asymptotic potential. Under general conditions, we show the convergence of the proposed approach by stability analysis and show that applying kernel regression compensation for consensus control leads to synchronization of the agents within high probability error bounds.

KEYWORDS

Leader-follower consensus, high-probability guarantees

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1 INTRODUCTION

Multi-agent systems (MAS) have received considerable attention in recent years, particularly due to their potential for solving tasks that are beyond the scope of a single agent [12]. The distributed nature of MAS requires, however, dedicated techniques to work efficiently.

In the field of MAS, there are many problems of interest, such as formation control, where the task for the group of agents is to achieve or maintain some desired state [8], and distributed estimation, in which agents are used to model some unknown phenomenon based on noisy observations [1, 13].

In this paper, we consider the leader-follower consensus problem, one of the most common tracking problems related to multi-agent systems. In this setting, we distinguish an individual (physical or virtual) leader and its followers, whose goal is to track the leader's behavior. Nevertheless, only a fraction of all followers can observe the leader's state directly. Hence, it is necessary to implement a collaborative control policy. In the literature, one can find papers raising various issues related to leader-follower consensus. For instance, in [9], the problem of agent synchronisation is considered when the network topology changes over time. In [5], the authors discuss the coordination problem in the presence of time delays in the communication between the agents. One may also consider numerous assumptions regarding the dynamics of agents, such as the influence of nonlinear components, time-invariant [6] or time-varying [15] leader states, and continuous [7] or discrete [10] time settings.

Our work focuses on achieving *practical consensus*, which may differ from the final (asymptotic) consensus by not more than a given small (known) value, *cf.* [2]. We consider the scenario where the follower dynamics are unknown and influence the behavior of the agents. Similar work has been presented in [14], where Gaussian processes were used for estimation.

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2 PROBLEM FORMULATION

We consider a network of M homogeneous followers and a single independent leader. The i-th follower dynamics is assumed to be $\dot{x}_i = f(x_i) + u_i,\ i = 1, 2, \ldots, M$, where $x_i = [x_{i1}, \ldots, x_{id}]^\top \in \mathbb{X} \subset \mathbb{R}^d$ denotes the state vector, the mapping $f\colon \mathbb{X} \to \mathbb{R}^d$ characterizes the latent dynamics of the follower, and u_i is the control input.

The leader dynamics is given by $\dot{x}_l = f_l(x_l, t)$, where $x_l = [x_{l1}, \ldots, x_{ld}]^\top \in \mathbb{X}$ is the state vector, and $f_l\colon \mathbb{X} \times \mathbb{R} \to \mathbb{R}^d$.

Assumption 1. *The follower's latent dynamics $f\colon \mathbb{X} \to \mathbb{R}^d$ is a Lipschitz continuous mapping, with a known constant $0 \le L < \infty$.*

Assumption 2. *The leader's nonlinear dynamics $f_l(x_l, t)$ is a continuous and bounded function, i.e., there exists a positive constant $\bar{f}_l$, for which $\|f_l(x_l, t)\| < \bar{f}_l$, for all x_l and t.*

We model the connections between the followers as an adjacency matrix $A = \{a_{ij}\}$, where $a_{ij} = 1$ if the i-th and j-th followers can share their states, and 0 otherwise. To describe their connections with the leader, we use the diagonal matrix $B = \operatorname{diag}\{b_{11}, b_{22}, \ldots, b_{MM}\}$, where $b_{ii} = 1$ if the leader can share its state with the i-th follower, and 0 otherwise.

The tracking error between the i-th follower and the leader is defined as $e_i = x_i - x_l$. Furthermore, due to the distributed character of the followers, we also define the consensus error $\xi_i = \sum_{j=1}^{M} a_{ij}(x_i - x_j) + b_{ii}(x_i - x_l)$.

We consider the control input consisting of a proportional consensus error gain k_i, as commonly used in the control theory for multi-agent systems [3], and a model of the follower's nonlinear dynamics, $\hat{\mu}_i(x_i)$, which we discuss in the next sections. Thus, we have $u_i = -k_i\xi_i - \hat{\mu}_i(x_i)$.

