Group-fair Facility Location Games with Externalities

Extended Abstract

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ABSTRACT

We study facility location games with externalities where agents are located on a real line and divided into groups. The cost of an agent is affected by the facility location and their group members. The goal is to design mechanisms to locate a facility to approximately optimize group-fair objectives while eliciting the agents' locations truthfully. We consider two types of group interactions: competitive and collaborative, and two group-fair objectives, minimizing the maximum total group cost and minimizing the maximum average group cost. For each scenario, we analyze classic mechanisms, presenting their approximation ratios, and introduce new mechanisms that achieve improved approximation ratios. Additionally, we establish tight lower bounds for each setting, demonstrating that our mechanisms are the best possible.

KEYWORDS

Facility Location; Group Fairness; Externality

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1 INTRODUCTION

Approximate mechanism design in facility location (facility location games) is first studied by Procaccia and Tennenholtz [12]. Recently, there is an increasing number of researchers studying the interactions of agents within groups, recognizing their significant impact on decision-making and collective outcome. Wang et al. [13], Zhou [14] studied facility location games with collaborators where the agent utility is achieved by using the facility and interacting with other group members. Peng and Zhou [11] studied facility location games with competitors where locating the facility closer to some other group members' locations may increase the cost of an agent. All of them proposed novel strategyproof mechanisms for both utilitarian and egalitarian objectives.

Motivated by the importance of ensuring group fairness and equity among groups of agents in our society, we consider the group-fair facility location games with different types of externalities, where the set of agents is partitioned into groups based on criteria (e.g., gender, race, or age) and aim to design strategyproof mechanisms to locate the facility to serve groups of agents to ensure some desired forms of group fairness and the truthfulness of agents. Zhou et al. [16] first studied classical facility location games with group-fair objectives. Zhou et al. [15] studied altruistic agents in facility location games with group-fair objectives. As we will show later, both settings are special cases of ours. Moreover, Li et al. [8] studied group-fair objectives in obnoxious facility location games where every agent wants to be far away from the facility.

1.1 Related Work

1.1.1 Facility Location Games with Externalities. Li et al. [9] first studied facility location games with externalities where agents have effects on each other. However, as they did not consider groups, they only considered the location misreporting. Moreover, they did not study the competitive relationship between agents. Zhou [14] studied facility location games with group externalities where there is one group activity with particular internal connections accessible to agents, and two types of agents are separated based on whether or not they participate in the group activity, which could be regarded as a special case of multiple groups. Wang et al. [13] extended the results to multiple groups. Both of them studied intra-group cooperation rather than competition. Peng and Zhou [11] studied intra-group competition in facility location games. All above studied classic objectives such as the social cost and the maximum cost.

1.1.2 Fairness in Facility Location Games. A growing body of research is exploring fairness in facility location games. Procaccia and Tennenholtz [12] examined the fairness objective of minimizing the maximum cost among agents. More recently, various envy-related concepts are investigated, including minimax envy [4, 6]

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and envy ratio [7, 10]. These concepts aim to minimize the maximum normalized cost difference between any pair of agents and the maximum ratios of utility for any pair of agents. A very recent study Zhou et al. [16] incorporated group fairness into the facility location games while adopting the approximate mechanism design approach. Following this, Zhou et al. [15] studied altruistic facility location games with group-fair objectives where each agent cares about their groups instead of themselves. Li et al. [8] studied obnoxious facility location games with group-fair objectives. Beyond the objective-centric mechanism design, Aziz et al. [1, 2, 3] investigated mechanism design for the proportional fairness notions (Individual Fair Share and Unanimous Fair Share) for both classic and obnoxious facility location games.

More works in facility location games can be found in a recent survey [5].

2 PROBLEM STATEMENT

Let $N = \{1, 2, \ldots, n\}$ be a set of agents on a normalized closed interval $I = [0, 1]$. Each agent $i \in N$ has a location x_i and belongs to a group $g_i \in [m]$. A collection of agents with $g_i = j$ is denoted as G_j, so we have $\bigcup_{j\in[m]} G_j = N$ and $G_{j1} \cap G_{j2} = \emptyset$ for all $j1 \neq j2$. We denote the profile of agent i as $r_i = (x_i, g_i)$ and denote the profile set as $\mathbf{r} = \{r_1, \cdots, r_n\}$. A mechanism is a function f which maps a profile set $\mathbf{r}$ to a facility location $y \in I$. We take $d(a, b) = |a - b|$ to represent the distance between a and b. The externality factor within group G_j is denoted by α_j.

