Agent-based Modeling and Simulation of Ambiguity in Catastrophe Insurance Markets

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ABSTRACT

Pricing covers for catastrophes is challenging for insurers due to uncertainty in loss probabilities. This paper addresses this so-called ambiguity problem in competitive catastrophe insurance markets through three key approaches. First, it introduces ambiguity in premium pricing and capital holdings. Second, it develops an Agent-based Model simulator to mimic general insurance markets and the Lloyd's market. Third, it applies Empirical Game-Theoretical Analysis to explore insurers' ambiguity preferences in different markets. The study evaluates the effects of ambiguity by analyzing their impact on individual companies, differences between small and large companies, and overall market performance. Simulation results reveal that the simulator effectively captures underwriting cycles and insurers' strategic shifts following catastrophes. In markets with equally sized insurers, competition mitigates the negative effects of ambiguity by stabilizing premiums and increasing the number of underwritten risks. In markets with varying-sized insurers, large insurers gain market power while small insurers adopt aggressive ambiguity strategies to compete. In contrast, Lloyd's lead-follow mechanism encourages conservative ambiguity strategies and reduces bankruptcy.

KEYWORDS

Insurance Markets; Agent-based Modeling; Agent-based Simulation; Empirical Game-Theoretic Analysis

ACM Reference Format:

Yu Bi, Lingxiao Zhao, Jinyun Tong, Zhe Feng, and Carmine Ventre. 2025. Agent-based Modeling and Simulation of Ambiguity in Catastrophe Insurance Markets. In *Proc. of the 24th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2025), Detroit, Michigan, USA, May 19 – 23, 2025*, IFAAMAS, 9 pages.

1 INTRODUCTION

Catastrophes, such as hurricanes, terrorist attacks, and financial crises, are low frequency and high impact risks [16, 27]. Over recent decades, due to various reasons such as climate change and unstable geopolitics, their frequency and the resulting losses have increased significantly [1, 13, 20, 34]. This rise of catastrophes has led to more victims needing compensation. Among diverse compensations, the insurance industry plays an important role. For example, in 2007, the property damage in the United States caused by 335 catastrophes amounted to over $70 billion, with insurance covering about one-third [13]. Consequently, a great deal of attention is increasingly paid to catastrophe insurance [13].

The catastrophe insurance market faces the unique challenge of the rare historical data to get an accurate estimation of the loss probability distribution [5, 9, 16]. This market is characterized by ambiguity, defined as uncertainty about the probability of loss [9, 27]. To address this, insurers rely on the catastrophe models to estimate the potential losses [15, 20]. However, due to imperfect scientific knowledge and competing theories, the loss probability based on these models can deviate from the actual outcome, which is another source of ambiguity [10, 11, 35]. To respond to ambiguity, insurers often set higher premiums and maintain higher holding capitals, which can result in thin markets or market failure [4, 5, 9, 18]. Lloyd's of London is the world's largest catastrophe insurance market, known for its strong capacity and flexibility due to its lead-follow mechanism, where multiple insurers share the same risk [20]. This mechanism allows Lloyd's market to cover catastrophes, such as terrorist attacks, political violence, commercial property damage, etc [20, 22]. These risks are currently highly ambiguous lacking sufficient data and fundamental models [11, 26].

Ambiguity is a critical challenge in catastrophe insurance markets. Previous research has examined its impact on market participants' behavior [3, 18], and insurance pricing either by adopting "α-maxmin" expected utility representation [9, 14], or through model distortion technology [12, 17, 28]. However, there is a lack of studies exploring the effects of ambiguity in competitive insurance markets, particularly in Lloyd's market. This paper fills this gap by investigating how ambiguity influences individual and systemic performance in competitive catastrophe insurance markets. Unlike previous studies that model insurers' competition as a normal form game [21, 24, 30, 33, 34], which is unsuitable for the complex dynamics of multi-agent interactions in real world, this paper is the first to model insurers' competition as an empirical game, where payoffs are generated through simulation. Additionally, this paper provides the first analysis of ambiguity in Lloyd's market.

Proc. of the 24th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2025), Y. Vorobeychik, S. Das, A. Nowé (eds.), May 19 – 23, 2025, Detroit, Michigan, USA. © 2025 International Foundation for Autonomous Agents and Multiagent Systems (www.ifaamas.org).

There are three main challenges in studying ambiguity in catastrophe insurance markets and our contribution addresses these challenges.

i) **Diverse Insurer Sizes and Strategies.** Insurers differ in capital size and risk appetite. We introduce different ambiguity levels to insurers based on an α-maxmin model [10] to study the impact of ambiguity. In this model, ambiguity-averse (ambiguity-seeking) insurers place more (less) weight on higher probabilities of ruin, hold more (less) capital, and charge a higher (lower) premium.

ii) **Complex System and Market Dynamics.** Insurance markets including Lloyd's market are characterized by complex interactions among participants [22]. To understand these dynamics, we develop a simulator based on agent-based model (ABM) to study the behavior of individual participants. Our simulator is designed to be modular and flexible to fit both the general insurance market and Lloyd's market by incorporating a lead-follow mechanism.

iii) **Competitive Market.** The competitive nature of the market means that insurers' strategies are influenced by others. We model insurers' interactions as a competitive game by allowing them to choose ambiguity strategies according to the equilibrium status derived from empirical game-theoretical analysis (EGTA) [32].

The paper is organized as follows: Section 2 explains how we introduce ambiguity in insurance pricing and capital holding. Section 3 details the simulator and empirical game modeling. The main experiment results and their analysis are shown in Section 4. The last section summarizes our findings and suggests future research.

2 AMBIGUITY IN INSURANCE

In this section, we integrate ambiguity attitudes into the insurance underwriting model. We review related work on ambiguity in insurance, highlighting the approaches that link insurers' level of ambiguity to pricing strategies. Building on the "α-maxmin" approach, we explain how ambiguity is incorporated into pricing and capital holding models while aligning with European Union insurance Solvency II risk management requirements [8].

