Temporal Network Creation Games: The Impact of Non-Locality and Terminals

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ABSTRACT

We live in a world full of networks where our economy, our communication, and even our social life crucially depends on them. These networks typically emerge from the interaction of many entities, which is why researchers study agent-based models of network formation. While traditionally static networks with a fixed set of links were considered, a recent stream of works focuses on networks whose behavior may change over time. In particular, Bilò *et al.* (IJCAI 2023) recently introduced a game-theoretic network formation model that embeds temporal aspects in networks. More precisely, a network is formed by selfish agents corresponding to nodes in a given host network with edges having labels denoting their availability over time. Each agent strategically selects local, i.e., incident, edges to ensure temporal reachability towards everyone at low cost.

In this work we set out to explore the impact of two novel conceptual features: agents are no longer restricted to creating incident edges, called the *global setting*, and agents might only want to ensure that they can reach a subset of the other nodes, called the *terminal model*. For both, we study the existence, structure, and quality of equilibrium networks. For the terminal model, we prove that many core properties crucially depend on the number of terminals. We also develop a novel tool that allows translating equilibrium constructions from the non-terminal model to the terminal model. For the global setting, we show the surprising result that equilibria in the global and the local model are incomparable and we establish a high lower bound on the Price of Anarchy of the global setting that matches the upper bound of the local model. This shows the counter-intuitive fact that allowing agents more flexibility in edge creation does not improve the quality of equilibrium networks. Finally, in contrast to Bilò *et al.* (IJCAI 2023) where the authors restrict the model to single labels per connection, all of our results hold for the restricted case and the generalized case where every edge can have multiple labels.

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KEYWORDS

network creation games; temporal graphs; nash equilibria; price of anarchy; temporal spanner; reachability

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1 INTRODUCTION

Networks are an integral part of our everyday lives, playing a key role in almost every aspect of human existence. Prominent examples include transportation networks (road networks, train tracks, airplaine routes, etc.), communication networks (e.g. the Internet), neural networks (both biological and artificial), biological networks (e.g. protein-protein interaction networks) and many more. With the growing digitization of society, networks, in particular communication networks and (online) social networks, came more and more into the focus of computer science research over the last decades. Many different topics have been studied ranging from the formation of social networks [2] over information diffusion [15] and generating synthetic social networks with real-world properties [24, 29] to uncover their underlying geometry [31].

To understand how social networks (and many other types of networks) emerge, one must understand the mechanisms and principles that govern the formation of networks among several non-cooperative agents [30]. This sparked the investigation of game-theoretic network formation models like the *Network Creation Game* (NCG) [18]. In this model, selfish agents act as nodes of a network which can form costly connections to others to gain a central position in the arising network. In particular, each agent can build connections only locally, i.e., via creating incident edges. Since then, many variations and extensions of this model have been formulated and studied, e.g., variants with non-uniform edge cost [6, 10, 13, 26], robustness considerations [11, 16, 20, 27], or geometric aspects [7, 17, 28].

Although all these models aim to capture time-dependent processes of network formation, in practice, they consider networks that, once formed, are static. This is in contrast to many real-world networks in which temporal aspects play a prominent role. We highlight two motivating examples to make this more evident.

One example is the commercial airline network: each time an airline company wants to serve a new route, the company also has to take into account connecting flights with their corresponding departure and arrival times. Planning the routes carefully can ensure reachability: customers can get from any airport to any other airport by taking a sequence of flights, possibly of different airlines, with ascending departure and arrival times. Here, the airlines are the selfish agents that can establish new connections to enable their customers to travel anywhere.

For another example, consider the supply chain network of companies that are participating in the production of a particular product X. Assume that company A wants to make product X and sell it. Unless company A owns every part of the production chain (which is highly unlikely in today's world), they want to have a connection to other companies in order to send materials and use their means of production that are missing from their production chain. As such, they want to guarantee that they have the logistical infrastructure to send their parts to all other companies participating. But company A may want to combine deliveries. For example, load a vehicle with parts that goes to company B, and then the vehicle loads up parts from company B and moves them to company C. In order for this behaviour to be accurately portrayed, the scheduling of the connections must happen in ascending order (time-wise).

Other examples of network formation that include temporal properties are scheduling problems in which jobs have an order of preferences, neural networks where neurons forming a chain are serially activated one after the other, navigation networks in which the travel time of roads changes over time (e.g. due to traffic, or roadblocks), as well as pathways in biological networks which are series of actions among molecules in a cell that lead to a certain product or a change in the cell. These examples motivate, that understanding network formation of temporal networks is crucial.

Recently, Bilò et al. [4] made a first step towards incorporating temporal aspects into NCGs. In their model, the game is played on an underlying *temporal host network* that defines the time steps in which the bought edges will be available and each agent can only build incident edges. However, this setting might not be general enough to represent real-world networks. Let us consider our two previous motivating examples again.

In the airline route network, the 5th Freedom Right[1] allows airline companies to create connections among countries that do not necessarily include the country the airline is based at. Meanwhile, a company is not interested in reaching every possible destination in other countries, but it mainly serves the hubs and cities which are in high demand for its customers. Finally, an airline company may want to have multiple connections between two countries on each day.

Similarly, in the supply chain network, company A will send parts to company B for processing and then may want to use its own transport vehicles to transfer the processed parts to company C afterwards. Additionally, company A may not need to have a connection to the whole supply chain network, but only to particular other companies. Finally, company A may want to establish more than one connection between two factories during a day, due to a multitude of logistical reasons.

In this work, we extend the model by Bilò et al. [4] to cope with the three raised issues. First, we introduce the *terminal model* in which nodes want to reach only a subset of the nodes, called terminals. The second addition is the *global setting* in which we allow each agent to build connections anywhere in the network, i.e., agents can create non-incident edges. Finally, in contrast to Bilò et al. [4] where the authors restrict the model to single labels per connection, we study the restricted case and also generalize to multiple labels per connection.

Before giving an overview of our contribution, we introduce our model and some notation.

1.1 Model and Notation

We first introduce temporal graphs, then we move on to the game-theoretic definition of our model.

Temporal Graphs and Temporal Spanners. A *temporal graph* $G = (V_G, E_G, \lambda_G)$ consists of a set of nodes V_G, a set of undirected edges $E_G \subseteq \{\{u, v\} \subseteq V_G \mid u \neq v\}$, and a labeling function $\lambda_G : E_G \to P(\mathbb{N}) \setminus \emptyset$, where, for each edge $e \in E_G$, the term $\lambda_G(e)$ denotes the set of *time labels* of e. Informally, the labeling function λ_G describes the time steps in which edge e is available. We sometimes write $\lambda_G(e) + c$ for some $c \in \mathbb{N}$ to denote the set $\{\lambda + c \mid \lambda \in \lambda_G(e)\}$. We define the set Λ_G of *time edges* as the set of tuples of edges and each of their time labels, i.e. $\Lambda_G := \{(e, \lambda) \mid e \in E_G, \lambda \in \lambda_G(e)\}$. For nodes $u, v \in V_G$ and a time label λ, we sometimes abuse notation and write (u, v, λ) instead of $(\{u, v\}, \lambda)$. Furthermore, we call the largest label $\lambda_G^{max} := \max_{e \in E_G} \max_{\lambda \in \lambda_G(e)} \lambda$ the *lifetime* of G. If the graph G is clear from context, we might omit the subscript G to enhance readability. We call a temporal graph simple if there is exactly one time label on each edge. For simple graphs G, we sometimes treat $\lambda_G(e)$ as a number instead of a set for easier notation.

