Liquid Welfare and Revenue Monotonicity in Adaptive Clinching Auctions

Extended Abstract

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ABSTRACT

This study explores the monotonicity of adaptive clinching auctions – a key mechanism in budget-constrained auctions – with respect to fluctuations in the number of bidders. Specifically, we investigate how the addition of new bidders affect efficiency and revenue. In a symmetric setting, where all bidders have equal budgets, we show that while the allocated goods and payments for many bidders decrease, overall both liquid welfare and revenue weakly increase. Our analysis also extends to scenarios where bidders arrive online during the auction. In contrast, for asymmetric budgets, we provide counterexamples showing that these monotonicity properties no longer hold. These findings contribute to a better theoretical understanding of budget-constrained auctions and offer insights into the behavior of adaptive clinching auctions in social networks, where new bidders emerge through information diffusion.

KEYWORDS

Auctions; Budget Constraints; Social Network; Clinching Auctions; Monotonicity; Liquid Welfare

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1 INTRODUCTION

Budget-constrained auctions are a fundamental setting in auction theory, particularly for real-world applications such as ad auctions. The *adaptive clinching auction* proposed by Dobzinski et al. [3] has played an important role in this context. Built on Ausubel's clinching framework [1], it is still the only budget-feasible mechanism that satisfies *incentive compatibility* (IC), *individual rationality* (IR), and *Pareto optimality* (PO). Consequently, clinching auctions are now widely accepted as standard mechanisms in auctions with budgets and have inspired extensive additional research.

Recently, auctions with *information diffusion* [7] have gained significant attention, as transactions through *social networks* have become increasingly common. These auctions are characterized by incentive design that encourages participants to spread information.

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Table 1: Summary of studies on the monotonicity of adaptive clinching auctions

	Input Parameter Change	Monotonicity Focus
Bhattacharya et al. [2]	Decrease in Budget	Utility
Goel et al. [5]	Increase in Supply	Allocation
This Study	Addition of Bidders	LW and Revenue

Although the participation of new bidders is expected to improve outcomes, it also increases competition. Following this research trend, Xiao et al. [9] introduced budget-constrained auctions in social networks and proposed an ascending auction that builds on the adaptive clinching auction. In their framework, when a bidder drops out, their neighboring potential bidders are invited to participate, which can be viewed as the *online arrival* of new bidders. The incentive design for information diffusion is effective in their mechanism, satisfying IC, IR, and non-wastefulness – an efficiency concept introduced by Kawasaki et al. [6].

While non-wastefulness is valuable, it is relatively weak, thereby allowing for further improvement in the theoretical guarantees of the mechanism. Ideally, as more bidders engage through information diffusion, both efficiency and revenue should increase, ensuring that the spread of information leads to better outcomes. To assess whether efficiency and revenue remain monotonic with the arrival of new bidders, the effect of participation on clinching auctions must be analyzed. This question is not only practically and fundamentally important, but it also extends previous research on the monotonicity of clinching auctions by Bhattacharya et al. [2] and Goel et al. [5], as summarized in Table 1. In this study, we investigate the monotonicity of adaptive clinching auctions in the presence of new bidders.

2 OUR MODEL

Consider a market with n bidders ($n \geq 2$) and a seller selling a single unit of a divisible good. We often denote a singleton $\{i\}$ by i and a set $\{1, 2, \dots, k\}$ by $[k]$. Each bidder i has a valuation of $v_i \in \mathbf{R}_+$ for one unit of the good and strategically reports a bid $v'_i \in \mathbf{R}_+$. Each bidder also has a public budget of $B_i \in \mathbf{R}_+$, which represents the maximum possible payment. Define $N := [n]$, $v := \{v_i\}_{i \in N}$, $v' := \{v'_i\}_{i \in N}$, and $B := \{B_i\}_{i \in N}$. Bidders' valuations are assumed to be different and they are listed in descending order.

An *outcome* (x, π) consists of an *allocation* $x := (x_i)_{i \in N}$ and a *payment* $\pi := (\pi_i)_{i \in N}$, where $x_i \in \mathbf{R}_+$ is the amount of good allocated to bidder i and $\pi_i \in \mathbf{R}_+$ is the payment of i.[1] The outcome must satisfy the following conditions: $\sum_{i\in N} x_i = 1$ and $\pi_i \leq B_i$ for each $i \in N$. A *mechanism* $\mathcal{M}$ is a map that determines the outcome based on the bids. The *utility* u_i of bidder i is given by $u_i(v', \mathcal{M}) := v_i x_i - \pi_i$ if $\pi_i \leq B_i$ and $-\infty$ otherwise, where (x, π) is the outcome from the mechanism $\mathcal{M}$ under v'.

We consider a budget-feasible mechanism $\mathcal{M}$ that satisfies the following desirable properties:

- **Incentive Compatibility (IC):** It is the best strategy for each bidder to report their true valuation, i.e., $u_i((v_i, v'_{-i}), \mathcal{M}) \geq u_i(v', \mathcal{M})$ for each i and v'.
- **Individual Rationality (IR):** There is a bid such that each bidder receives non-negative utility. If IC holds, IR is expressed as $u_i((v_i, v'_{-i}), \mathcal{M}) \geq 0$ for each i and v'.

These properties ensure truthful bidding and voluntary participation of bidders, respectively. Given an outcome (x, π), the objectives focused in this paper are described as follows:

- **Social Welfare (SW):** A standard efficiency objective, defined by $\mathrm{SW}(x) := \sum_{i\in N} v_i x_i$. It represents the sum of willingness-to-pay (i.e., valuations) of bidders for their allocated good.
- **Liquid Welfare (LW):** A natural extension of SW that incorporates the ability-to-pay (i.e., budgets), defined by $\mathrm{LW}(x) := \sum_{i\in N} \min(v_i x_i, B_i)$. It represents the sum of admissibility-to-pay of bidders for their allocated good.
- **Revenue:** The seller's revenue, denoted by $\mathrm{REV}(\pi) := \sum_{i\in N} \pi_i$.

