Single-Deviation Stability in Additively Separable Hedonic Games with Constrained Coalition Sizes

Extended Abstract

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ABSTRACT

We study stability in additively separable hedonic games when coalition sizes have to respect fixed size bounds. We consider four classic notions of stability based on single-agent deviations, namely, Nash stability, individual stability, contractual Nash stability, and contractual individual stability. For each stability notion, we consider two variants: in one, the coalition left behind by a deviator must still be of a valid size, and in the other there is no such constraint. We provide a full picture of the existence of stable outcomes with respect to given size parameters. Additionally, when there are only upper bounds, we fully characterize the computational complexity of the associated existence problem. In particular, we obtain polynomial-time algorithms for contractual individual stability and contractual Nash stability, where the latter requires an upper bound of 2. We obtain further results for Nash stability and contractual individual stability, when the lower bound is at least 2. An extended version of this paper is available at https://arxiv.org/pdf/2510.12641.

KEYWORDS

Algorithmic game theory; coalition formation; hedonic games

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1 INTRODUCTION

Imagine you are tasked with splitting undergraduate students into groups for a collaborative coursework project. Clearly, the groups should not be too large. Moreover, students may additionally need to gain experience in working as a team, and thus there may also be a lower bound on the size of each group. Ideally, the partition of the students into groups should also be robust to students wishing to switch groups, as any such switch contributes to administrative overhead. Similar challenges arise when assigning desks to faculty in a department with multi-person offices, assigning participants to hiking groups on Duke of Edinburgh expeditions (which are required to have groups of four to seven participants), organizing seating plans for conference dinners and so on.

Such scenarios can be captured by the framework of *hedonic games* [17]. These involve partitioning a set of agents into coalitions. Agents have preferences over coalitions; the term 'hedonic' conveys that agents are only concerned with the composition of their own coalition. The book chapters by Aziz and Savani [4] and Bullinger et al. [9] offer introductions to this field. Hedonic games have received a steady stream of attention, starting with the seminal paper of Bogomolnaia and Jackson [6], which introduced additively separable hedonic games (ASHGs). In ASHGs, preferences naturally arise from utilities encoded by a weighted, directed graph: An agent's utility for a coalition is simply the sum of the weights of edges from her towards other members of the coalition.

In our motivating examples there are constraints on possible coalition sizes. We study prominent stability concepts in ASHGs that are based on deviations by single agents, in the presence of size constraints. Specifically, we assume that, for each coalition, its size s satisfies a global lower bound $\lambda \in \mathbb{N}$ and a global upper bound $\mu \in \mathbb{N}$, where $\lambda \leq \mu$.

Stability is measured in terms of incentives for single agents to deviate by joining another coalition or forming a singleton coalition. An agent finds a deviation desirable if it improves her utility. A partition is Nash-stable if no such deviation exists [6]. A drawback of Nash stability (NS) is that it disregards other agents involved in the deviation. This has motivated the definition of individual stability (IS), contractual Nash stability (CNS), and contractual individual stability (CIS), which require consent by the target coalition, the abandoned coalition, or both [6, 22]. Note that NS implies IS and CNS, which in turn imply CIS.

In the presence of size constraints, we need to adjust the definitions of the common stability concepts defined above. First, we only allow deviations into coalitions of size $s < \mu$, i.e., the target coalition must remain feasible after the deviation, which is a natural assumption. However, it is not as clear whether the size of the

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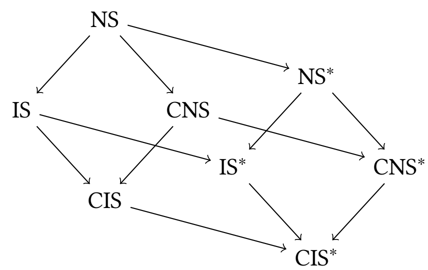

Figure 1: Logical relationships of stability concepts.

abandoned coalition should be constrained in the same way: On the one hand, a selfish agent might not care about the feasibility of the abandoned coalition, while, on the other hand, in some settings such deviations may be prohibited (e.g., students are only allowed to leave their project group if the remaining group is large enough). To capture this, we introduce two variants of each stability notion distinguished by an asterisk: a *standard* notion (e.g., NS) where deviations (e.g., a Nash deviation) may render the abandoned coalition infeasible and a *feasible* notion (e.g., NS*) where a deviation must maintain the feasibility of the abandoned coalition. Since standard notions allow a larger set of deviations, they are stronger solution concepts than their feasible counterparts. We depict the logical implications amoing the stability concepts in Figure 1.