A *temporal path* is a sequence of nodes $v_0, \ldots, v_\ell \in V$ that form a path in G, such that there exists an increasing sequence of time labels $\lambda_0 \leq \cdots \leq \lambda_{\ell-1}$ with $\lambda_i \in \lambda(\{v_i, v_{i+1}\})$ for every $i = 0, \ldots, \ell - 1$. We define ℓ to be the length of the temporal path. Note that we do not require the labels on the temporal path to increase strictly. We say that a node $u \in V_G$ *reaches* $v \in V_G$ if there is a temporal path from u to v in G. Observe that, even though the edges are undirected, a temporal path from u to v does not necessarily imply the existence of a temporal path from v to u. Moreover, we define $R_G(v) \subseteq V_G$ as the set of nodes that node v can reach in G. We call the graph G *temporally connected* if $R_G(v) = V_G$ for every $v \in V_G$.

We define a *temporal host graph with terminals* (or *host graph* for short) as $H = (V_H, E_H, \lambda_H, T_H)$, where (V_H, E_H, λ_H) is a complete temporal graph, i.e. $E_H = \{\{u, v\} \subseteq V_H \mid u \neq v\}$, while $T_H \subseteq V_H$

[1] https://www.icao.int/Pages/freedomsAir.aspx

is a set of *terminal nodes* (or terminals), which is the same for all agents. W.l.o.g., we assume that, for every $\tau = 1, \ldots, \lambda_H^{\max}$, there is an edge $e \in E_H$ with $\tau \in \lambda_H(e)$.[2]

A temporal subgraph of H is a temporal graph G such that (V_G, E_G) is a subgraph of (V_H, E_H) and $\lambda_G(e) \subseteq \lambda_H(e)$ for every $e \in E_G$. A *terminal spanner* of H is a temporal subgraph G of H, with $V_G = V_H$, where every node reaches all the terminals, i.e., $T_H \subseteq R_G(v)$ for every $v \in V_H$. Note that each terminal also needs to reach all the other terminals. Furthermore, for $k = n$ this is the definition of a *temporal spanner*.

Game-Theoretic Model. We introduce the game-theoretic model that we study in this paper. Let H be a temporal host graph with terminals that serves as a host graph for our game. Each node $v \in V_H$ is a selfish agent whose *strategy* $S_v \subseteq \Lambda_H$ corresponds to the set of time edges that agent v buys. We distinguish two settings: *Global edge-buying*, where agents have no restrictions on the time edges they can buy, and *local edge-buying* where agents can only buy incident time edges, i.e. $S_v \subseteq \{(\{v, u\}, \lambda) \mid u \in V_H \setminus \{v\}\}$. We denote by $\mathbf{s} = \bigcup_{v \in V_H} \{(v, S_v)\}$ the *strategy profile* and by $G(\mathbf{s})$ the temporal graph formed by the agents. Formally, the graph $G(\mathbf{s})$ is a temporal subgraph of H with $V_G = V_H$ and $\Lambda_{G(\mathbf{s})} = \bigcup_{(v,S_v) \in \mathbf{s}} S_v$. Note that $E_{G(\mathbf{s})}$ and $\lambda_{G(\mathbf{s})}$ are implicitly defined when $\Lambda_{G(\mathbf{s})}$ is known. In figures, we sometimes display edges as directed to illustrate the edge ownership. Such edges are bought by the node they originate in and can still be used in both direction for the purpose of temporal reachability. In the global setting this simplification does not always work. In this case we write onto the edge who buys it. For simple temporal graphs we sometimes talk about buying edges instead of time edges as they are equivalent in this case.

Each agent $v \in V_H$ aims at reaching all terminals while buying as few time edges as possible. Formally, agent v wants to minimize its costs given by

$$c_H(v, \mathbf{s}) = |S_v| + C \cdot |T_H \setminus R_{G(\mathbf{s})}(v)|.$$

where $C > 1$ is a large constant ensuring that reaching any terminal is more beneficial than not buying a single edge. Indeed, as H is a complete temporal graph, each agent v can always reach all terminals in T_H by buying, for example, an arbitrary time edge for each edge of the form $\{v, u\}$, with $u \in T_H$. We call the defined models *global edge-buying k-terminal Temporal Network Creation Game* (global k-tNCG) and *local edge-buying k-terminal Temporal Network Creation Game* (local k-tNCG), respectively.

Before defining the solution concepts, we need some more notation regarding strategies. Let $\mathbf{s}$ be a strategy profile and consider any agent $v \in V_H$. We define $\mathbf{s}_{-v} := \mathbf{s} \setminus \{(v, S_v)\}$ as the strategy profile without the strategy of agent v. Now, consider an alternative strategy $S'_v \neq S_v$ for agent v. We denote by $\mathbf{s}_{-v} \cup S'_v$ the strategy profile $\mathbf{s}_{-v} \cup \{(v, S'_v)\}$. If $c_H(v, \mathbf{s}_{-v} \cup S'_v) < c_H(v, \mathbf{s})$, we say that $\mathbf{s}_{-v} \cup S'_v$ is an *improving response* for v (w.r.t. $\mathbf{s}$). If additionally, the strategies S_v and S'_v differ by at most one element (i.e. v either adds or removes a single time edge), we call this a *greedy improving response*[3]. We call $\mathbf{s}$ a *best response* of agent v (resp., a *greedy best response*) if there is no improving response (resp., greedy improving response) for agent v.

We can now introduce our solution concepts. A strategy profile $\mathbf{s}$ is a *Pure Nash Equilibrium (NE)* (resp., *Greedy Equilibrium (GE)*) if no agent has an improving response (resp., greedy improving response). As every greedy improving response is also an improving response, we have that every NE is also a GE. Furthermore, every NE (and thus every GE) guarantees pairwise disjoint strategies, since any agent can trivially remove the intersection of its strategy and some other agent's strategy without affecting its reachability. Moreover, our definition of the cost function directly implies that the created graph $G(\mathbf{s})$ must be a terminal spanner.