3 OUR RESULTS

We consider the continuous form of the adaptive clinching auction by Bhattacharya et al. [2]. This mechanism is an ascending auction inspired by Ausubel's clinching framework [1]. Intuitively, in this mechanism, the price clock gradually increases from zero, and at each price, each bidder wins (or *clinches*) the good if the total demand of other bidders is less than the amount of remaining good. For each bidder i, let $x_i(p)$ denote the amount of their allocated good and $b_i(p)$ denote their remaining budget at price p in the mechanism. Initially, the former is set to zero and the latter to B_i. They are updated according to their transactions.

Consider a symmetric setting where all bidders have equal budgets, i.e., $B_i = \beta$ for each i and some $\beta > 0$. We fix the input and let $(x^{\mathrm{f}}, \pi^{\mathrm{f}})$ be the final outcome by the mechanism. Our first result is to provide an explicit formula for the allocation and remaining budget at any price using the *clinching interval* [4], the price interval where the good is actually traded. Define p_s and p_f by $p_s := \inf\{p\colon \exists i \in N,\ x_i(p) > 0\}$ and $p_f := \inf\{p\colon \sum_{i\in N} x_i(p) = 1\}$. Also, define $\kappa := |\{i\colon \pi_i^{\mathrm{f}} = \beta\}| + 1$. Then, the following holds:

Theorem 3.1. *For each $i \in [\kappa]$, if $p_s < p_f$, then it holds*

$$x_i(p) = \frac{1}{\kappa} - \frac{(\kappa-1)(\kappa\beta - p_s)(p_s)^{\kappa-1} p^{-\kappa}}{\kappa} \quad (p_s \leq p < p_f),$$
$$b_i(p) = (\kappa\beta - p_s)(p_s)^{\kappa-1} p^{-(\kappa-1)} \quad (p_s \leq p < p_f).$$

Combined with the existing structural property [2], it is shown that the execution of the mechanism is completely described by the triple (p_s, p_f, κ). This result is of independent interest.

[1] For a vector $x \in \mathbf{R}_+^N$, we often denote $x(i)$ by x_i, and write as $x = (x_i)_{i\in N}$. We also define x_{-i} by $x_{-i} := (x_j)_{j\in N\setminus i}$.

Now suppose a new bidder θ is added to the auction at the start. Let $(\tilde{x}^{\mathrm{f}}, \tilde{\pi}^{\mathrm{f}})$ be the final outcome by the mechanism under the same input with the addition of the new bidder. We investigate the change in these outcomes and establish their relationship:

Theorem 3.2. *In the adaptive clinching auction [2] under the symmetric setting, the following relationship holds:*

(i) For each bidder $i \in N \setminus \kappa$, it holds $x_i^{\mathrm{f}} \geq \tilde{x}_i^{\mathrm{f}}$ and $\pi_i^{\mathrm{f}} \geq \tilde{\pi}_i^{\mathrm{f}}$.
(ii) It holds $\mathrm{LW}(x^{\mathrm{f}}) \leq \mathrm{LW}(\tilde{x}^{\mathrm{f}})$ and $\mathrm{REV}(\pi^{\mathrm{f}}) \leq \mathrm{REV}(\tilde{\pi}^{\mathrm{f}})$.

Property (i) captures the monotonicity of each bidder's allocation and payment, while property (ii) reflects the monotonicity of key auction outcomes, such as LW and revenue. By repeatedly applying this result, it can be generalized to the addition of multiple bidders. In the proof, we detected changes in the clinching interval and use Theorem 3.1 to compare the outcomes.

Our next focus is to understand how broadly the monotonicity holds across different scenarios. Consider the following symmetric setting where new bidders $\Theta := \{\theta_1, \theta_2, \ldots, \theta_t\}$ arrive online:

- Every new bidder θ_k arrives at $p = \gamma_k$ and immediately reports their bid v'_{θ_k} $(> \gamma_k)$.
- All bidders in N^Θ have equal budgets, i.e., $B_i = \beta$ $(i \in N^\Theta)$.
- The auctioneer has no prior knowledge of $\Gamma := \{\gamma_1, \gamma_2, \ldots, \gamma_t\}$.

Note that the mechanism can accommodate the online arrival of bidders and still satisfies IC and IR. Based on this, we can assume that bidders report their valuations truthfully as their bids. Then, we show that Theorem 3.2 can even be extended to this setting:

Theorem 3.3. *Theorem 3.2 remains valid even when new bidders in Θ arrive online.*

This result suggests that information diffusion improves efficiency and revenue in the adaptive clinching auction with symmetric bidders. In this online setting, the remaining budgets of bidders might be different and thus Theorem 3.1 cannot be directly applied. In the proof, we extended this theorem to incorporate with the online arrival of bidders in some critical cases.

We further explore the asymmetric setting where the budgets of bidders are different. In this scenario, we must consider the addition of two parameters: the valuation and the budget of the new bidder. The interaction of these parameters influences the outcome in various ways. Notably, we find counterexamples showing that neither each bidder's outcome nor LW is monotonic. In case of revenue monotonicity, the current evidence suggests that it is still an open question. Thus, we provide a counterexample for the case of *indivisible* goods, which is more relevant for applications.

For more details of our results, please check the full paper [8].

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