2.1 Related Work

The ambiguity problem in catastrophe insurance markets has been studied across three main aspects. First, studies focus on the effects of ambiguity on the behavior of market participants. It is found that insurance decision-makers are ambiguity averse and would set premiums significantly higher for risks under ambiguity [18]. Higher premiums and lower coverage will degrade the performance of the insurance market, reduce the demand [3] and lead to market failure [18]. Insurers charge even higher premiums when ambiguity stems from conflict theories rather than model imprecision [7]. Second, studies explore the ambiguity in insurance pricing. In [14], authors adopt the well-known "α-maxmin" expected utility representation of choice under ambiguity into the insurance and establish a clear connection between ambiguity and the pricing of (re)insurance. In their model, the insurer's attitude to ambiguity affects the premium through the amount of capital it chooses to hold against the risk of ruin. In [9], authors use two real data sets to demonstrate the practical use of quantifying the insurer's attitude to ambiguity. The impact of ambiguity aversion on insurance pricing is explored in [35], with closed-form pricing formulae incorporating ambiguity aversion into mortality risk and property risk. Finally, the third aspect concentrates on the premium based on model distortion technology to cope with ambiguity. Research explores closed-form solutions for the extreme case risk measures [28], optimal insurance contracts under distortion risk measures [17], and Wasserstein distance-based ambiguity measurement [12].

2.2 Risk Management

Exposure management is a critical function for all insurance firms to quantify the impact of worst-case scenarios on their portfolio [22]. In our model, we keep the design of n peril regions in [16] and calculate risk exposure and balance portfolio settings for each region. We use the Value-at-Risk (VaR) to quantify the risk of the insurers in each peril region. For a random variable X representing the losses in the insurer's portfolio of risks under study, the VaR with exceedance probability $\theta \in [0, 1]$ is a θ-quantile (defined in Equation (1)).

$$VaR_\theta(X) = \inf\{x \in \mathbb{R} : P(X > x) \leq \theta\}. \tag{1}$$

Under Solvency II, insurers are required to have 99.5% confidence they can cope with the worst expected losses over a year [8]. This means that the capital that the insurer is required to hold can be computed with the $VaR_{0.005}(X)$. The VaR calculation is simplified in two ways: (i) considering the VaR due to individual catastrophes, and (ii) considering the VaR separately by peril region because it is not required to compute the VaR for all peril regions and over the entire year for the ambiguity problem [16].

2.3 Premium Pricing and Capital Holding

In our design, the ambiguity comes from the inaccuracy of catastrophe models and $\mathbf{\Pi}$ is the set of convex and compact models that encompasses all probability measures the insurer believes might characterize its uncertainty correctly. At time t, insurer j determines whether to underwrite a contract by assessing if its capital, Z_t^j, is sufficient to cover the combined VaR of both new and existing contracts within the peril region. We introduce a parameter p_{dev} to indicate the deviation of the model $\pi \in \mathbf{\Pi}$ from the true loss distribution. The capital required for the insurer must be at least $VaR_{0.005}(X) \cdot p_{dev} \cdot \epsilon \cdot v$, where ϵ is a margin of safety and v is the value of risk brought by brokers. Additionally, insurers aim to maintain a diversified portfolio by ensuring approximately equal VaR across all the n peril regions.

In terms of the insurer's capital setting and pricing, both the pre-existing insurance portfolio and the new contract to be potentially added to this portfolio are regarded as a book $f \in \mathcal{F}$. An insurance book is a $\mathcal{B}$-measure mapping from state space $\mathcal{S}$ to $\mathbb{R}$ [10], where $\mathcal{B}$ means the Borel σ-algebra on $\mathcal{S}$, and $\mathcal{S}$ consists of all possible states that are relevant to the performance of the book. Based on the insurance pricing model that maximizes expected profit subject to a survival constraint, there is an insurer that, given any book $f \in \mathcal{F}$, sets its capital holding $Z_f = min\{x : P_f(-x) \leq \theta\}$. That is, Z_f represents the minimum holding such that the probability of losses x exceeding it is no more than a benchmark level θ. Here, $P_f(E)$ is used for $P_f(x : x < E)$ representing the probability that portfolio f pays out any amount less than E, which is calculated among all $\mathbf{\Pi}$. Given $P_f(\cdot)$ is defined, one can alternatively think of

x as the VaR of book f with respect to the confidence level "$1-\theta$" [10]. Consequently, the insurer focuses on the single probability of its book paying out less than its capital holding.

This framework is extended to allow the capital holding to depend on both the range of models $\mathbf{\Pi}$ and the insurer's attitude to ambiguity $\hat{\alpha}$ about the risk of ruin. Specifically, insurer sets

$$\begin{aligned} Z_f &= \min\{x : \hat{\alpha} \cdot [max_{\pi \in \mathbf{\Pi}} P_f^{\pi}(-x)] \\ &\quad + (1-\hat{\alpha}) \cdot [min_{\pi \in \mathbf{\Pi}} P_f^{\pi}(-x)] \leq \theta\} \\ &= \min\{x : \hat{\alpha} \cdot [p_{dev}^{max} \cdot P_f(-x)] \\ &\quad + (1-\hat{\alpha}) \cdot [p_{dev}^{min} \cdot P_f(-x)] \leq \theta\} \end{aligned} \tag{2}$$

where $\hat{\alpha} \in [0,1]$, $x = VaR_{0.005}$, and the measure P_f^{π} for each $\pi \in \mathbf{\Pi}$ on $\mathcal{B}_{\mathbb{R}}$ is defined as $P_f^{\pi}(E) = \pi(f^{-1}(E))$. The weight factor $\hat{\alpha}$ functions similarly to the ambiguity attitude parameter α in the well-known α-maxmin expected utility (α-MEU) representation of decision making under ambiguity, as described in [14].