Lastly, we introduce a measure for the well-being of all agents combined. Let H be a host graph and let $\mathbf{s}$ be any strategy profile. The *social cost* of $\mathbf{s}$ on H is then defined as

$$\mathrm{SC}_H(\mathbf{s}) = \sum_{v \in V_H} c_H(v, \mathbf{s}).$$

Note that $\mathrm{SC}_H(\mathbf{s}) = |\Lambda_{G(\mathbf{s})}|$ for every NE or GE $\mathbf{s}$. A strategy profile of minimum social cost for the given host graph H is called *social optimum* and denoted as $\mathbf{s}^*_H$. When considering the efficiency of equilibria, we will compare their social costs to the social optimum. For $n, k \in \mathbb{N}$ with $k \leq n$, let $\mathcal{H}_{n,k}$ be the set of all host graphs containing n nodes and k terminals. Furthermore, for a host graph H, let $\mathsf{NE}_H^{\mathsf{lo}}$, $\mathsf{NE}_H^{\mathsf{gl}}$, $\mathsf{GE}_H^{\mathsf{lo}}$ and $\mathsf{GE}_H^{\mathsf{gl}}$ be the sets of Nash Equilibria and Greedy Equilibria in the local edge-buying and the global edge-buying setting, respectively. We define the *Price of Anarchy (PoA)* for the local edge-buying setting with respect to Nash Equilibria as the ratio of the socially worst equilibrium and the social optimum

$$\mathsf{PoA}_{\mathsf{NE}}^{\mathsf{lo}}(n, k) := \max_{H \in \mathcal{H}_{n,k}} \max_{\mathbf{s} \in \mathsf{NE}_H^{\mathsf{lo}}} \frac{\mathrm{SC}_H(\mathbf{s})}{\mathrm{SC}_H(s^*_H)}.$$

We define $\mathsf{PoA}_{\mathsf{NE}}^{\mathsf{gl}}$, $\mathsf{PoA}_{\mathsf{GE}}^{\mathsf{lo}}$, and $\mathsf{PoA}_{\mathsf{GE}}^{\mathsf{gl}}$ analogously. If a result holds for both settings (local and global), we omit the superscript. If a result holds for both GE and NE, we omit the subscript.

Lastly, we define the *Price of Stability* as

$$\mathsf{PoS}_{\mathsf{NE}}^{\mathsf{lo}}(n, k) := \max_{H \in \mathcal{H}_{n,k}} \min_{\mathbf{s} \in \mathsf{NE}_H^{\mathsf{lo}}} \frac{\mathrm{SC}_H(\mathbf{s})}{\mathrm{SC}_H(s^*_H)}.$$

Again, we define $\mathsf{PoS}_{\mathsf{NE}}^{\mathsf{gl}}$, $\mathsf{PoS}_{\mathsf{GE}}^{\mathsf{lo}}$, and $\mathsf{PoS}_{\mathsf{GE}}^{\mathsf{gl}}$ analogously.

1.2 Our Contribution

The main contribution of this work is the generalization of the model introduced by Bilò et al. [4] and its game-theoretic analysis. We introduce the concepts of terminals, global edge-buying and multiple labels. To the best of our knowledge terminals have not been considered yet on any network creation model. While the terminal version is just a generalization of the normal model, we show that the global edge-buying leads to a completely different model with an incomparable set of equilibrium graphs. Our results for the generalized model work for both single label graphs and multi label graphs. Note that our techniques can be used to extend

[2]Indeed, as long as some value of τ, with $1 \leq \tau \leq \lambda_H^{\max}$, is missing, we can decrease by 1 all the edge labels that are strictly larger than τ.

[3]Note that, in the literature [25], a greedy improving response also allows a swap, i.e. removing one edge and adding one edge simultaneously. However, in our game, every improving response consisting of a swap also implies an improving response that only adds an edge and omits the remove part. This is because a swap is an improving response for an agent only when the number of reached terminals increases. This means, we can disregard swaps for our definition of greedy improving responses.

Table 1: An overview of our results (yellow) and comparison with the existing results from [4]. Here, n is the number of nodes, m the number of (time) edges, k the number of terminals, and λ^{max} the largest label in the host graph.

	(local n-)TNCG	local k-TNCG	global k-TNCG
Optimum	min temporal spanner	min terminal spanner	min terminal spanner
Equilibria	$\lambda^{max} = 2$: spanning tree $m \leq \sqrt{6}n^{\frac{3}{2}}$	$\lambda^{max} = 2$: spanning tree [2.5] $k = 2$: exists [2.6] $m \leq \sqrt{6k}n + n$ [2.7]	$\lambda^{max} = 2$: spanning tree [2.5] $k = 2$: exists [2.6] GE: exists [2.9]
PoA	$O(\sqrt{n})$ $O(\lambda^{max})$ $\Omega(\log n)$ $\text{PoA}_{\text{GE}} \leq O(\log(n))\text{PoA}_{\text{NE}}$	$O(\sqrt{k})$ [3.5] $O(\lambda^{max})$ [3.3] $\Omega(\log k)$ [3.4]	$O(k)$ [3.6] $O(\lambda^{max})$ [3.3] $\Omega(\sqrt{k})$ [3.8] $\text{PoA}_{\text{GE}} \in \Theta(k)$[3.6,3.7]
PoS	?	?	1 (for GE) [3.2]

the results of Bilò et al. [4] to the multi label model. Table 1 gives an overview of our results in comparison with the results of [4].

In Section 2, we study the structure and properties of equilibria. First, we introduce a special kind of graph product, see Definition 2.1, that allows us to take any two host networks and respective equilibria and construct a new host graph together with a new equilibrium. This can then be used to construct lower bound examples for the PoA for a wide range of numbers of nodes n and numbers of terminals k by constructing only a few initial equilibria. Additionally, we show that, in the local setting, many structural properties of equilibria and bounds on the price of anarchy derived by [4] that seemed to be dependent on the number of nodes in the graph are actually dependent on the number of terminals instead. Moreover, we show that for two terminals in the local and global setting, Greedy and Nash Equilibria always exist. We also show that for the global setting, Greedy Equilibrium graphs are exactly the set of inclusion minimal temporal spanners. We conclude the section by showing that the set of equilibrium graphs in the global setting are incomparable to the ones from the local setting.

In Section 3, we analyze the efficiency of equilibria. For the global setting, many results carry over from the local setting but there are notable differences. Our findings show that allowing the agents to buy non-incident edges does not improve the efficiency of equilibria but in fact might make them even worse. For the case of Greedy Equilibria, we show that the PoA in the global setting is in $\Omega(k)$, in contrast to the upper bound of $O(\sqrt{k})$ that exists for the local setting. We also show that for Nash Equilibria, the PoA is in $O(\sqrt{k})$ in the local setting, while it is in $\Omega(\sqrt{k})$ for the global setting. While it is still possible that those bounds match asymptotically, we conjecture that the actual PoA is much closer to the lower bound of $\Omega(\log k)$ in the local setting.

For more detailed proofs, we refer to the full version [8].

Simple Host Graphs. As mentioned before, all our results also hold for the special case where the host graph is a simple temporal graph, i.e. every edge has exactly one time label. For all results from Definition 2.1 to Lemma 2.4 this is true since given simple host graphs, the constructions in turn admit simple host graphs. All remaining results are either general upper bounds/statements, and therefore, they also hold for the special case of simple graphs or constructions that are already simple graphs.

