Environmental Policies within Cournot Oligopoly

Extended Abstract

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ABSTRACT

We consider how to effectively regulate environmental policies with clear penalties and rewards, through a game-theoretical point of view. To this end, we use social welfare as the primary metric for evaluation. We demonstrate that the best possible social welfare can be achieved through policies that incorporate both linear taxation and subsidies in a Cournot competition model. To make it constructive, we propose efficient algorithms to find optimal policies in a Cournot competition model. Our work can be seen as the first step towards obtaining the optimal environmental policy through the lens of computation.

KEYWORDS

Cournot Game; Social Welfare; Environmental Policies

ACM Reference Format:
Liang Shan, Zhengyang Liu, Haoqaing Huang, and Zihe Wang ✉. 2025. Environmental Policies within Cournot Oligopoly: Extended Abstract. In *Proc. of the 24th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2025), Detroit, Michigan, USA, May 19 – 23, 2025*, IFAAMAS, 3 pages.

1 INTRODUCTION

The interplay between economic development and environmental impact has been a critical focus since Pigou's early work [15]. As industries grow, managing environmental consequences becomes increasingly important [1]. Two regulatory tools in environmental policy are emission taxes and Cap-and-Trade (CAT) systems.

Emission taxes, widely adopted globally [2, 10, 13, 17], increase firms' costs for emissions, incentivizing them to reduce pollution. Meanwhile, CAT set an overall pollution cap and allow firms to trade emission permits, as seen in initiatives like the U.S. Acid Rain Program and the EU Emissions Trading System [1]. However, CAT often faces fairness issues due to free initial permit allocations.

This study focuses on taxation policies within the Cournot model, where firms differ in costs and emissions. Previous research, from two-firm scenarios [3, 17, 18] to multi-firm analyses [1, 5, 6, 9, 16], has explored environmental policies. While heterogeneous taxes [6, 16] may raise fairness concerns, homogeneous taxes [1, 8] offer simpler alternatives but often neglect subsidies.

We propose combining taxes with subsidies, a common governmental tool [6, 11, 14]. For instance, subsidies are provided for electric vehicles in China [4] and semiconductor manufacturing in the U.S. [12]. Our approach proves that joint tax-and-subsidy schemes maximize social welfare, even with linear taxation, and provides efficient algorithms for identifying optimal policies under different subsidy settings. Building on prior studies, we rigorously compare policies and establish tight upper bounds for welfare ratios across subsidy structures. This framework demonstrates the efficiency of combined strategies in achieving optimal outcomes.

Proc. of the 24th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2025), Y. Vorobeychik, S. Das, A. Nowé (eds.), May 19 – 23, 2025, Detroit, Michigan, USA. © 2025 International Foundation for Autonomous Agents and Multiagent Systems (www.ifaamas.org).

2 PRELIMINARIES

We consider a market with n utility-maximizing firms and a pollution regulator. Each firm $i \in [n]$ operates with a linear inverse demand function $P_i(\boldsymbol{q}) = A - q_i - r\sum_{j\neq i} q_j$, where A denotes market size, $\boldsymbol{q} = (q_1, \ldots, q_n)$ represents production quantities, and $r \in [0,1]$ measures product substitution. A higher r indicates greater homogeneity among products. A firm survives if $q_i > 0$.

Firms are heterogeneous in their production costs, denoted by $c_1, \ldots, c_n$, and pollution emissions per unit of production, $e_1, \ldots, e_n$. Pollution taxes, $\mathsf{tax}(e_i q_i)$, are imposed based on emissions and are modeled as non-decreasing, convex functions. Simultaneously, firms receive subsidies proportional to production, represented as $s_i q_i$, where s_i is the subsidy per unit. The utility for firm i is:

$$u_i(\boldsymbol{q}) := q_i(P_i(\boldsymbol{q}) - c_i) - \mathsf{tax}(e_i q_i) + s_i q_i. \quad (1)$$