The capital holding influences the premium charged on a new contract added to the existing portfolio in the following manner. An insurer with book f who agrees to an additional contract k will have a new book $f' = f+k$. Consequently, the insurer must increase its capital holding by $Z_{f'} - Z_f$. Let y denote the opportunity cost of capital, which is set to 10% [9]. If k is competitively priced it must be computed according to Equation (3), where L_k is the expected loss on k [10]:

$$p_k = L_k + y(Z_{f'} - Z_f). \tag{3}$$

In [16], for the sake of simplicity, it is assumed that insurance premiums oscillate around the fair premium $p_{f,t}$ that would on average offset the damages and thus lead to zero profits and zero losses. Considering the premium calculation in Equation (4), $p_{f,t}$ can be seen as the compensation of expected loss L_k for each monetary unit of the risk. To introduce ambiguity to the premium, firstly, we add the capital holding cost for each monetary unit of risk to the fair premium. Then, this price is multiplied by e^m, where m is an underwriter log markup, attempting to model the price elasticity of demand in the market [22]. Hence, ambiguity premium $p_{a,t}$ in our model is calculated in Equation (5) below. Secondly, to avoid unrealistically high volatility, we set hard upper L_{max} and lower L_{min} bounds to the premium proportional to the ambiguity premium $p_{a,t}$. The slope measures two key factors: i) The total available capital Z_t^J divided by total market loss, which is calculated by multiplying the number of risks available in the market H by the expected damage by risk $\hat{D}$. ii) The impact of an insurer j's share of the market capital on the premium. It is designed such that the greater the insurer's share of total capital $\frac{Z_0^j}{Z_t^J}$, the higher the premium it can charge [16]. γ represents the premium sensitivity parameter in this slope. Equations (5) and Equation (6) below are used to get the premium at the time t for each insurance firm j. The temporary premium $p'_{j,t}$ is calculated with the slope first, then this value will be compared with the $p_{a,t} \cdot L_{max}$ and $p_{a,t} \cdot L_{min}$ to get the final premium $p_{j,t}$.

$$p_{a,t} = (p_{f,t} + y(Z_{f'} - Z_f)/v_i) \cdot e^m \tag{4}$$

$$p'_{j,t} = p_{a,t} \cdot L_{max} - \frac{\gamma \cdot Z_t^J}{Z_0^j \cdot \hat{D} \cdot H} \tag{5}$$

$$p_{j,t} = \begin{cases} p_{a,t} \cdot L_{max} & if\ p_{a,t} \cdot L_{max} \leq p'_{j,t} \\ p'_{j,t} & if\ p_{a,t} \cdot L_{min} \leq p'_{j,t} \leq p_{a,t} \cdot L_{max} \\ p_{a,t} \cdot L_{min} & if\ p'_{j,t} \leq p_{a,t} \cdot L_{min} \end{cases} \tag{6}$$

3 SIMULATOR DESCRIPTION

Our ABM-based simulator is designed to be modular and includes three modules: market participants, market environment, and market management. The market environment is implemented within the Gym environment and our code is available on GitHub [1]. Compared to the closest work [22], which also developed an ABM-based simulator for Lloyd's market, our simulator incorporates the ambiguity preference and a competitive environment for insurers.

3.1 Market Participants

Insurance market participants are modeled as different types of agents embedding behaviors and actions reflective of real-world concepts.

i) **Brokers**: Brokers acting as intermediaries between the customers and insurers, bring new risks to the market. At time t, the number of brokers is B_t. We adopt the assumption from [16] that the value of insurable risks i brought by broker b, v_i is normalized to 1 monetary unit each. This assumption is fairly realistic for property insurance, which constitutes a significant portion of catastrophe insurance. These brokers are responsible for accepting insurance contracts, paying premiums and asking for claims.

ii) **Insurers**: Insurers are responsible for pricing risks brought by brokers, receiving premiums, paying claims, and paying dividends. In Lloyd's market, groups of private individuals or corporate investors who underwrite risks are called syndicates and they also need to decide which line size (i.e., a portion of risk to be covered – see Section 3.2.1) to give [20, 22]. At time t, the number of insurers is J_t. They offer standard insurance contracts with a 12-month duration (t_c). Each insurer j starts with an initial capital Z_0^j. Its income comes from premiums and interests on capital Z_t^j at interest rate ξ and its profits can be obtained from $Z_t^j - Z_0^j$. The insurer exits the market if it becomes bankrupt ($Z_t^j < 0$).

iii) **Capital holders**: In this model, insurers pay dividends to capital holders annually provided they are profitable [22]. The dividends D can be calculated as $D = max(0, \delta \cdot profits)$, where δ represents the portion of profits that is paid as dividends.

3.2 Market Environment

The market serves as an environment encompassing all the initialized agents and generated risks and events. It is also responsible for the information collection and log process.

3.2.1 *Lead-follow Mechanism.*

In the general insurance market, a single insurer typically underwrites and assumes full responsibility for covering the customer's risk. In contrast, the Lloyd's market operates on a lead-follow mechanism, where multiple syndicates share the risk. The lead syndicate negotiates terms with a broker and accepts the largest portion of the risk, which is called lead line size l_{size}, while other syndicates (followers) agree to take smaller shares of the risk, called as follow line size f_{size} until 100% of the

[1] https://github.com/teresa-bi/Simulator-SpecialtyInsurance

coverage required is achieved, and adhere to the leader's terms [22]. This structure enables Lloyd's to handle larger and more complex risks by spreading the exposure among several participants. In our model, the l_{size} is set to 1 for the general insurance market and 0.5 for Lloyd's market.

3.2.2 Catastrophe. A catastrophe is a low-frequency, high-severity event affecting multiple risks simultaneously, causing significant drops in insurers' capitals and premium increases. Inspired by [16], each catastrophe in this model is associated with a peril region and affects all the risks in this region and leaves policies issued in other perils regions unaffected. Claims occur in clusters within each peril region. Catastrophe follows a Poisson distribution with variable λ_c. The total damage inflicted by every catastrophe follows the Pareto probability distribution with a shape parameter τ, and the value of the damage follows the truncated distribution designed in [16]. The individual loss d_o is calculated as in [16].