Our main goal is to show that utilizing the proposed control policy results in practical consensus, that is,

$$\lim_{t\to\infty} \|x_i(t) - x_l(t)\| \leq \epsilon \quad i = 1, 2, \ldots, M, \tag{1}$$

where ϵ is some prescribed positive constant.

3 KERNEL REGRESSION MODELING

In the considered setting, for every agent i we define the training data set D_i, which contains N corresponding data pairs $(x_{i,n}, y_{i,n})$, where

$$y_{i,n} = f(x_{i,n}) + \eta_{i,n}, \quad n = 1, 2, \ldots, N, \tag{2}$$

and $\eta_{i,n}$ is additive noise, subject to the following assumption:

Assumption 3. *The disturbance $\{\eta_t \in \mathbb{R}^d : t \in \mathbb{N}\}$ is a sub-Gaussian stochastic process.*

Following the training set D_i, we introduce the kernel regression estimator $\hat{y}_i(\psi) := \sum_{n=1}^N \frac{K_h(\psi, x_{i,n})}{\kappa_{i,N}(\psi)} y_{i,n}$, where $\kappa_{i,N}(\psi) := \kappa_{i,N}(\psi, h) = \sum_{n=1}^{\top} K_h(\psi, x_{i,n})$, with $K_h(\psi, x) := K(\|\psi - x\|/h)$, and where K and h are the kernel function and the bandwidth parameter, respectively.

Assumption 4. *The kernel $K : \mathbb{R} \to \mathbb{R}$ is such that $0 \leq K(v) \leq 1$ for all $v \in \mathbb{R}$. Also, $K(v) = 0$ for all $|v| > 1$.*

To achieve practical consensus, we need to ensure that the models of the follower's dynamics are reliable with high probability, uniformly among all the agents and the domain $\mathbb{X}$. To this end, we utilise the nonasymptotic bounds for kernel regression proposed in [13], extended to the multivariate setup in [4].

To establish the uniformity over the state space, firstly, we over-approximate $\mathbb{X}$ with a hypercube Ω with an edge length $r_\Omega = \max_{x,x' \in \mathbb{X}} \|x - x'\|$. Furthermore, we introduce a finite set that ρ-covers Ω.

Lemma 1. *Let Assumptions 1, 2 and 4 be in force and Ω be a hypercube over-approximation of $\mathbb{X} \subset \mathbb{R}^d$. Consider the estimator $\hat{y}_i \in \mathbb{R}^d$, with fixed bandwidth h. Pick a parameter $\rho > 0$ and define a finite set $\bar{\mathbb{X}}$, with cardinality $|\bar{\mathbb{X}}|$, that ρ-covers Ω. Then, with probability at least $1 - \delta$, for all M agents, and for all $x \in \mathbb{X}$, there exists a $\bar{x} \in \bar{\mathbb{X}}$ such that*

$$\|\hat{y}_i(\bar{x}) - f(x)\|_2 \leq \beta_i(\bar{x}), \;\; \beta_i(\bar{x}) := L(h + \rho) + 2\sigma \frac{\alpha_i(\bar{x}, \delta)}{\kappa_i(\bar{x})}, \tag{3}$$

$$\alpha_i(\bar{x}, \delta) := \begin{cases} \sqrt{\log(M|\bar{\mathbb{X}}|\delta^{-1} 2^{d/2})}, & \text{for } \kappa_i(\bar{x}) \leq 1 \\ \sqrt{\kappa_i(\bar{x}) \log\left(M|\bar{\mathbb{X}}|\delta^{-1}\left(1 + \kappa_i(\bar{x})\right)^{d/2}\right)}, & \text{for } \kappa_i(\bar{x}) > 1. \end{cases} \tag{4}$$

Note that, even though the cardinality of a covering set may not be known *a priori*, it can be easily upper bounded, *cf.* [11].