The government sets pollution taxes and subsidies to maximize social welfare, which incorporates environmental damage and consumer surplus. Environmental damage, caused by pollution from all firms, is defined as: $\mathsf{ED}(\boldsymbol{q}) = \alpha\left(\sum_{i=1}^n e_i q_i\right)^2, \alpha \in [0,1]$. Consumer surplus, denoted $\mathsf{CS}(\boldsymbol{q})$, measures the aggregate difference between consumers' willingness to pay and the price. It is given by: $\mathsf{CS}(\boldsymbol{q}) = \frac{r}{2}\left(\sum_{i=1}^n q_i\right)^2 + \frac{1-r}{2}\sum_{i=1}^n q_i^2$. Social welfare, $\mathsf{SW}(\boldsymbol{q})$, integrates production surplus, consumer surplus, pollution tax revenue, subsidies, and environmental damage [1, 19, 20]:

$$\mathsf{SW}(\boldsymbol{q}) = \sum_{i=1}^n (u_i(\boldsymbol{q}) + \mathsf{tax}(e_i q_i) - s_i q_i) + \mathsf{CS}(\boldsymbol{q}) - \mathsf{ED}(\boldsymbol{q}). \quad (2)$$

Since tax payments and subsidies are internal transfers between firms and the government, they are excluded from the final social welfare calculation. Finally, we establish the following result:

Theorem 2.1. *If* $\mathrm{tax}(\cdot)$ *is non-decreasing and convex, a unique Nash equilibrium exists.*

3 EFFECTIVENESS OF LINEAR TAXES AND SUBSIDIES

This section demonstrates that the highest possible social welfare can be achieved through a combination of taxes and subsidies. To characterize this, we consider a scenario where the government fully controls the production quantities q_i of all firms, bypassing firms' independent utility maximization. Prices are determined by the market's inverse demand function, and the government solves the following optimization problem, when $q_i \geq 0, \forall i \in [n]$:

$$\max \sum_{i=1}^{n} q_i\Big(A - c_i\Big) - \frac{r}{2}\Big(\sum_{i=1}^{n} q_i\Big)^2 - \frac{1-r}{2}\sum_{i=1}^{n} q_i^2 - \alpha\Big(\sum_{i=1}^{n} e_i q_i\Big)^2,$$

where $q_1, q_2, \ldots, q_n$ are decision variables. The optimal value of this problem, denoted by $\mathrm{SW}_{T\&S}$, serves as an upper bound for social welfare under any regulatory scheme. We prove that $\mathrm{SW}_{T\&S}$ can be realized with appropriately designed linear taxes and subsidies.

Theorem 3.1. *Social welfare* $\mathrm{SW}_{T\&S}$ *can be achieved if the government adopts a linear tax rate equal to the marginal environmental damage and an optimal subsidy rate.*

Theorem 3.1 shows that linear taxes and subsidies are sufficient to achieve maximum social welfare. Setting the tax equal to marginal environmental damage aligns firms' utilities with environmental objectives. Additionally, the optimal subsidy s_i^* ensures that a firm's production benefits the welfare of other firms and consumer surplus: $\frac{\partial}{\partial q_i}\Big(\sum_{j\neq i} u_j(\boldsymbol{q}) + CS(\boldsymbol{q})\Big) = s_i^*, \forall i$. By integrating these elements, the policy harmonizes individual firms' interests with social welfare. While tiered tax structures exist in practice [1, 7, 19], our focus is on implementing a *linear* tax scheme.

4 OPTIMAL POLICY

We analyze the optimal policy using linear taxes and subsidies. When the substitution parameter r is low, firms' production has minimal impact on each other's prices. However, for homogeneous products ($r = 1$), uniform subsidies are more equitable. We derive optimal designs for both uniform and non-uniform subsidies.

4.1 Optimal Policy with Subsidies

To determine the optimal policy, we solve the problem introduced above, a quadratic optimization problem, using the KKT conditions to identify the set of surviving firms. By enumerating all possible sets, the solution is obtained efficiently.