3.2.3 Attritional Loss. Attritional Loss (high-frequency, low-severity events) is defined as those losses which are generally uncorrelated with each other in both space and time and are fairly predictable [22]. The attritional loss claim also follows a Poisson distribution with λ_a, and the severity of the loss is defined by the gamma distribution with mean μ_a and coefficient of variation cov_a to get the scale $\mu_a \cdot cov_a^2$ and shape $\frac{1}{cov_a^2}$ of the gamma distribution in [22].

3.2.4 Events. In the designed simulator, three types of events are included that trigger the interaction among market participants. Detailed explanations are provided next.
i) **Bring Risk Event**: Risks are brought by each broker following a Poisson distribution with parameter λ_r. Once a broker brings a risk, the risk-related information is sent to insurers, who calculate premiums based on their current capital, portfolio and ambiguity level. In the general insurance market, the broker selects the insurer who offers the lowest price. In Lloyd's market, the insurer offering the lowest price becomes the leader, and the other insurers who are willing to cover this risk are selected as followers randomly until this risk is fully covered. If the risk cannot be covered fully by the above strategy, the leader covers the rest of the risk. If no insurer offers coverage, this risk remains uncovered.
ii) **Pay Premium Event**: At the beginning of the contract, the broker will pay premiums to the lead insurer according to its lead line size $p_{lead} = p_{j,t} \cdot l_{size} \cdot v_i$, and to follow insurers according to their follow line size $p_{follow} = p_{j,t} \cdot f_{size} \cdot v_i$.
iii) **Ask Claims Event**: Throughout contracts, if any catastrophic or attritional losses occur, the broker initiates a claim request. If the claim is approved and paid, the claim record and the capital of insurers will be updated. Claim received by the insurer j from risk i is computed by Equation (7) below, where h_i is the excess of the insurance contract, $d_{o,i}$ is the individual loss affected by the catastrophe or attritional event o on risk i, v_i is the value of risk, and Q_i is the deductible.

$$Claims_{o,j} = \sum_i \begin{cases} \min(h_i, d_{o,i} \cdot v_i) - Q_i & Q_i \le d_{o,i} \cdot v_i \\ 0 & d_{o,i} \cdot v_i \le Q_i \end{cases}. \tag{7}$$

The insurers holding affected contracts will pay claims to brokers according to their lead line size $Claims_{lead} = Claims_{o,j} \cdot l_{size}$, or to their follow line size $Claims_{follow} = Claims_{o,j} \cdot f_{size}$.

3.3 Market Management

The management algorithm is included in this simulator, which gets access to the status of brokers, insurers, and events and assists insurers in making decisions.

3.3.1 Game Formulation. In this section, we explore how ambiguity impacts the general insurance market and Lloyd's market using game theory. By adding ambiguity to insurance pricing, insurers can select their preferred ambiguity level $\hat{\alpha}$ and adjust their premiums to achieve higher expected returns. However, selecting the optimal ambiguity parameters is challenging because each insurer's utility depends not only on its own strategy but also on the strategies of others. This creates a game where the ambiguity parameters are taken as strategies. In our game model, given a strategy profile, insurers' payoffs are determined by their own expected utilities (i.e., capital changes during the evaluation period) estimated by the simulator, thus inducing an empirical game [23]. EGTA methodology is developed to examine multi-agent interactions in complex systems [31, 32] and has been applied in many practical settings in financial markets [6, 19, 25, 29]. We use the EGTA method to solve for the equilibrium of this game involving multiple insurers. Based on the obtained payoff matrix, we use the α-rank algorithm [23] to compute the equilibrium in which no insurer has the incentive to switch its strategy and the system reaches a stable final state under incentive-compatible conditions. According to the equilibrium, the values of ambiguity parameters are eventually determined in an incentive-compatible way.

The simulation is modeled as a repeated game and for each game, the period is one year (12 time steps). The detailed game design is as follows.

At time $t = 0$, initiate B number of brokers b, and J number of insurers j with initial capital Z_0^j, generate risks, catastrophes and attritional losses, market settings including l_{size}, f_{size}, etc. All insurers are players in this game and each of them can play an ambiguity strategy from $S = \{0, 0.5, 1\}$. Let $S_{Joint} = S^J$ be the space of the joint strategy profile. Each insurer receives a payoff $U_j : S_{Joint} \to \mathbb{R}$. For period $T_{game} = 1, 2, .., T_{sim}/12$, repeat the following five steps:

- *i)* Update the market whenever a catastrophe or attritional loss occurs, brokers ask for claims, and insurers make payments. Update the insurers' capital by incorporating premiums, interest earnings, and dividend payouts.
- *ii)* Calculate the payoff $U_T^j = \hat{Z}_{T+1}^j - Z_T^j$, which is the change of the capital $\hat{Z}_{T+1}^j$ predicted at the end of this year T and the capital Z_T^j at the beginning of this year, for traversing all the strategies combination $|S_j|$ from the ambiguity strategy set S for all insurers in the market. Then compile all the payoff data into a payoff matrix.
- *iii)* Use α-rank algorithm to score the strategies profile via the stationary distribution π of the ensuing Markov chain [23]. The strategy profile with the highest score, indicating the best stability, is selected as the equilibrium strategy of the induced empirical game: $\mathbf{s}_T^\star = (s_{1,T}^\star, s_{2,T}^\star, \ldots, s_{J,T}^\star)$, where $s_{j,T}^\star$ denotes the strategy chosen by insurer j at equilibrium in year T.

