Different Models for Fair and Efficient Resource Allocation

Doctoral Consortium

Bin Sun
Institut für Informatik, TU Clausthal
Clausthal-Zellerfeld, Germany
bin.sun@tu-clausthal.de

ABSTRACT

Computing fair and efficient allocations is a very important topic in the area of fair allocation of indivisible resources. There are different models of resource allocation, each applicable to distinct contexts. My research focuses on designing and analyzing various allocation models that are tailored to specific scenarios, as well as their fairness and efficiency. My current research interests include several areas: allocations with costs, allocations and groups, allocations and externalities, allocations allowing sharing, and allocations allowing selling.

KEYWORDS

resource allocation, fairness, social welfare, indivisible resources, computational complexity, algorithms

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1 INTRODUCTION

In general, an allocation consists of a set of resources and a set of agents. In addition, each agent expresses their preference for each resource usually through a utility function. An allocation assigns each agent a subset of resources, where no resource is assigned to more than one agent.

Computing fair and efficient allocations of indivisible resources is a crucial issue with diverse applications across various fields. However, there are multiple definitions of fairness or efficiency. Among the most notable properties are envy-freeness [8] and Pareto-efficiency [9]. Envy-freeness ensures that no agent strictly prefers the resources that are allocated to a different agent over their own. In addition, an allocation is Pareto-efficient when it is impossible to make any agent happier without making at least one agent less happy.

The mathematical models employed for allocation exhibit a considerable degree of diversity. Different allocation scenarios have been explored, including the division of divisible goods [13], the challenges of fair allocation under ternary valuations [15], and achieving Pareto efficient and envy-free resource allocation with few agents or resources [5]. In addition, an allocation may involve shared resources [11] or conflicting items [16] i.e. items that should not be allocated to the same agent.

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2 RESEARCH DIRECTIONS

I will delve into several ideas for researching resource allocation.

2.1 Allocations with Costs

The study of resource allocation and its associated costs is very important, as the actual distribution may require time or effort, which should not be seen as limitless. Costs can vary significantly depending on factors like location and distance. For instance, supercomputers generate substantial heat, requiring effective cooling mechanisms. Placing them in colder regions with abundant water resources can lead to lower maintenance costs compared to warm, arid areas. Even if warmer regions have a greater demand for such resources, allocating them to cooler regions might offer a more practical and economically sensible solution. In Bredereck et al. [10], we discussed a new model for the adjusting winner winner considering that selling resources will result in costs.

To manage these complexities, we can design different frameworks that account for costs in resource allocation. One approach involves translating the cost of a resource into a negative utility, which can be balanced by the benefits of using that resource. Alternatively, another approach bypasses this translation and instead sets limits on the total allowable costs in the allocation process. This way, the focus is on keeping costs within acceptable boundaries, promoting a fair and efficient allocation strategy.

2.2 Allocations and Groups

We can also consider that allocation may occur on multiple levels. In practice, agents might belong to different groups. There are a branch of papers discussing about fair allocations among groups, exploring how fairness is defined among groups in different ways. [1, 4, 14, 19]. In this context, I introduce a new perspective: We compare intra-group envy-freeness with global envy-freeness. The former refers to a situation where members within the same group do not experience envy towards one another, while the latter ensures that there is no envy among all members across different groups. The idea of intra-group envy-freeness is similar to the graph-envy-freeness mentioned in Bredereck et al. [12], as both examine envy-freeness among certain pairs of agents, such as inside groups or neighbors, rather than universally across all pairs.

We examine a collection of groups, each with distinct agents and resources. Firstly, we seek to achieve fair and efficient resource allocations within each group. Secondly, we also explore a scenario where a centralized authority mandates resource redistribution across groups to enforce global fairness. This can lead to utility losses for group members as they lose access to their established resources, potentially causing inefficiencies.

For example, within a country, the economic development levels of different states can vary, prompting the federal government to redistribute resources that are indivisible yet transferable, such as medical units and research institutes. However, not all resources are suitable for relocation—consider a research institute focused on wind energy that clearly cannot be effectively moved to a region with little to no wind. Such mismatches can lead to inefficiencies and losses. Consequently, pursuing global fairness versus intra-group fairness can lead to different levels of total wealth loss, highlighting the complexity and challenges in achieving each. This is an example of how we can only seek relative fairness among members across different groups, rather than achieving absolute envy-freeness, thereby necessitating a primary focus on fairness within each group first.

