Equilibrium Selection via Cheap-talk Partition

Extended Abstract

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ABSTRACT

Coordinating the behaviour of self-interested agents in the presence of multiple Nash equilibria is a major research challenge for multi-agent systems. Pre-game communication between all the players can aid coordination in cases where the Pareto-optimal payoff is unique, but can lead to deadlocks when there are multiple payoffs on the Pareto frontier. We consider a communication partition where only players within the same coalition can communicate with each other. We show that under some natural assumptions about symmetry and the conditions under which players within the same coalition can reach an agreement about their joint actions, certain communication partitions can induce socially optimal outcomes in singleton congestion games. This game is a reasonable model for a decentralised, anonymous system where players are required to choose from a range of identical resources, and incur costs that are increasing and convex in the total number of players sharing the same resource. The communication partition can be seen as a mechanism for inducing efficient outcomes in this context.

KEYWORDS

Equilibrium selection; Cheap-talk; Symmetry; Communication Partition; Congestion Game

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1 INTRODUCTION

Equilibrium selection in the presence of multiple equilibria has been a long-standing problem in the game theory literature. A natural solution, and one that often takes place in practice, is that players would communicate with each other before their individual actions are chosen, which enables them to coordinate on a jointly beneficial outcome.

We consider a general mechanism where players are partitioned into coalitions, and are only able to communicate with those within the same coalition. We do not explicitly model how players communicate with each other. Rather we assume that where there is an agreement (an action profile among those within the same coalition) that satisfies three conditions: envy-freeness, credibility, and Pareto-optimality, then such an agreement can be reached. If there are multiple agreements satisfying these conditions, then such communication would enable players to coordinate on one of the qualified candidates.

Because players cannot communicate across coalitions, they need to form beliefs about the behaviour of other coalitions. We propose a simple *symmetry principle*, which says that when faced with uncertainty over symmetric outcomes, players attribute the same probability to each outcome. This principle reflects the fact that there is no basis for placing one outcome as being more likely than another. This symmetry principle is applicable to the Singleton Congestion Games (SCG) where all resources are identical.

Under these assumptions, we show that in SCGs with identical resources, we can find a communication partition where players within each coalition can reach an agreement that is envy-free, credible, Pareto-optimal, and that the outcome of such agreements across all coalitions is socially optimal. These socially optimal outcomes cannot be achieved if players cannot communicate (i.e., each player is in a singleton coalition), or have unrestricted communicate (i.e., all players are in a grand coalition). Thus, the communication partition can be seen as a generalisation of these two standard setups.

Using a partition to represent a communication network between players was first introduced in [13], which differs from our work in its focus on cooperative games with transferable utility, and the role of communication as a means of negotiating for shares of the value of the coalition. Our work can also be seen as an alternative to the well-studied concept of correlated equilibrium, as developed in [3, 4], which relies on a trusted third-party mediator for coordination.

In congestion games, the seminal work of [15], and later [12], established important properties of the game. Further works on the 'price of anarchy' (PoA) of atomic congestion games (e.g., [1, 6–10]) study the gap between the worse equilibrium outcome and the socially optimal outcome. On symmetry, [14] established the result that every finite symmetric game has a symmetric mixed-strategy Nash equilibrium, which we consider to be the baseline outcome in the absence of communication as a natural consequence of the symmetry principle. We also rely on the concept of epistemic uncertainty [2, 5] to model a player's belief about the behavior of other players.

This paper briefly sets out our model, key assumptions and main findings; for a full exposition of the ideas covered in this paper, see [11].

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2 PRELIMINARIES

We consider a singleton congestion game (SCG) with n players, N, and m resources, M. Players simultaneously choose one of the resources, thus M also represents the action set of each player. Each player incurs a cost $f : \mathbb{N} \to \mathbb{R}$ that is strictly increasing and weakly convex in the total number of players choosing the same resource. I.e., given some pure-strategy profile (also referred to as an outcome) $a \coloneqq (a_1, ..., a_n) \in M^n$, the cost incurred by player i is $c_i(a) \coloneqq f(n_{a_i}(a))$, where $n_j(a)$ denotes the number of players that choose resource j in a, and $f' > 0$, $f'' \geq 0$, where these conditions characterise a increasing and convex function. The game can be specified by $G = \langle n, m, f \rangle$.

A partition π of N represents the mutually exclusive and collectively exhaustive subsets of players that can freely communicate with one another. For example, [12|34|5] represents a partition where players 1 and 2 form a communicating coalition, and similarly between players 3 and 4, while player 5 does not communicate with any other player. The partition is assumed to be common knowledge.

An SCG augmented by a communication partition game is specified by $G^\pi = \langle n, m, f, \pi \rangle$ and played over two phases: In the communication phase, players within a coalition communicate with each other for an indefinite period of time. Once players are satisfied with their communication, the game proceeds to the action phase, where players choose their actions simultaneously, unbound by any agreement reached during the communication phase.

To analyse how rational players behave in the absence of communication, we make the natural assumption that players abide by what we term the *symmetry principle*, which says that when faced with symmetric uncertainties, a player places equal probability on each outcome. This does not imply that the player expects others to play stochastic strategies, but rather that the probability reflects its own epistemic uncertainty about the action that is played by others.