Table 1: Parameter Settings for Simulation [16, 22]

Symbol	*Variable*	*Value*
T	Total simulation time steps	600
B	Number of brokers	30
J	Number of insurers	6
Z_0^j	Initial capital of insurers	40,000
δ	Dividends as share of profit	0.4
Q	Deductibles of insurance value	0
ξ	Monthly interest rate	0.001
v_i	Value of risk	5000
σ	Tail exponent of damage distribution	2
n	Number of catastrophe regions	4
ϵ	Margin of safety	2
m	Underwriter Markup	0.2
L_{min}	Lower premium limit factor	0.85
L_{max}	Upper premium limit factor	1.2
γ	Premium sensitivity parameter	1.29×10^{-9}
θ	Risk exposure balance requirement parameter	0.1
λ_r	Poisson distribution of risk brought by broker per month	1.8
λ_a	Attritional Loss yearly claim frequency	0.1
$mean_a$	Attritional Loss mean	60,000
cov_a	Attritional Loss cov	1
l_{size}	lead line size	$\{0.5, 1\}$
f_{size}	follow line size	$\{0.1, 0\}$
λ_c	Poisson distribution of the catastrophe	3/100
t_c	Mean contract runtime	12
$\hat{\alpha}$	ambiguity level	$\{0, 0.5, 1\}$
y	cost of capital	0.1
p_{dev}	catastrophe probability deviation factor	$[0.1, 1]$

- *iv)* Calculate the premium $p_{j,T}$ using $\hat{\alpha} = s^{\star}_{j,T}$ for each insurer, the one offering the lowest premium will be the leader for this risk and cover the corresponding lead line size l_{size} fraction.
- *v)* Insurers who also offer premiums for the risk will be randomly selected as followers until the 100% risk is covered, otherwise, the risk remains uninsured. This step is omitted in the general insurance simulation and applies only to Lloyd's market simulation.

4 SIMULATION AND RESULTS

4.1 Simulation Settings

This experiment analyzes the impact of ambiguity on the general insurance market and Lloyd's market. Since anecdotal evidence says that there are 10 largest syndicates in Lloyd's market and the top 6 syndicates and top 30 brokers can dominate the market, the number of agents is set accordingly $J_0 = 6$ and $B_0 = 30$. Simulation settings are detailed in Table 1. Using EGTA, we approximate equilibrium through 100 simulations with different random seeds. Each simulation has 3^6 tasks for all the strategy combinations. We run 100 tasks in parallel, and each simulation takes approximately 27 hours. While catastrophes are random, they occur at the same time steps for the different replications to provide meaningful comparisons.

4.2 Results Analysis

In this section, we first assume that insurers can independently determine their own ambiguity parameters and fix their ambiguity strategies during the simulation period. Then, for all ambiguity strategy profile space, insurers use the simulator with EGTA to estimate their expected utilities and choose the top-ranked ambiguity strategies as equilibrium strategies.

4.2.1 Market of Same Size Insurers. In the first experiment where insurers have the same initial capital of 40,000, we compare the simulation results of *i)* the ambiguity parameter is fixed at 0 for all the insurers, *ii)* the ambiguity parameter is fixed at 0.5 for all the insurers, *iii)* the ambiguity parameter is fixed at 1 for all the insurers, and *iv)* the ambiguity parameter is decided from the equilibrium strategies solved by the EGTA method in the genral insurance market and Lloyd's insurance market.

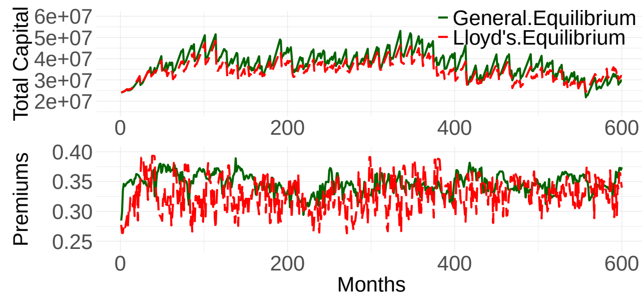

Figure 1: Underwriting Cycles for Same Size Insurers

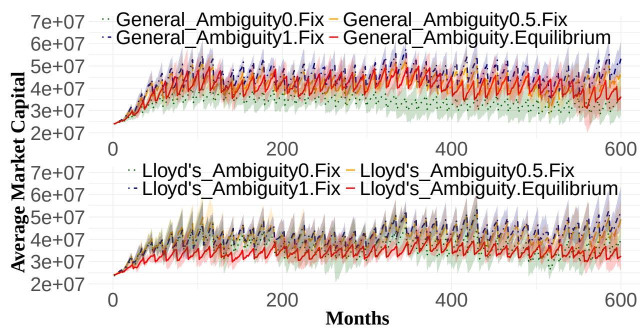

Figure 2: Average Market Capital for Same Size Insurers

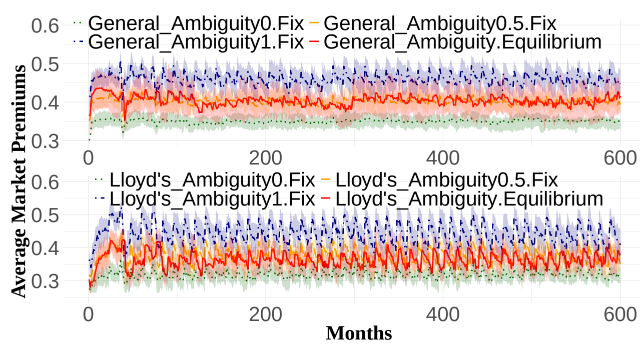

Figure 3: Average Market Premiums for Same Size Insurers

Figure 1 validates the capability of our simulator to mimic the insurance market because the variations in capital and premium reflect the critical insurance market phenomenon known as the "underwriting cycle" [22]. It compares capital and premium time series from a single simulation in a competitive environment. For both general and Lloyd's insurance markets, we observe that during a soft market, the average market capital is high, indicating that insurers have sufficient money. On the contrary, during a hard