Facility Location Games with Collaborators. We first consider the case of cooperative interactions within groups. The cost of agent i is defined as $c_i^p(y, \mathbf{r}) = d(y, x_i) + \alpha_{g_i} \sum_{k\in G_{g_i};k\neq i} d(y, x_k)$, where $\alpha_j \in [0, 1]$ for all $j \in [m]$. From this definition, agent i's cost will decrease if the facility is closer to the majority of the group members as opposed to just being close to himself. If $\alpha_j = 0$ for all $j \in [m]$, this setting coincides with classic facility location games [12]. If $\alpha_j = 1$ for all $j \in [m]$, the cost function degenerates to the altruistic total cost in Zhou et al. [15].

Facility Location Games with Competitors. In the case, the relationship between group members is competitive. The cost of agent i is defined as $c_i^n(y, \mathbf{r}) = d(y, x_i) + \alpha_{g_i} \sum_{k\in G_{g_i};k\neq i}(1 - d(y, x_k))$, where $\alpha_j \in [0, 1/(|G_j|-1)]$ for all α_j,which implies that the cost of an agent will be incurred by not only going to the facility (the first term), but also competing with their group members (the second term).

The goal is to design strategyproof mechanisms in both games. To simplify the description, we use c_i instead of c_i^p and c_i^n in the remaining part of this section.

Definition 1. *A mechanism f is **strategyproof** (SP) if an agent can never benefit by reporting a false location, regardless the strategies of the other agents. More formally, given any profile set $\mathbf{r} = \{r_1, ..., r_n\}$ and any profile set $\mathbf{r}' = \{r'_1, ..., r'_n\}$ reported by n agents where $r'_i = (x'_i, g_i)$. We have $c_i(f(r_i, \mathbf{r}'_{-i}), \mathbf{r}) \leq c_i(f(\mathbf{r}'), \mathbf{r})$ where $\mathbf{r}'_{-i}$ is a collection of reported profiles of n agents except agent i.*

Group-fair Objectives. We consider two group-fair objectives proposed by Zhou et al. [16], the maximum total group cost (mtgc), $\mathrm{mtgc}(y, \mathbf{r}) = \max_{j\in[m]} \left\{\sum_{i\in G_j} c_i(y, \mathbf{r})\right\}$, and the maximum average group cost (magc), $\mathrm{magc}(y, \mathbf{r}) = \max_{j\in[m]} \left\{\frac{\sum_{i\in G_j} c_i(y,\mathbf{r})}{|G_j|}\right\}$. We measure the performance of a mechanism f by comparing the objective that f achieves and the objective achieved by the optimal solution. If there exists a number α such that for any profile set $\mathbf{r}$, the output from f is within α times the objective achieved by the optimal solution, then we say the approximation ratio of f is α.

2.1 Our Contribution

Our main results are summarized in Table 1.

Facility Location Games with Competitors. For minimizing the mtgc, we first establish that any deterministic strategyproof mechanism has an approximation ratio of at least 2. We then show the approximation ratios for several classic strategyproof mechanisms. Specifically, locating the facility at the leftmost agent's location (Left-M) has an approximation ratio of n. Locating the facility at the median agent location (Med-M) achieves an approximation ratio of m. Locating the facility at the median agent in the largest group (Major-M) attains an approximation ratio of 3. Building on these results, we propose a novel strategyproof mechanism (CGA-M) that not only leverages the group information but also takes advantage of agents who are not the group median, yielding an improved approximation ratio of 2.

For minimizing the magc, we show that Left-M has an approximation ratio of n, while both Med-M and Major-M achieve an approximation ratio of 3. We then adapt CGA-M to this setting by using normalization, which attains an approximation ratio of 2. Finally, we establish a lower bound of 2 for any deterministic strategyproof mechanism for this objective, demonstrating that CGA-M is the best possible mechanism for this objective.

Facility Location Games with Collaborators: We first show that there is no strategyproof mechanism with a bounded approximation ratio for both group-fair objectives. Then we consider the strategyproofness in equilibrium (SP-E) and show that neither Left-M nor Med-M satisfies SP-E. Moreover, Major-M has an approximation ratio of $n + 1$ for both objectives.

For minimizing mtgc, we propose a group-based SP-E mechanism by carefully setting weights for each group, which achieves an approximation ratio of 3. We complement our result by establishing a lower bound of 3 for this objective.

For minimizing magc, we establish a lower bound of 3 for any deterministic SP-E mechanism. We complement our result by introducing a new group-based SP-E mechanism that achieves an approximation ratio of 3.

Problem	Objective	Upper bound	Lower Bound
Competitor	mtgc	2	2
	magc	2	2
Collaborator	mtgc	3	3
	magc	3	3

Table 1: Results of group-fair facility location games with externalities.

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