1.3 Related Work

As mentioned in the introduction, this paper extends the temporal network creation game proposed by Bilò et al. [4], which studies the all-pairs reachability in the local edge-buying model. In particular, in [4], the authors first prove the existence of NE for host graphs with lifetime $\lambda_H^{\max} = 2$ and show that, for every host graph with lifetime $\lambda_H^{\max} \geq 2$, the problems of computing a best response strategy and the problem of deciding whether a strategy profile is a NE are both NP-hard. The authors then consider upper and lower bounds to the PoA w.r.t. both NE and GE. In particular, they show that the PoA w.r.t. NE is in between $\Omega(\log n)$ and $O(\sqrt{n})$. Moreover, they connect GE with NE by showing that the PoA w.r.t. GE is no more than a $O(\log n)$ factor away the PoA w.r.t. NE.

Besides the paper by Bilò et al. [4] which, to the best of our knowledge, is the only one that combines temporal aspects with network formation games, there has been an extensive line of research on related games in the last decades. One of the earliest models which is close to our work is by Bala and Goyal [3], where selfish agents buy incident edges and their utility increases with the number of agents they can reach while it decreases with the number of edges bought. For the version where undirected edges are formed, the authors prove that equilibria always exist forming either stars or empty graphs, and that improving response dynamics quickly converge to such states. They also show how to efficiently compute a best response strategy as well as deciding if a given state is in equilibrium. Goyal et al. [20] extended this model to a setting with attacks on the formed network, where the objective is to maintain post-attack reachability. This variant is more complex, yet Friedrich et al. [19] proved that best response strategies can still be computed efficiently. Recently, Chen et al. [12] studied a variant where the attacks are probabilistic. Eidenbenz et al. [17] studied the different, yet related, topology control game, where the agents are points in the plane and edge costs are proportional to the Euclidean distance between the endpoints. A similar game was studied by Gulyás et al. [21], with the difference that agents are points in hyperbolic space and using greedy routing. Regarding the

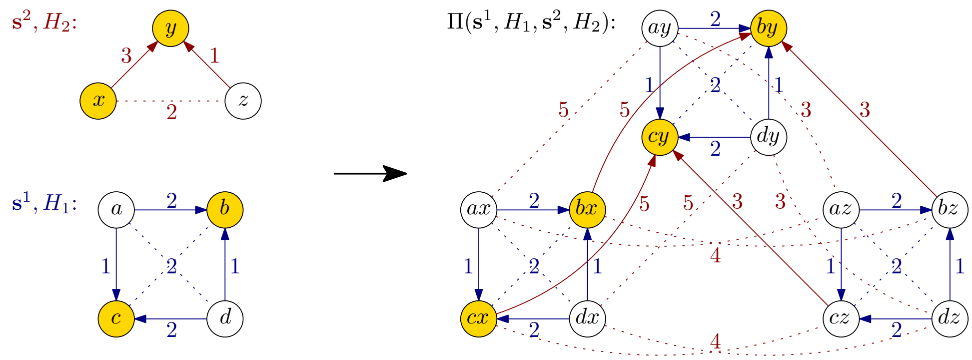

Figure 1: This figure shows two host graphs H_1 (blue) and H_2 (red) and two respective strategy profiles s^1 and s^2 (solid lines) on the left. Yellow nodes are terminals and all edges are bought by the nodes where they originate. On the right, you can see the resulting graph product $\Pi(\mathbf{s}^1, H_1, \mathbf{s}^2, H_2)$ according to Definition 2.1. Again, the terminals are yellow and the strategy profile consists of the solid lines. For clarity, all edges in the host graph with label 6 (the diagonal edges) are not displayed.

idea of using global edge-buying in network creation games, the model by Demaine et al. [14] is related. There, coalitions of agents can buy costshares of any edge in the network.

From a centralized algorithmic perspective, starting from the work by Kempe et al. [22], a lot of research has been devoted to the problem of computing sparse spanners in temporal graphs. More precisely, temporal cliques admit sparse temporally connected spanners [9], even when we seek for all-pairs temporal paths of bounded length [5]. In contrast, there exist very dense temporal graphs that are not complete whose temporal spanners are all dense [1]. Closely related to the reachability problem, Klobas et al. [23] study the problem of finding the minimum number of labels required to achieve temporal connectivity in a graph.

2 EQUILIBRIA

In this section, we analyze the structure and properties of equilibria. We introduce a tool that constructs host graphs with arbitrary size and number of terminals that contain equilibria. In particular, we define a graph product similar to the cartesian product that transfers equilibria from the input graphs to the product graph. This allows us to translate PoA lower bounds between different numbers of terminals. We begin with a description of the construction.

Intuitively, given two host graphs H_1 and H_2, this operation creates one copy of H_1 for each node in H_2 and connects the nodes inside those copies to their counterparts in other copies according to H_2 such that all edges inside a copy have smaller labels than the edges between copies, filling the gaps with high labeled edges. This leads to temporal paths first travelling inside a local copy of H_1 before using edges from H_2 to reach the destination copy of H_1. Strategy profiles $\mathbf{s}^1$ and $\mathbf{s}^2$ for H_1 and H_2 are transformed such that the resulting graph contains all time edges inside the copies of H_1 that are present in $G(\mathbf{s}^1)$ and all time edges between those copies corresponding to time edges in $G(\mathbf{s}^2)$ but only if the connected nodes correspond to a terminal in H_1. See Figure 1 for an example.

Definition 2.1 (graph product). Let H_1, H_2 be host graphs with n_1, n_2 nodes and k_1, k_2 terminals, respectively. We define the *product* of H_1 and H_2 as a host graph $\Pi(H_1, H_2)$ such that

$$V_{\Pi(H_1,H_2)} \coloneqq V_{H_1} \times V_{H_2} \quad \text{and}$$
$$T_{\Pi(H_1,H_2)} \coloneqq T_{H_1} \times T_{H_2} \quad \text{and}$$
$$\forall e = \{(x_1,x_2),(y_1,y_2)\} \in E_{\Pi(H_1,H_2)}:$$
$$\lambda_{\Pi(H_1,H_2)}(e) \coloneqq \begin{cases} \lambda_{H_1}(\{x_1,y_1\}) & \text{if } x_2 = y_2, \\ \lambda_{H_2}(\{x_2,y_2\}) + \lambda_{H_1}^{max} & \text{if } x_1 = y_1, \\ \{\lambda_{H_1}^{max} + \lambda_{H_2}^{max} + 1\} & \text{else.} \end{cases}$$

Now, we extend this definition to include strategies. Let $\mathbf{s}^1$ and $\mathbf{s}^2$ be strategy profiles for H_1 and H_2, respectively. We define the *product* of the two strategy profiles and their host graphs $\Pi(\mathbf{s}^1, H_1, \mathbf{s}^2, H_2)$ as a pair $(\mathbf{s}^\times, H_\times)$ such that $H_\times \coloneqq \Pi(H_1, H_2)$ and $\mathbf{s}^\times$ is a strategy profile for $H_\times$ such that for all $(v_1, v_2) \in V_\times$ we have

$$S^\times_{(v_1,v_2)} \coloneqq \{((x,v_2),(y,v_2),\lambda) \mid \{x,y,\lambda\} \in S^1_{v_1}\} \cup \{((v_1,x),(v_1,y),\lambda + \lambda^{max}_{H_1}) \mid (x,y,\lambda) \in S^2_{v_2} \wedge v_1 \in T_{H_1}\}.$$

Note that, if $\mathbf{s}^1$ and $\mathbf{s}^2$ are local (i.e. every node only buys incident edges), then $\mathbf{s}^\times$ is local, too. Furthermore, if H_1 and H_2 are simple, $H_\times$ is simple, too.