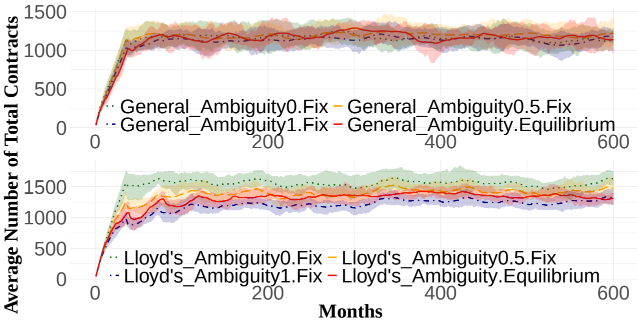

Figure 4: Average Total Contracts for Same Size Insurers

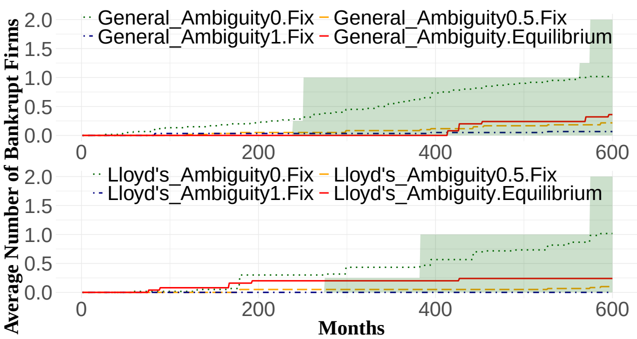

Figure 5: Average Bankrupt Firms for Same Size Insurers

market, many insurers may face financial tightness or even bankruptcy and premiums rise. It can be shown that from $t = 450$ to $t = 550$, the market turns to a hard market, increasing average premiums by approximately 0.03. Compared to the general insurance market, Lloyd's market experiences less fluctuation and maintains lower average premiums, suggesting that its risk-sharing mechanism in Lloyd's enhances stability. Even with aggressive ambiguity strategies and lower premiums, the overall Lloyd's market remains stable.

Figure 2 and Figure 3 illustrate the average market capital and average market premiums of 100 simulation processes over 50 years. The trends reveal that, on average: 1) In the same market, a higher ambiguity parameter leads to a conservative insurer with increased premium and capital. Specifically, insurers with ambiguity parameter $\hat{\alpha} = 1$ have the highest premium and capital (presented by blue dotted dash lines), followed by $\hat{\alpha} = 0.5$ (presented by orange long-dashed lines), while $\hat{\alpha} = 0$ (presented by green dotted lines) with the lowest average premium and capital. 2) When insurers choose the same ambiguity parameter, they have higher premiums and capital in the general insurance market compared to Lloyd's market. For example, the blue dotted dash line in the top sub-figure of Figure 3 representing the general insurance market fluctuates around 0.47, which is larger than the 0.45 seen in the bottom sub-figure in Lloyd's market. 3) Insurers setting $\hat{\alpha} = 0$ in the general insurance market demonstrate the poorest performance. The time series exhibits the most fluctuating and low capital due to a low premium and lack of risk-sharing mechanism, which cannot prevent bankruptcy (as shown in Figure 5). Consequently, it results in significant capital fluctuations.

Figure 4 compares the average number of total contracts under different conditions. Lloyd's market can underwrite more risks compared to the general insurance market. Specifically, the average number of contracts in the general insurance market is below 1,250. In contrast, in Lloyd's market, even the least effective method exceeds 1,250 contracts (insurers set ambiguity as 1 presented by blue dotted dash line), while the most effective method surpasses 1,500 contracts (insurers set ambiguity as 0 presented by green dotted line).

By comparing the results of an average number of bankrupt firms (Figure 5), it can be concluded that insurers with higher ambiguity value (i.e., conservative) yield lower bankruptcies in both insurance markets and the risk-sharing mechanism in Lloyd's can also reduce the bankruptcies. Insurers with the highest fixed $\hat{\alpha} = 1$ (blue dotted dash line) demonstrate no bankruptcies in Lloyd's market. Conversely, insurers with the lowest fixed $\hat{\alpha} = 0$ (green dotted dash line) in the general insurance market perform the worst, with the earliest bankruptcy and the highest average number of bankrupt firms.

After we introduced the competition to the market, several observations can be drawn: 1) The high premiums caused by ambiguity (≥ 0.5) can be effectively reduced by the competition (lower value of red line than blue dotted dash line and orange dash line shown in Figure 3), which results in increased number of total contracts (higher value of red line than blue dotted dash line shown in Figure 4). 2) Compared to the method with a fixed ambiguity strategy of 0 (green dotted line in all figures), equilibrium strategies from the competitive game bring significant benefits, notably reducing bankruptcies, raising premiums and raising capital. 3) Bankruptcy is an avoidable problem for all the methods. Although we set the annual profits as the utility in our game, competition can still cause bankruptcy. 4) Fixed $\hat{\alpha} = 1$ can be seen as the upper threshold in our analysis, achieving the highest capital. However, this scenario is unrealistic in the competitive market. Because insurance companies cannot maintain an ambiguity strategy of 1, as there will inevitably be competitors who lower their ambiguity strategies, lower their premiums, underwrite more risks, and seek to make profits.

4.2.2 Market of Various Size Insurers. In the second experiment, six insurers are initialized with varying levels of capital and ambiguity levels. The insurers are divided into two groups. 1) Large-size insurers: three insurers start with 40, 000, each assigned a different ambiguity parameter from 0, 0.5 and 1. 2) Small-size insurers: three insurers start with 20, 000, each assigned a different ambiguity parameter from 0, 0.5 and 1. Their market performance is compared across two scenarios: a general insurance market with a lead line size of 1 and Lloyd's market with a lead line size of 0.5.

Figure 6 and Figure 7 compare average total capital and bankruptcies in the general insurance and Lloyd's market. Despite the average capital in the general insurance market being 1.5 times higher than that in Lloyd's market, it faces a higher risk of severe bankruptcies. In contrast, Lloyd's lead-follow mechanism significantly improves the overall market performance, contributing to greater stability and enhancing resistance to insolvency.