Theorem 4.1. *Given production costs* c_i *and pollution emissions* e_i*, the optimal policy and social welfare* $\mathrm{SW}_{T\&S}$ *can be computed in* $O(n^3)$ *time.*

4.2 Optimal Policy with Uniform Subsidies

When restricted to uniform subsidies, firms' actions form a Nash equilibrium as per Theorem 2.1. The optimization problem becomes:

$$\begin{aligned} &\max \mathrm{SW}(\boldsymbol{q}) \\ &\quad \text{s.t. } u_i(\boldsymbol{q}) \geq u_i(q_i', \boldsymbol{q}_{-i}), \forall q_i' \geq 0,\ \forall i, \\ &\qquad q_i \geq 0,\ \forall i. \end{aligned}$$

Here, $\mathrm{SW}(\boldsymbol{q})$ and $u_i(\boldsymbol{q})$ are defined in Eqs. (2) and (1), respectively.

Definition 4.2 (Sublevel Set). Set B is a *sublevel set* of an array $(z[i])_{i\in[n]}$ if there exists θ such that $B = \{i : z[i] \leq \theta\}$.

The indices of surviving firms correspond to a sublevel set of Γ_t for some t.

Theorem 4.3. *Given* c_i *and* e_i*, the optimal taxes and uniform subsidies policy can be computed in* $O(n^3)$ *time.*

For tax-only policies ($s = 0$), the solution remains efficient. For subsidy-only ($t = 0$), fewer firms survive, reducing complexity.

Corollary 4.4. *The optimal tax-only policy can be found in* $O(n^3)$ *time.*

Corollary 4.5. *The optimal subsidy-only policy can be found in* $O(n^2)$ *time.*

5 APPROXIMATION GUARANTEES

When both taxes and subsidies are used, the government achieves better outcomes than with taxes alone, as subsidies lower firms' costs, reduce prices, attract buyers, and enhance social welfare. This section quantifies the effectiveness of uniform subsidies, providing theoretical guarantees and worst-case approximation ratios. To derive the upper bound on $\frac{\mathrm{SW}_{T\&US}}{\mathrm{SW}_T}$, we characterize cost profiles $(c_i)_{i\in[n]}$ under specific conditions. Firms are labeled such that $c_1 \leq \cdots \leq c_n$, and in the extreme case where $\frac{\mathrm{SW}_{T\&US}}{\mathrm{SW}_T}$ is maximized, all firms in the survival set E share identical production costs.

Lemma 5.1. *To maximize* $\frac{\mathrm{SW}_{T\&US}}{\mathrm{SW}_T}$*, we have* $E \subseteq E'$*. For the case where* $E \subsetneq E'$*,* $|E' - E| - 1$ *firms in* $E' \setminus E$ *have cost per unit of production equaling to* $\frac{(2-r)A + \sum_{i\in E} c_i}{2-r+|E|}$*, and the other firm in* $E' \setminus E$ *has cost no less than* $\frac{(2-r)A + \sum_{i\in E} c_i}{2-r+|E|}$*.*

Theorem 5.2. *If* $e_i = 0$ *for all firms, then* $\frac{\mathrm{SW}_{T\&US}}{\mathrm{SW}_T} \leq \frac{4}{3}$*.*

For $\frac{\mathrm{SW}_{T\&S}}{\mathrm{SW}_{T\&US}}$ under $e_i = 0$ and $r = 1$, we analyze cost profiles maximizing this ratio.

Lemma 5.3. *When* $e_i = 0$ *and* $r = 1$*, all firms in* E' *except the one with the lowest production cost share identical costs.*

Theorem 5.4. *When* $e_i = 0$ *and* $r = 1$*, we have* $\frac{\mathrm{SW}_{T\&S}}{\mathrm{SW}_{T\&US}} \leq \frac{3}{2}$ *as* $n \to \infty$*.*

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (Grant No. 62172422 and No. 62472029) and Key Laboratory of Interdisciplinary Research of Computation and Economics (Shanghai University of Finance and Economics), Ministry of Education.

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