The next theorem shows how we can translate equilibria from the original two host graphs into an equilibrium for the graph product.

Theorem 2.2. *Let H_1 and H_2 be host graphs and $\mathbf{s}^1$ and $\mathbf{s}^2$ equilibria of the same type (NE or GE) for a chosen setting (local or global). Further, let $(\mathbf{s}^\times, H_\times) = \Pi(\mathbf{s}^1, H_1, \mathbf{s}^2, H_2)$. Then $\mathbf{s}^\times$ is an equilibrium for $H_\times$ for the chosen game setting and equilibrium type.*

Proof sketch. The main idea of this proof is to exploit the order of time labels in $H^\times$: The labels of edges inside a copy of H_1 are smaller than the labels of edges between copies of H_1 that connect nodes corresponding to the same original node in H_1 (those edges come from H_2). These, in turn, are smaller than edges connecting nodes in different copies of H_1 that do not correspond to the same original node in H_1. We call the last type of edges *diagonal edges* and show that it is not beneficial for the nodes to buy them. Thus, a best response for some node in $H^\times$ will only buy time edges originating

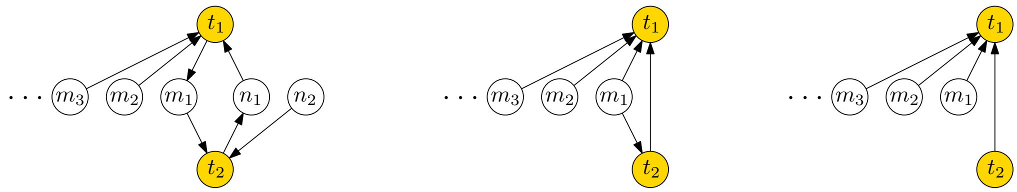

Figure 2: This figure illustrates the equilibrium constructions for the proof of Theorem 2.6. On the left, we have the case where $M \neq \emptyset$ and $N \neq \emptyset$. The middle shows the case where $N = \emptyset$ and $\min \lambda(m_1, t_1) > \min \lambda(t_1, t_2)$ and the right illustrates the case where $N = \emptyset$ and $\min \lambda(m_1, t_1) < \min \lambda(t_1, t_2)$.

from H_1 or H_2, not diagonal ones. This best response can then be partitioned into those two kinds of edges which can than be used to construct an improving response either for $\mathbf{s}^1$ or $\mathbf{s}^2$. The details of the proof can be found in the full version [8]. □

With the product graph construction, we can get host graphs and equilibria for some combinations of n and k. To obtain equilibria for (almost) any combination of n and k, we show some useful properties. The following corollary shows that if we we have an equilibrium for k terminals and nodes, we can scale up the example to arbitrarily high number of nodes while keeping the ratio between the time edges in the equilibrium and the number of nodes. That enables us to derive PoA bounds from single examples. The scaled up graphs are created by using the graph product on the original example and a graph with only one terminal.

Corollary 2.3. *Let H_1 be a host graph on k nodes which are all terminals and $\mathbf{s}^1$ be an equilibrium for H_1 with m_1 time edges. Then, for each $c \in \mathbb{N}$, there is a host graph H with $n := ck$ nodes and k terminals, and an equilibrium for H containing $m := cm_1 + (c-1)k$ time edges. Additionally, if $k \geq 2$ and $c \geq 3$, the host graph H contains a spanning tree consisting of edges with label λ_H^{max}.*

The following lemma lets us fill the gaps that are left from the previous corollary. Corollary 2.3 lets us create examples with a specific ratio between time edges in the equilibrium and nodes in the graph for arbitrary multiple of the size of a given example. With the following lemma we also get examples for numbers of nodes that are not multiples of the original by adding nodes (and maybe terminals) one by one without changing the number of edges in the equilibrium too much.

Lemma 2.4. *Let H be a host graph on n nodes and k terminals and $\mathbf{s}$ be an equilibrium (Nash or Greedy) creating m time edges in the local or global setting. Then there is*

(1) a host graph H_1 on $n+1$ nodes and $k+1$ terminals and an equilibrium $\mathbf{s}^1$ (of the same type and for the same setting as $\mathbf{s}$) creating at least $m+1$ time edges and
(2) a host graph H_2 on $n+1$ nodes and k terminals and an equilibrium $\mathbf{s}^2$ (of the same type and for the same setting as $\mathbf{s}$) creating $m+1$ time edges.

Additionally, if H contains a spanning tree consisting of edges with label λ_H^{max}, then H_1, and H_2 contain spanning trees consisting of edges with label $\lambda_{H_1}^{max}$ and $\lambda_{H_2}^{max}$, respectively.

The previous lemmas give us a way to create graphs with equilibria of specific sizes. In the next part, we analyze for which kind of host graphs equilibria exist.

The first observation is that when the host graph only contains two distinct labels then an equilibrium in the form of a spanning tree exists. This follows the same way as for the local setting in [4].

Corollary 2.5. *Let H be a host graph with $\lambda^{max} = 2$. Then there is a NE $\mathbf{s}$ for H. Furthermore, $G(\mathbf{s})$ is a spanning tree.*

Interestingly, we can also show the existence of equilibria if we restrict the host graph to only two terminals.

Theorem 2.6. *Let H be a host graph with $k = 2$ terminals. Then there is a NE $\mathbf{s}$ for the local and the global setting containing at most n (time) edges.*

Proof sketch. For simplicity, we assume distinct edge labels in the host graph. Let $T = \{t_1, t_2\}$ be the set of terminals. We partition all remaining nodes into

$$M := \{v \in V \setminus T \mid \min \lambda(t_1, v) < \min \lambda(v, t_2)\} \qquad \text{and}$$
$$N := \{v \in V \setminus T \mid \min \lambda(t_1, v) > \min \lambda(v, t_2)\}.$$

Let further $m_1, \dots, m_p \in M$ be the nodes in M sorted descendingly by $\min \lambda(t_1, m_i)$ and $n_1, \dots, n_q \in N$ be the nodes in N sorted descendingly by $\min \lambda(n_i, t_2)$. We now have to consider three cases.

Case $M \neq \emptyset$ and $N \neq \emptyset$: Buying the edges as shown on the left in Figure 2 is a NE.

Case w.l.o.g. $N = \emptyset$ and $\lambda(m_1, t_1) < \lambda(t_1, t_2)$: Buying the edges as shown on the right in Figure 2 is a NE.