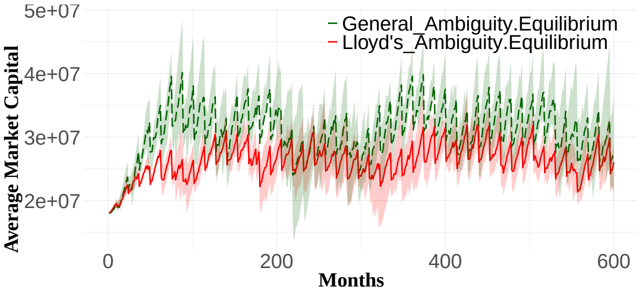

Figure 6: Average Market Capital for Various Size Insurers

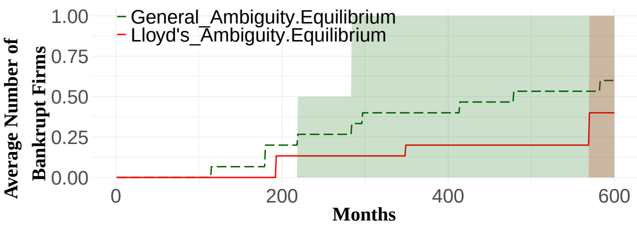

Figure 7: Average Number of Bankrupt Firms for Various Size Insurers

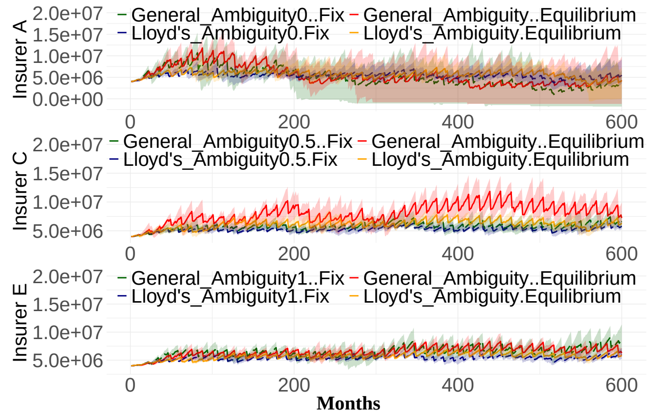

Figure 8: Average Capital for Individual Large Insurers

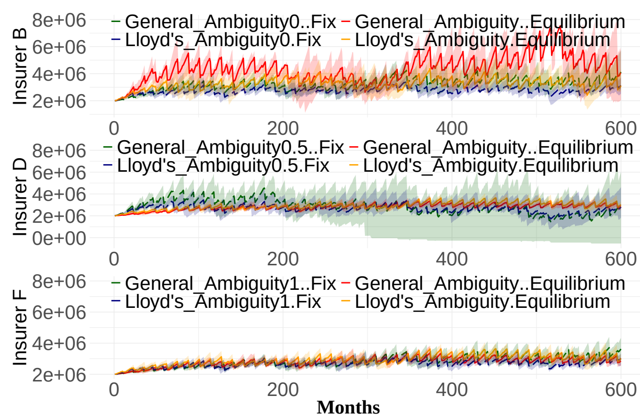

Figure 9: Average Capital for Individual Small Insurers

Figure 8 and Figure 9 show the effects of ambiguity on the individual insurer capital performance. From Figure 8, it can be shown that 1) large aggressive insurer holding ambiguity level at 0 (Insurer A) is more prone to bankruptcy regardless of the market types; 2) competition enhance the performance of large insurers by increasing their capital and reduce bankruptcy risk, notably for Insurer C, whose capital doubles in the general insurance market; 3) large conservative insurer holding ambiguity level at 1 (Insurer E) performs similarly in both insurance market, with and without competition. From Figure 9, it can be concluded that 1) small-size insurers with high (Insurer F) or low (Insurer B) ambiguity levels are more likely to survive compared to the ambiguity-neutral one (Insurer D), because small insurers mitigate risks by either being conservative (charging higher premiums and underwriting fewer contracts) or being aggressive to make more profits to resist bankruptcies during catastrophes; 2) small aggressive insurer (Insurer B) can significantly increase its capital, while small conservative insurer (Insurer F) achieves steady but less dramatic profits; 3) both the risk-sharing mechanism in Lloyd's market and the competition introduced by the game can greatly improve the small insurers' performance, for instance, Insurer D experiences fewer bankruptcies in Lloyd's market and also gains stability from competition in both insurance markets.

To understand the role of the game on various size syndicates, the average capital and average ambiguity strategies for large-size and small-size insurers are plotted in Figure 10 (a) and Figure 10 (b). Several meaningful conclusions need to be highlighted here. Firstly, in general, insurers are more ambiguity-seeking, choosing lower $\hat{\alpha}$ value on higher probabilities of ruin, holding less capital, and setting lower premiums in the general insurance market (represented by green and blue dash lines) compared to those in Lloyd's market (represented by red and orange dash lines). Because insurers are more aggressive in the general insurance market, their capital performance are more fluctuating and more bankruptcies happen compared to Lloyd's market. Secondly, compared to the small-size insurers, large-size insurers are more ambiguity-averse, selecting higher $\hat{\alpha}$ value on higher probabilities of ruin, holding more capital, and setting higher premiums (represented by the higher value of green lines than blue lines in the general insurance market, and higher value of red lines than orange lines in Lloyd's market). These results show that the competition among insurers can enable large-size insurers to leverage their market power and set higher premiums for risks. Small-size insurers are forced to take more aggressive strategies in the competition to make profits, however, it may expose them to un-afforded risks during catastrophe events and increase their risk of bankruptcy. Thirdly, the lead-follow mechanism can reduce the competition among various size insurers because risks are shared among them and the difference between the red line and the orange line is smaller than the difference between the green line and blue line in Figure 10 (b).