After the statements of Lemma 1, we are in a position to introduce the model $\hat{\mu}_i$, for $\hat{\mu}_i$ defined as $\hat{\mu}_i(x) = \hat{y}_i(\bar{x})$, where $\bar{x}$ corresponds to the nearest element of x from $\bar{\mathbb{X}}$.

4 LYAPUNOV STABILITY

Denote the *global* tracking error as $\boldsymbol{e} = \left[e_1^\top, \ldots e_M^\top\right]^\top$ and consider the following Lyapunove candidate $V = \frac{1}{2}\boldsymbol{e}^\top(\tilde{L} \otimes I_d)\boldsymbol{e}$, where $\tilde{L} = L + B$ and L is the Laplacian matrix. Next, observe that $\dot{V} = \boldsymbol{e}^\top(\tilde{L} \otimes I_d)\,\dot{\boldsymbol{e}}$, which can be upper bounded with probability $1 - \delta$ by $\dot{V} \leq -\frac{3}{4}k^*\lambda_{\min}^2(\tilde{L})\,\|\boldsymbol{e}\|^2 + \frac{v}{k^*}$, where $k^* := \min\{k_1, \ldots, k_M\}$, $v = \sum_{i=1}^M \left(\beta_i + \bar{f}_l\right)^2$ and $\lambda_{\min}$ denotes the minimal eigenvalue.

Consider the multi-agent system with M followers and an individual leader with dynamics $\dot{x}_i$, $\dot{x}_l$, where $i = 1, \ldots, M$. Let Assumptions 1–4 be in force and apply the control u_i. Then, it can be shown, that for a given $\delta \in (0, 1)$, the norm of the global tracking error, $\|\boldsymbol{e}\|$, converges, with probability $1 - \delta$, to the ball centered at the origin with radius

$$r \leq \frac{2\sqrt{v}}{\sqrt{3}k^*\lambda_{\min}(\tilde{L})}. \tag{5}$$

The parameter v depends on the model quality and the upper bound of the leader dynamics. However, the model quality may be improved by increasing the number of training data samples. Also, the control gain k^* is a crucial user-dependant parameter, which can significantly improve the bound.

5 NUMERICAL EXPERIMENTS

Let us consider a network of 4 followers and an individual leader l. The dynamics of the leader is given by $\dot{x}_{l1} = \sin(0.01\pi + x_{l2})$, $\dot{x}_{l2} = \cos(0.01\pi + x_{l1})$, whereas the followers have the dynamics $\dot{x}_{i1} = 1.5x_{i1}\sin(x_{i2})$, $\dot{x}_{i2} = x_{i1}\cos(x_{i2})$, $i = 1, \ldots, 4$.

Every agent has access to an individual training data set consisting of 500 measurements randomly distributed on the domains $[-1, 4]$ and $[-2, 2]$ for x_{i1} and x_{i2}. The training data samples are perturbed by an additive Gaussian noise $\mathcal{N}(0, 0.5)$. The estimates are calculated on an evenly spaced grid within their domains (grid density 0.2).

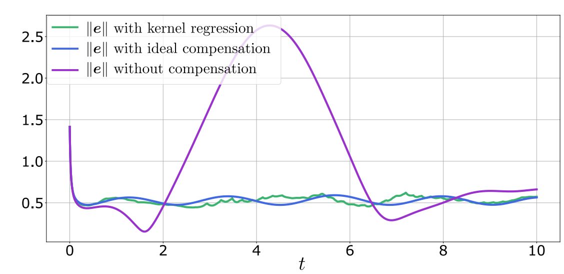

Figure 1: Comparison of the global tracking error for $k^* = 20$.

Applying kernel regression modeling for dynamics compensation allows to achieve similar results as in the scenario where the dynamics are completely known and can be fully compensated.

6 CONCLUSIONS

In this paper, we have proposed a new approach for compensating unknown nonlinear dynamics in consensus control. Due to the rather mild *a priori* knowledge required for the applied kernel regression smoothing method, the modeling process is well suited for real-world applications.

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