Case w.l.o.g. $N = \emptyset$ and $\lambda(m_1, t_1) > \lambda(t_1, t_2)$: Either buying the edges as shown in the middle in Figure 2 is a NE or m_1 can sell both its edges and buy another edge instead. We let m_1 make this change and look at m_2 as the new candidate buying edges towards both terminals. We can do this iteratively until the resulting graph is either stable with the configuration shown in the middle or we arrive at the previous case (on the right) which is also stable. For the details, we refer to the full version [8]. □

2.1 Local Edge-Buying k-Terminal TNCG

In this section, we prove an upper bound on the number of edges in an equilibrium in the local setting. This is mainly an adaption of a proof from [4]. The details can be found in the full version [8].

Theorem 2.7. *Let H be a host graph with $|V_H| = n$ agents and k terminals and let $\mathbf{s}$ be a strategy profile in the local setting. If $G := G(\mathbf{s})$ contains at least $\sqrt{6kn} + n$ time edges, then G is not a GE.*

2.2 Global Edge-Buying k-Terminal TNCG

In this section, we analyze properties of equilibria for the global setting. We start by showing some differences between the structural properties of Greedy and Nash Equilibria. We finish with our main result for this section which shows that equilibria in the global and local setting are incomparable.

The set of inclusion minimal temporal spanners and the set of Greedy Equilibria coincide.

THEOREM 2.8. *Let H be a host graph in the global setting.*
(i) For every GE $\mathbf{s}$, the graph $G(\mathbf{s})$ is an inclusion minimal terminal spanner of H.
(ii) For every inclusion minimal terminal spanner G of H, there is a GE $\mathbf{s}$ with $G(\mathbf{s}) = G$.

COROLLARY 2.9. *For every host graph H a GE exists in the global setting.*

For NE, equilibria and minimal temporal spanners do not coincide. We show that by providing a minimal temporal spanner that does not admit an equilibrium regardless of the edge assignement.

LEMMA 2.10. *There exist simple host graphs H with $k = n$ terminals and inclusion minimal temporal spanners G for H such that there is no NE s with $G(s) = G$ in the global setting.*

The next theorem compares local and global equilibria, showing that those settings are really different and no setting is a generalization of the other in terms of the generated equilibrium graphs.

THEOREM 2.11. *For a host graph H with $k = n$ terminals, let $\mathsf{NE}_H^{\mathsf{gl}}$ be the set of Nash equilibria in the global setting on H and let $\mathsf{NE}_H^{\mathsf{lo}}$ be the set of Nash equilibria in the local setting. Then the set of graphs defined by those equilibria are incomparable. In particular:*

- *There exists a simple host graph H_1 and a global equilibrium $A_g \in \mathsf{NE}_{H_1}^{\mathsf{gl}}$ such that for all $B_l \in \mathsf{NE}_{H_1}^{\mathsf{lo}}$ holds $G(A_g) \neq G(B_l)$.*
- *There exists a simple host graph H_2 and a local equilibrium $A_l \in \mathsf{NE}_{H_2}^{\mathsf{lo}}$ such that for all $B_g \in \mathsf{NE}_{H_2}^{\mathsf{gl}}$ holds $G(A_l) \neq G(B_g)$.*

3 EFFICIENCY OF EQUILIBRIA

In this section, we analyze the efficiency of equilibria by comparing their social cost to the social cost of socially optimal networks. In particular, we derive several bounds on the Price of Anarchy and Price of Stability. We start by bounding the size of a social optimum. The bound follows directly from the fact that social optima are minimum temporal spanners and the $O(n \log(n))$ upper bound on the size of minimum temporal spanners from Casteigts et al. [9].

COROLLARY 3.1. *Let H be a host graph with $|V_H| = n$ agents and $\mathbf{s}^*$ be a social optimum for H. Then $\mathrm{SC}_H(\mathbf{s}^*) \in O(n \log(n))$.*

For GE in the global setting we exactly know the PoS as the social optimum is also an equilibrium.

COROLLARY 3.2. $\mathsf{PoS}_{\mathsf{GE}}^{\mathsf{gl}}(n, k) = 1$.

The rest of the section analyzes the Price of Anarchy in multiple settings. We first give an upper bound on the PoA in all settings based on the maximum lifetime. It follows from the fact that there will not be a cycle of any label in an equilibrium which means that equilibria can contain at most $\lambda^{max}(n-1)$ time edges.

THEOREM 3.3. *For host graphs with a maximum lifetime of λ^{max}, it holds that* $\mathsf{PoA} \in O(\lambda^{max})$.

In the local setting we bound the PoA from both directions dependant on k. For the lower bound we first use the graph product from Definition 2.1 to create graphs with k nodes and terminals that contain spanning trees with a single label and equilibria that are hyper cubes with $\Theta(k \log k)$ time edges. We then use Corollary 2.3 and Lemma 2.4 to blow up the examples to arbitrarily large graphs that still have $\log k$ times as many time edges in an equilibrium than in the social optimum.

THEOREM 3.4. $\mathsf{PoA}^{\mathsf{lo}}(n, k) \in \Omega(\log k)$.

The $O(\sqrt{k})$ upper bound follows directly from Theorem 2.7.

COROLLARY 3.5. $\mathsf{PoA}^{\mathsf{lo}}(n, k) \in O(\sqrt{k})$.

In the global setting we get a much higher upper bound on the PoA. It follows from the simple observation that for each terminal a spanning tree suffices to reach it. Hence all nodes together only buy at most $n-1$ time edges per terminal. As a social optimum needs at least $n-1$ edges, the PoA is upper bounded by k.

THEOREM 3.6. $\mathsf{PoA}^{\mathsf{gl}}(n, k) \leq k$.

For Greedy Equilibria this upper bound is asymptotically tight. This follows from a construction from Axiotis and Fotakis [1] that creates graphs with $\Theta(n^2)$ minimum temporal spanners. We extend those graphs to get temporal cliques with minimal temporal spanners of size $\Theta(n^2)$, which by Theorem 2.8 implies GEs of the same size. Using Corollary 2.3 and Lemma 2.4 again generalizes the bound to arbitrarily large graphs with fixed number of terminals k.

THEOREM 3.7. $\mathsf{PoA}_{\mathsf{GE}}^{\mathsf{gl}}(n, k) \in \Omega(k)$.

We conclude with one of our main results, establishing a high lower bound on the Price of Anarchy of NEs in the global setting.

THEOREM 3.8. $\mathsf{PoA}_{\mathsf{NE}}^{\mathsf{gl}}(n, k) \in \Omega(\sqrt{k})$.