Figure 10 (c) shows the timing and value of catastrophe events. By comparing Figure 10 (a), Figure 10 (b), and Figure 10 (c), we

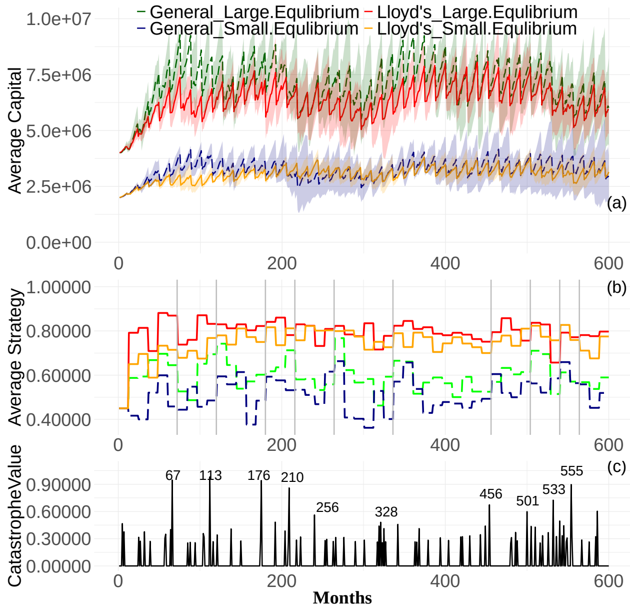

Figure 10: Average Capital and Average Ambiguity Strategies for Large and Small Insurers under Catastrophes over 50 Year

can draw the impact of catastrophes on strategy selection and underwriting cycles. From $t = 0$ to $t = 210$, a period of soft market, individual catastrophe can affect the strategies. For example, after the catastrophe happening at $t = 113$ and $t = 176$, insurers in both markets become more conservative by raising their ambiguity strategies (higher $\hat{\alpha}$), holding more capital and charging higher premiums. It seems that $t = 67$ is the opposite example and it can be caused that at this period, the number of underwriting risks is still small and catastrophe can only affect a small number of contracts and will not influence the strategies of insurers. Following a series of catastrophes, the market shifts to a hard period. Catastrophes in this period are not severe but can also encourage insurers to adopt more conservative strategies. Notably, around $t = 256$ and $t = 328$, smaller but frequent catastrophes can lead insurers to raise their ambiguity strategy levels significantly. From $t = 400$ to $t = 501$, fewer severe catastrophes occur, and the market turns to a soft state. However, after three severe catastrophes at $t = 501$ and $t = 533$ and $t = 555$, the market transits to a hard period again. For the first two catastrophes, insurers immediately raise their ambiguity strategies after them. Interestingly, the third catastrophe at $t = 555$ does not result in increased conservatism but rather more aggressiveness, which is likely due to prior bankruptcies diminishing this catastrophe's impact. These trends indicate that both high-value and frequent catastrophes drive insurers to adopt higher ambiguity strategies and hold more capital. Infrequent catastrophes and more aggressive strategies (lower $\hat{\alpha}$ ambiguity strategies) can turn the market from hard to soft. However, frequent severe catastrophes can lead to bankruptcies, even with conservative strategies.

Inspired by the results in Figure 10, ambiguity strategies at each time step T_{game} can be influenced by three key factors: 1) the capital of the insurance company at the start of each game $Z^j_{T_{game}}$, 2) all

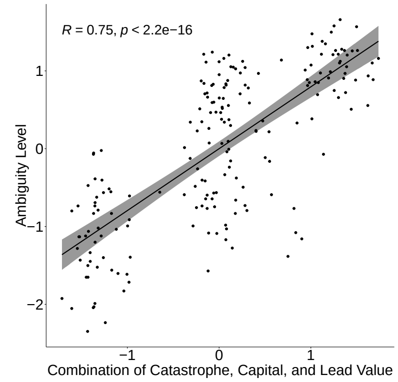

Figure 11: Correlation Between Ambiguity Levels and Affecting Factors

the catastrophe values in the previous game $d_o^{T_{game}-1}$, and 3) the lead line size in different competition scenarios, whether it is a fully competitive general insurance market with $l_{size} = 1$, or Lloyd's market with cooperation risk-sharing mechanism where $l_{size} = 0.5$. We take the Principal Component Analysis (PCA) [2] to determine the optimal combination of these factors that most strongly correlates to the ambiguity preferences. After standardizing data of capital, catastrophe value, lead size and ambiguity level to a mean of 0 and a variance of 1, the optimal combination was found to be $-0.13 \cdot d_o^{T_{game}-1} + 0.71 \cdot Z^j_{T_{game}} - 0.70 \cdot l_{size}$. This combination shows a strong positive correlation of 0.75 to ambiguity levels. Insurers can use this relationship to adjust their ambiguity strategies in different competition scenarios to enhance performance.

5 CONCLUSION

This paper addresses the ambiguity problem in the catastrophe insurance market and takes Lloyd's market as a case study. By introducing ambiguity preferences for insurers to calculate capital holdings and premiums, insurers can strategically respond to catastrophe and competition. The market performance is enhanced threefold: 1) Competition reduces the higher premiums caused by the ambiguity, enabling more risks to be underwritten; 2) Large-size insurance companies can leverage market power to set higher premiums in competition, while smaller insurance companies can adopt conservative strategies in Lloyd's market to avoid insolvency; 3) Insurance companies can adjust their ambiguity strategies to manage potential losses and ensure market stability in response to severe or frequent catastrophes. Compared to general insurance markets, the lead-follow mechanism in Lloyd's market reduces market volatility and bankruptcy risk. Future research could include parameter-sensitive experiments, the application of reinforcement learning for centralized and decentralized management, and an exploration of equilibrium dynamics between insurers and brokers.

ACKNOWLEDGMENTS

This research was delivered through the Accenture-Turing Strategic Partnership, and we acknowledge the generous financial and in-kind support of Accenture and The Alan Turing Institute.

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