2.3 Allocations with Externalities

Resource allocation with externalities has been discussed in many papers [17, 18, 20]. In this section, I will focus on the following approaches.

This approach, as discussed was Aziz et al. [3], considers that the allocation itself does not generate flexible, divisible profits. Instead, assigning a resource to one individual can lead to positive or negative utility for others. Here, allocating an item to one person affects the utility of all individuals differently. We will revisit the concepts of fairness and efficiency based on this premise. An illustrative example can be easily provided: a city might prioritize affordable energy and therefore strongly support the establishment of a nuclear power plant within its limits. However, neighboring cities may have concerns regarding the safety of such a facility. Consequently, while constructing the nuclear power plant would generate significant utility for the city in question, it would result in negative utility for the surrounding cities. In this situation, to achieve fairness and efficiency, it's important to consider the negative impact caused by neighbors. In some works like Aziz et al. [2], Bogomolnaia et al. [7], resources with negative utilities are referred to as chores. Here, we consider resources which can bring positive utility to some agents but negative utilities to others.

2.4 Allocations Allowing Sharing

Here, we allow a single resource to be allocated to multiple individuals, up to a certain limit. This means that a fixed number of people can share the same resource. Under this premise, we will revisit the notions of fairness and efficiency. This scenario is extremely common in real life; for example, resources purchased by a research group, such as computers, small water heaters, and small printers, are often shared among several people. Sharing resources clearly improves their utilization efficiency. Therefore, developing a rational allocation strategy based on this concept is undoubtedly an important area of study.

Bredereck et al. [11] introduced a new research idea. This idea aims to improve allocation by allowing resources to be shared with social network neighbors of the resource owners, without reallocating the resources. This model allows agents to form pairs which may share a limited number of resources. This paper analyzes the complexity of allocation allowing resources, finding that while improving social welfare (measured in both the egalitarian and utilitarian ways) via sharing can be achieved in polynomial time, reducing envious agents remains NP-hard. In addition, sharing a resource can come with costs or result in a loss of utility.

2.5 Allocations Allowing Selling

In this section, we consider the scenario where the sale of resources is permitted. When resources are difficult to divide fairly, a natural approach is to sell a portion of the resources to obtain a reward. This reward can then be converted into positive utilities according to a specific rate. Of course, selling resources incurs time costs and potential losses, so there are limitations on the quantity of resources that can be sold or the amount of reward obtained. This idea was discussed in Bredereck et al. [10].

Consider the following example: Three artists, Adam, Bill, and Chad, have decided to divide their ten collaborative paintings, denoted as $\{r_1, \ldots, r_{10}\}$, after a disagreement. Among these paintings, r_1 is exceptionally valuable, while r_{10} is considered less successful. The valuations of the seven paintings by the three artists are shown in Table 1. It is easy to see that selling r_{10} is a good idea even if its reward is 0. In this case, we can assign r_1 to Adam, $r_2, \ldots, r_5$ to Bill and $r_6, \ldots, r_9$ to Chad. Clearly, aside from the resource r_{10} being sold, all other resources were allocated to the agent who values them the most. Furthermore, this allocation is envy-free and not Pareto-dominated by any other envy-free allocations.

Table 1: Valuations of the paintings by 3 Artists

	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}
Adam	57	13	13	13	13	13	13	13	13	1
Bill	60	15	15	15	15	10	10	10	10	2
Chad	60	10	10	10	10	15	15	15	15	2
reward	40	5	5	5	5	5	5	5	5	0

This topic shares similarities with some studies [6] on resource allocation that permit donation (without rewards); however, our research delves deeper into the realm of divisible rewards. By redistributing the reward, we can enhance the allocation to further promote fairness and efficiency.

3 CONCLUSION

There are numerous real-world applications for investigating different models of resource allocation. In my research, I will focus on how algorithms distribute indivisible resources among agents to achieve both fairness and efficiency, with an emphasis on analyzing computational complexity. It's important to recognize that different models can exhibit vastly different computational complexities and approximation properties, offering a rich field for theoretical exploration. In many cases, finding allocations that simultaneously satisfy fairness and efficiency criteria is inherently NP-hard. Thus, it's often necessary to relax the strict definitions of some concepts to develop algorithms that operate within polynomial time. Consequently, the way we define fairness and efficiency becomes a crucial aspect of this study, influencing both the practical outcomes and theoretical advancements in the field.

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