PROOF SKETCH. To show this, we construct a class of simple temporal graphs containing k terminals and nodes and $\Omega(k^{\frac{3}{2}})$ edges, which we term *dense cycle graphs* (see the following Definition 3.9 and Figure 3). We then embed it into a host graph and show that the edges can be assigned to the agents such that the resulting strategy profile is an equilibrium in the global setting. The main idea is that there are $\Theta(\sqrt{k})$ bags arranged in cyclic fashion each containing $\Theta(\sqrt{k})$ nodes. The labeling is such that each node has exactly one temporal path towards the bag on the opposite side enforcing it to buy the whole path of length $\Theta(\sqrt{k})$. We then use Corollary 2.3 and Lemma 2.4 to create examples with $\Theta(\sqrt{k})$ bought edges per node for arbitrarily large number of nodes n. The full sequence of proofs can be found in the long version [8]. □

Definition 3.9. Let $x \in \mathbb{N}$ be an even number. A *dense cycle graph* is a simple temporal graph G consisting of $2x$ bags $B_0, \dots, B_{2x-1}$, $B_{2x} = B_0$ each containing $\frac{x}{2}$ pairs of nodes, i.e.

$$\forall 0 \leq i < 2x \colon B_i \coloneqq \{v_{i,0}, v'_{i,0}, v_{i,1}, v'_{i,1}, \dots, v_{i,\frac{x}{2}-1}, v'_{i,\frac{x}{2}-1}\}$$

$$V_G \coloneqq \bigcup_{0 \leq i < 2x} B_i.$$

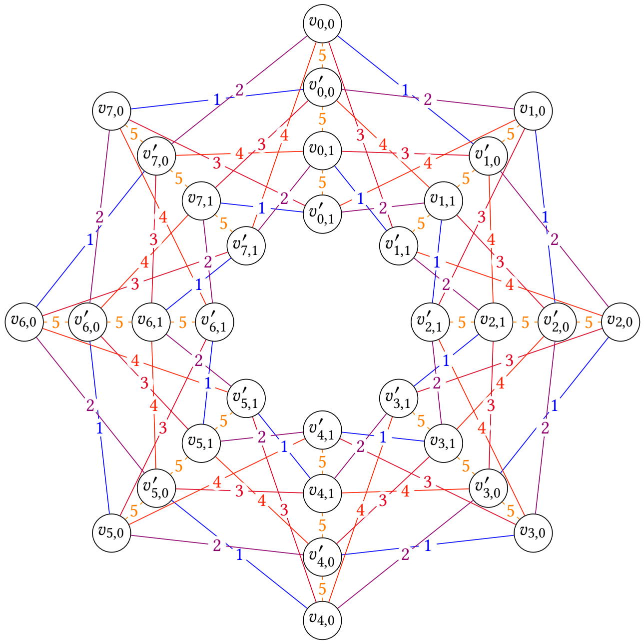

Figure 3: This figure shows a dense cycle graph for $x = 4$.

We call all nodes $v_{i,j}$ *odd* and all nodes $v'_{i,j}$ *even*. The bags form a ring-shaped graph with $n = 2x^2$ nodes, where every two adjacent bags are densely connected in the following way:

$$E_G \coloneqq \{\{v_{i,j}, v'_{i+1,k}\} \mid 0 \le i < 2x \land 0 \le j,k < \frac{x}{2}\} \\ \cup \{\{v'_{i,j}, v_{i+1,k}\} \mid 0 \le i < 2x \land 0 \le j,k < \frac{x}{2}\}.$$

Notice, that between two bags, only nodes of different parity are connected. For the labels, we choose λ_G such that, for all $0 \le i < 2x$ and $0 \le j,k < \frac{x}{2}$, we have

$$\lambda_G(\{v_{i,j}, v'_{i+1,k}\}) \coloneqq (2(k-j) \bmod x) + 1 \text{ and}$$
$$\lambda_G(\{v'_{i,j}, v_{i+1,k}\}) \coloneqq ((2(k-j)+1) \bmod x) + 1.$$

We define a *connected* dense cycle graph G' as a cycle graph which contains an additional path with label $x+1$ inside each bag:

$$E_{G'} = E_G \cup \{\{v_{i,j}, v'_{i,j}\} \mid 0 \le i < 2x \land 0 \le j < \frac{x}{2}\} \\ \cup \{\{v'_{i,j}, v_{i,j+1}\} \mid 0 \le i < 2x \land 0 \le j < \frac{x}{2} - 1\}$$

For an example of a connected dense cycle graph, see Figure 3.

4 CONCLUSION AND OUTLOOK

We study a game-theoretic model where non-cooperative agents form a temporal network. We introduce two new dimensions to the problem: (i) the agents can build any edge in the network, (ii) the agents want to reach a subset of the other agents. We analyze the existence, the structure and the quality of equilibria. Our main results are upper and lower bounds on the PoA, along with a powerful tool that allows us to translate lower bounds between the non-terminal case to the terminal one. We show that the global and the local model are incomparable, which contradicts the intuition that more power to the agents would improve the quality of the equilibria. Additionally, all of our results hold for both the single label and multi label case.

There are various open problems that stem from our work. With regards to improving our results, it would be interesting to see whether equilibria exist for more than two terminals. We conjecture that this is true, but even the case with three terminals proves to be very challenging. It is also worthwhile to close the gaps between the upper and the lower bounds on the PoA for the local and the global setting. For Greedy Equilibria we showed that the PoA in the global setting is strictly worse than the PoA in the local setting. We believe that this is also true for Nash Equilibria. In particular, we believe that the PoA for NE is close to the current lower bound of $\Omega(\log(k))$ in the local setting while we conjecture it to be close to the current lower bound of $\Omega(\sqrt{k})$ in the global setting. This conjecture also implies that the gap between the PoA for NE and the PoA for GE is much larger in the global setting ($\Omega(\sqrt{k})$ versus $\Theta(k)$) than in the local setting (where the PoA for GE is at most a log factor larger).

While our paper provides a very general model for studying the formation of temporal networks by non-cooperative agents, there are still more extensions to be investigated. For example, agents might want to minimize the distance to the other agents or share the costs of buying edges. Structural properties of the host network could also be considered. For enhanced applicability, the edges could be directed, could have non-uniform buying costs, and/or non-instant traversal times.

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REFERENCES

[1] Kyriakos Axiotis and Dimitris Fotakis. 2016. On the Size and the Approximability of Minimum Temporally Connected Subgraphs. In *ICALP 2016*. 149:1–149:14. https://doi.org/10.4230/LIPIcs.ICALP.2016.149

[2] Lars Backstrom, Dan Huttenlocher, Jon Kleinberg, and Xiangyang Lan. 2006. Group formation in large social networks: membership, growth, and evolution. In *KDD 2006*. 44–54. https://doi.org/10.1145/1150402.1150412

[3] Venkatesh Bala and Sanjeev Goyal. 2000. A noncooperative model of network formation. *Econometrica* 68, 5 (2000), 1181–1229.