Proposition 1. *Let $\mu : \mathbb{N} \to [0, 1]$ be a probability distribution on the number of players u in $-C \coloneqq N \setminus C$ choosing some resource $x \in M$, and let $g_\mu(v) \coloneqq \sum_{u=0}^{\infty} \mu(u) \cdot f(u + v)$ be the expected cost to each player if an additional v players in C chooses x. Then g_μ is strictly increasing and weakly convex.*

Under the symmetry principle, each coalition can thus behave as if they are in the augmented subgame $\langle C, m, g, \pi \rangle$ where π is the grand coalition and g some increasing and convex cost function, without consideration of the behaviour of other coalitions.

We further assume that players within a coalition will agree on a joint action and adhere to it if three conditions are satisfied: envy-freeness, credibility, and Pareto-optimality.

Definition 2. Consider a subset of players $C \subseteq N$ and a common prior μ_{-C} about the behaviour of players in $-C \coloneqq N \setminus C$.

An agreement $a_C \in M^{|C|}$ is *envy-free* under μ_{-C} if all players incur the same expected cost, i.e. for all $i, j \in C$,

$$\mathbb{E}_{\mu_{-C}}[c_i(a_C)] = \mathbb{E}_{\mu_{-C}}[c_j(a_C)]$$

An agreement $a_C \in M^{|C|}$ is *credible* under μ_{-C} if no player can benefit from a unilateral deviation, i.e., for all $i \in C, x \in M$,

$$\mathbb{E}_{\mu_{-C}}[f(n_{a_i}(a_C))] \leq \mathbb{E}_{\mu_{-C}}[f(n_k(a_C) + 1)]$$

An agreement is *Pareto-optimal* under μ_{-C} if no player can be better-off without some player being worse-off. I.e. $\nexists a'_C \in M^{|C|}$ such that, for all $i \in C$,

$$\mathbb{E}_{\mu_{-C}}[c_i(a'_C)] \leq \mathbb{E}_{\mu_{-C}}[c_i(a_C)]$$

and, for some $j \in C$,

$$\mathbb{E}_{\mu_{-C}}[c_j(a'_C)] < \mathbb{E}_{\mu_{-C}}[c_j(a_C)].$$

An agreement a_C is *covering* if as many resources are used as possible. Concretely, this means if $|C| \geq m$, then $n_x(a_C) \geq 1$ for all $x \in M$. And if $|C| < m$, then $n_x(a_C) \leq 1$ for all $x \in M$.

These conditions are connected as follows: If we want to find an agreement that is Pareto-optimal, envy-free and credible, we need only check covering agreements that are envy-free.

Proposition 3. *An agreement a_C is covering if and only if it is Pareto-optimal; a covering agreement that is envy-free is credible; and a credible agreement is a covering agreement.*

3 EFFICIENT PARTITION & CONCLUSION

Below we establish that an outcome that is *evenly distributed* is $\bar{c}$-optimal and $\hat{c}$-optimal, and show that a *balanced* partition induces outcomes that are envy-free, credible, Pareto-optimal, and evenly distributed, and are therefore $\bar{c}$-optimal and $\hat{c}$-optimal.

Definition 4 (Even distribution). A joint action $a_S \in M^{|S|}$ of a subset of players $S \subseteq N$ is *evenly distributed* if for all $x, y \in M$, we have $|n_x(a_S) - n_y(a_S)| \leq 1$.

Lemma 5 (Efficient outcomes). *An outcome $a \in M^n$ is $\bar{c}$-optimal if and only if a is evenly distributed. An outcome $a \in M^n$ is $\hat{c}$-optimal if a is evenly distributed.*

Under the symmetry principle, efficient outcomes can be induced by partitioning players in a balanced way.

Definition 6 (Balanced partition). A partition π on N is *balanced* if at most one coalition has size $|C|$ such that $|C| < m$, with the remaining coalitions having sizes such that $|C| \mod m = 0$.

Definition 7 (π-induced strategy profile). A partition π *induces* a feasible strategy profile $s \in \Delta(M^n)$ if, under the symmetry principle, every outcome a in s with positive probability is composed of credible, envy-free and Pareto-optimal agreements a_C for all $C \in \pi$.

The idea of an induced strategy profile is that, given a partition, each coalition of players can reach an agreement that is envy-free, credible and Pareto-optimal, and agreements aggregated across all coalitions can be expressed as the induced strategy profile.

Finally, we establish that a balanced partition always induces a socially optimal outcome.

Theorem 8 (Optimal partition). *A partition π on N induces a $\bar{c}$-optimal strategy profile if and only if it is balanced. A partition π on N induces a $\hat{c}$-optimal strategy profile if it is balanced.*

Player communication is a natural consideration in congestion games, where players are self-interested but wish to avoid poorly coordinated outcomes. However, allowing all players to communicate with each other may be practically infeasible nor satisfactory for the players involved. The use of an appropriately designed communication partition can simplify the coordination problem for all players concerned, and avoid envy-induced deadlocks.

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