2 RELATED WORK

Explicit size bounds have thus far received little attention in ASHGs. Wright and Vorobeychik [25] consider mechanism design for hedonic games with upper and lower bounds, and group stability and welfare maximization have been studied for upper bounds [18, 20]. Furthermore, upper bounds on coalition sizes were also considered in the context of welfare maximization in online ASHGs [13, 19]. We complement these works by studying single-agent stability.

In a similar spirit, Bilò et al. [5] consider games with fixed coalition sizes. However, in this setting no agent can join a non-empty coalition, so stability is based on swaps instead of single-agent deviations. Another way to implicitly impose bounds on coalition sizes is to fix the number of coalitions in a partition [1, 15, 21]: in particular, when partitions have to be balanced, i.e., coalitions have to be of similar size, one obtains lower bounds on coalition sizes. However, in this model the coalition size bounds depend on the total number of agents, while our bounds are global parameters. Also, Li et al. [21] and Agarwal et al. [1] only consider simple, i.e., unweighted, ASHGs. Moreover, these works consider different solution concepts, mostly inspired by envy-freeness in the fair division literature. Beyond ASHGs, Darmann et al. [14] consider coalition bounds for group activity selection, a model related to anonymous hedonic games [6].

We conclude with a discussion of the complexity of our solution concepts in the absence of constraints on coalition sizes. The existence problems for NS, IS, and CNS are known to be NP-complete [8, 23], while CIS admits a polynomial-time algorithm [3]. Unfortunately, we identify an inaccuracy in this algorithm resulting in unstable outputs. We rectify this by providing a corrected algorithm in our treatment of CIS. Notions of group stability, as captured by the core, strict core, or popularity, are known to be Σ_2^p-complete [10, 24]. By contrast, one can obtain positive results for restricted valuation domains. E.g., symmetric valuations lead to Nash-stable outcomes [6], a well-known result in hedonic games, which we extend to the setting with coalition size bounds. Moreover, if the range of valuations only contains one non-positive or one non-negative value, then existence of IS and CNS outcomes is guaranteed [8]. These restrictions capture, in particular, subclasses of ASHGs where each agent classifies others into friends and enemies, for which even group-stable outcomes exist [16]. Notably, our counterexamples to existence hold even in this restricted model. Finally, IS and CNS outcomes exist and are efficiently computable in random ASHGs, while NS outcomes do not exist with high probability [11].

3 RESULTS

The full version of our paper makes the following contributions.

We begin with some preliminary considerations regarding the existence of coalition structures that satisfy the size constraints. We provide a simple characterization, and apply it to show that outcomes satisfying fixed size constraints always exist for a sufficiently large number of agents.

Our main objective is then to study the existence of *stable* outcomes and the computational complexity of deciding whether stable outcomes exist. We obtain a complete classification for all stability concepts we consider: as in similar settings [5, 6, 12], symmetric valuations lead to existence of NS* outcomes, the strongest feasible stability concept. In contrast, even for symmetric 0/1-valuations, CIS outcomes (the weakest standard stability notion) may not exist. Note that CIS deviations are Pareto improvements; while this guarantees existence of CIS partitions in the unconstrained case, in our setting CIS deviations may leave the space of feasible partitions. Thus, CIS partitions may not exist when all Pareto-optimal outcomes of the unconstrained game do not satisfy the size bounds. In addition, for non-symmetric valuations, only CIS* outcomes are guaranteed to exist.

Next, we establish a complete complexity picture for the setting where coalition sizes are only constrained by an upper bound μ. In particular, we provide a polynomial-time algorithm for CIS for arbitrary μ, amending the algorithm by Aziz et al. [3]. In addition, we present a polynomial-time algorithm for the construction of CNS outcomes when $\mu = 2$. For all other upper bounds (and a lower bound of $\lambda = 1$) and all other stability concepts, we show that deciding whether a stable outcome exists is NP-complete.

We conclude with results for a nontrivial lower bound $\lambda \geq 2$. We show NP-completeness for Nash stability as long as the size constraints satisfy $\mu \geq 4$ and $\lambda < \mu$. Finally, we present polynomial-time algorithms for CIS* for any λ and μ when the additively separable valuations are non-zero or non-negative.

4 OPEN PROBLEMS

Open problems concern the complexity of IS and CNS for a lower bound of at least 2, or CIS* on the complete domain of valuations. Other interesting avenues for future work within our bounded-coalition framework include investigating other classes of hedonic games, such as fractional hedonic games [2], or other solution concepts such as popularity [3, 7].

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