[4] Davide Bilò, Sarel Cohen, Tobias Friedrich, Hans Gawendowicz, Nicolas Klodt, Pascal Lenzner, and George Skretas. 2023. Temporal Network Creation Games. In *IJCAI 2023*. 2511–2519. https://doi.org/10.24963/IJCAI.2023/279

[5] Davide Bilò, Gianlorenzo D'Angelo, Luciano Gualà, Stefano Leucci, and Mirko Rossi. 2022. Sparse Temporal Spanners with Low Stretch. In *ESA 2022*. 19:1–19:16. https://doi.org/10.4230/LIPIcs.ESA.2022.19

[6] Davide Bilò, Tobias Friedrich, Pascal Lenzner, Stefanie Lowski, and Anna Melnichenko. 2021. Selfish Creation of Social Networks. In *AAAI 2021*. 5185–5193. https://doi.org/10.1145/3323165.3323199

[7] Davide Bilò, Tobias Friedrich, Pascal Lenzner, and Anna Melnichenko. 2024. Geometric Network Creation Games. *SIAM J. Discret. Math.* 38, 1 (2024), 277–315. https://doi.org/10.1137/20M1376662

[8] Davide Bilò, Sarel Cohen, Tobias Friedrich, Hans Gawendowicz, Nicolas Klodt, Pascal Lenzner, and George Skretas. 2024. Temporal Network Creation Games: The Impact of Non-Locality and Terminals. arXiv:2411.03973 [cs.GT] https://arxiv.org/abs/2411.03973

[9] Arnaud Casteigts, Joseph G. Peters, and Jason Schoeters. 2021. Temporal cliques admit sparse spanners. *J. Comput. Syst. Sci.* 121 (2021), 1–17. https://doi.org/10.1016/j.jcss.2021.04.004

[10] Ankit Chauhan, Pascal Lenzner, Anna Melnichenko, and Louise Molitor. 2017. Selfish Network Creation with Non-uniform Edge Cost. In *SAGT 2017*. 160–172. https://doi.org/10.1007/978-3-319-66700-3_13

[11] Ankit Chauhan, Pascal Lenzner, Anna Melnichenko, and Martin Münn. 2016. On Selfish Creation of Robust Networks. In *SAGT 2016*. 141–152. https://doi.org/10.1007/978-3-662-53354-3_12

[12] Yu Chen, Shahin Jabbari, Michael J. Kearns, Sanjeev Khanna, and Jamie Morgenstern. 2019. Network Formation under Random Attack and Probabilistic Spread. In *IJCAI 2019*. ijcai.org, 180–186. https://doi.org/10.24963/IJCAI.2019/26

[13] Andreas Cord-Landwehr, Alexander Mäcker, and Friedhelm Meyer auf der Heide. 2014. Quality of Service in Network Creation Games. In *WINE 2014*. 423–428. https://doi.org/10.1007/978-3-319-13129-0_34

[14] Erik D. Demaine, MohammadTaghi Hajiaghayi, Hamid Mahini, and Morteza Zadimoghaddam. 2009. The Price of Anarchy in Cooperative Network Creation Games. In *STACS 2009*. 301–312. https://doi.org/10.4230/LIPICS.STACS.2009.1839

[15] Benjamin Doerr, Mahmoud Fouz, and Tobias Friedrich. 2012. Why Rumors Spread So Quickly in Social Networks. *Communications of The ACM - CACM* 55 (06 2012), 70–75. https://doi.org/10.1145/2184319.2184338

[16] Hagen Echzell, Tobias Friedrich, Pascal Lenzner, and Anna Melnichenko. 2020. Flow-Based Network Creation Games. In *IJCAI 2020*. 139–145. https://doi.org/10.24963/ijcai.2020/20

[17] Stephan J. Eidenbenz, V. S. Anil Kumar, and Sibylle Zust. 2006. Equilibria in Topology Control Games for Ad Hoc Networks. *Mob. Networks Appl.* 11, 2 (2006), 143–159. https://doi.org/10.1007/s11036-005-4468-y

[18] Alex Fabrikant, Ankur Luthra, Elitza N. Maneva, Christos H. Papadimitriou, and Scott Shenker. 2003. On a network creation game. In *PODC 2003*. ACM, 347–351.

[19] Tobias Friedrich, Sven Ihde, Christoph Keßler, Pascal Lenzner, Stefan Neubert, and David Schumann. 2017. Efficient Best Response Computation for Strategic Network Formation Under Attack. In *SAGT 2017*. 199–211. https://doi.org/10.1007/978-3-319-66700-3_16

[20] Sanjeev Goyal, Shahin Jabbari, Michael J. Kearns, Sanjeev Khanna, and Jamie Morgenstern. 2016. Strategic Network Formation with Attack and Immunization. In *WINE 2016*. 429–443. https://doi.org/10.1007/978-3-662-54110-4_30

[21] András Gulyás, József J Bíró, Attila Kőrösi, Gábor Rétvári, and Dmitri Krioukov. 2015. Navigable networks as Nash equilibria of navigation games. *Nature communications* 6, 1 (2015), 1–10.

[22] David Kempe, Jon Kleinberg, and Amit Kumar. 2002. Connectivity and Inference Problems for Temporal Networks. *J. Comput. System Sci.* 64, 4 (2002), 820–842. https://doi.org/10.1006/jcss.2002.1829

[23] Nina Klobas, George B. Mertzios, Hendrik Molter, and Paul G. Spirakis. 2022. The Complexity of Computing Optimum Labelings for Temporal Connectivity. In *MFCS 2022*. 62:1–62:15. https://doi.org/10.4230/LIPIcs.MFCS.2022.62

[24] Dmitri Krioukov, Fragkiskos Papadopoulos, Maksim Kitsak, Amin Vahdat, and Marián Boguná. 2010. Hyperbolic geometry of complex networks. *Physical Review E* 82, 3 (2010), 036106.

[25] Pascal Lenzner. 2012. Greedy Selfish Network Creation. In *WINE 2012*. 142–155. https://doi.org/10.1007/978-3-642-35311-6_11

[26] Eli A. Meirom, Shie Mannor, and Ariel Orda. 2014. Network formation games with heterogeneous players and the internet structure. In *EC 2014*. 735–752. https://doi.org/10.1145/2600057.2602862

[27] Eli A. Meirom, Shie Mannor, and Ariel Orda. 2015. Formation games of reliable networks. In *INFOCOM 2015*. 1760–1768. https://doi.org/10.1109/INFOCOM.2015.7218557

[28] Thomas Moscibroda, Stefan Schmid, and Roger Wattenhofer. 2006. On the topologies formed by selfish peers. In *PODC 2006*. 133–142. https://doi.org/10.1145/1146381.1146403

[29] M. E. J. Newman, D. J. Watts, and S. H. Strogatz. 2002. Random graph models of social networks. *Proceedings of the National Academy of Sciences* 99, suppl_1 (2002), 2566–2572. https://doi.org/10.1073/pnas.012582999

[30] Christos H. Papadimitriou. 2001. Algorithms, games, and the internet. In *STOC 2001*. 749–753. https://doi.org/10.1145/380752.380883

[31] Fragkiskos Papadopoulos, Maksim Kitsak, M Ángeles Serrano, Marián Boguná, and Dmitri Krioukov. 2012. Popularity versus similarity in growing networks. *Nature* 489, 7417 (2012), 537–540.