Metric Distortion in Peer Selection[*]

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ABSTRACT

In the *metric distortion* problem, a set of voters and candidates lies in a common metric space, and a committee of k candidates must be elected. The objective is to minimize a social cost, defined as a function of the distances between voters and their chosen representatives, while the voting rule only has access to ordinal preferences. The *distortion* of a rule is the worst-case ratio between the social cost of its outcome and that of the optimal committee, taken over all consistent preferences and metrics.

We initiate the study of metric distortion in peer selection, where voters and candidates coincide. We consider four objectives, obtained by combining two aggregation rules with two types of social cost. Under *additive aggregation*, an individual's cost is the sum of their distances to all committee members; under *q-cost*, it is their distance to the qth closest member. The overall social cost is either *utilitarian*, given by the sum of all individual costs, or *egalitarian*, given by the maximum individual cost. Surprisingly, we find that even on the line metric, peer selection retains much of the hardness of the general case: Lower bounds remain strictly larger than one for all objectives, and cases where bounded distortion is impossible in general remain so here as well. On a positive note, cases with bounded distortion in the general setting achieve better constants in peer selection. For utilitarian cost, selecting the k middle agents achieves a distortion between 1 and 2 under additive aggregation. Under q-cost, we show positive results for $q = k = 2$, but impossibility results largely carry over. For egalitarian cost, selecting the extremes yields an optimal distortion of 2 under additive aggregation and for q-cost with $q > k/3$, while no bounded distortion is possible when $q \leq k/3$. Overall, our results show that peer selection on the line metric admits better constants than the general case, yet fundamental hardness barriers persist.

KEYWORDS

Multi-winner Voting; Metric Distortion; Peer Selection

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1 INTRODUCTION

A fundamental problem in social choice is the aggregation of individual preferences, expressed as rankings over a set of candidates, into a social preference consisting of a subset of elected candidates. For centuries, social choice theorists have proposed several axioms to capture desirable properties that these aggregation or *voting* rules should satisfy, usually leading to strong impossibility results [6, 21, 33, 51].

As an alternative approach, attempting to quantify the extent to which a certain voting rule can faithfully translate voters' preferences into the selected committee, Procaccia and Rosenschein [48] introduced the notion of *distortion* of a rule. The underlying assumption is that a voter's (dis)affinity with a candidate can be represented by a certain cost, and voters' rankings express these cardinal preferences. The cost of a committee for a voter is then defined by aggregating the costs of the committee members, and the overall *social cost* of the committee by aggregating the costs for all voters. The distortion corresponds to the worst-case ratio between the social cost of the selected committee and that of the optimal committee, over all possible preferences and consistent metrics.

The study of the distortion of voting rules has usually focused on two ways of modeling the social cost: utilitarian and egalitarian [13, 14, 35]. In the utilitarian case, the social cost is defined as the sum of the individual costs of the voters, so that all voters' costs contribute equally to the objective. In contrast, the egalitarian social cost considers the maximum individual cost among all voters, capturing a notion of fairness where no voter is excessively disadvantaged.

In voting theory, it is common to assume that voters' preferences are not fully arbitrary but exhibit some structural properties. A relevant line of work has sought structural restrictions that are natural and have powerful implications, such as single-peaked [9] or single-crossing [44]; see Elkind et al. [24] for a survey. A rather general framework among these is that of *spatial* or *metric voting*, where voters and candidates are assumed to lie in a common low-dimensional metric space and voters' costs correspond to their distance to each candidate [7, 25, 38, 43]. For instance, a line metric is commonly employed to capture political affinity on the left-right spectrum, whereas geographical preferences are typically modeled in two-dimensional spaces.

This structural assumption naturally fits in the metric distortion framework: the distances to candidates fully define the social cost of a committee, but the voting rules only receive their expression as preference rankings. With this structural restriction, a tight distortion bound of 3 has been shown for any single-winner deterministic voting rule [2, 34, 41]. Extending distortion to multi-winner elections requires defining how a voter's cost is aggregated over the selected committee. Two ways have been considered in the literature: the *additive cost*, where a voter's cost is the sum of their distances to all members of the committee [8], and the *q-cost*, where the cost is determined by their distance to their qth closest committee member [15, 20].

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Work on metric distortion has so far focused on the case where voters and candidates constitute disjoint sets, which forms a natural model for large-scale elections. However, in many decision-making scenarios, a group of agents seeks to elect a subset of their own members. One can think, for example, of a political organization selecting a committee. Each member ranks others according to their political affinity, and the organization aims to select a committee that represents the variety of preferences of its members. Since the voting rule only receives ordinal preferences, a small distortion constitutes a suitable objective to ensure a close-to-optimal outcome under this limited information. In general, this situation arises in the context of *peer selection*, where individuals evaluate each other to choose a group for governance, leadership, or resource allocation. Further examples include academic hiring and promotion, student representative elections, self-organized committees in cooperatives, and local governance selection.

While peer selection rules have been extensively studied in other contexts, particularly regarding the effect of strategic behavior [e.g. 1, 12, 37], little is known about their ability to accurately reflect agents' cardinal preferences. On the other hand, previous work on metric distortion for single-selection has often parameterized an election via its *decisiveness*, corresponding to the maximum ratio between a voter's distance to their top choice and to any other candidate [4, 34]. These works have motivated this parameter by the fact that it becomes zero in the peer selection setting, as each agent becomes their own top choice. However, directly modeling a common set of voters and candidates constitutes a structural change to the problem that has not been explored so far.

1.1 Our Contributions and Techniques

We initiate the study of metric distortion when the set of voters and candidates coincide and bound the distortion achievable by voting rules selecting k out of n agents on the line metric for several social costs. A summary of our results appears in Table 1.

We start by observing a simple yet powerful property of metric voting on the line with a single set of voters and candidates, which follows from previous work [8, 22]: we can fully compute the order of the agents from their rankings. This constitutes a powerful tool for the design of our mechanisms, as in the following we can always take this order as given.

Utilitarian Additive Cost. We first consider the utilitarian social cost, in which the social cost of a committee is defined as the sum of all individual costs. Intuitively, selecting k consecutive agents results in lower utilitarian social cost. In Section 3.1, we focus on the case of additive aggregation: the cost of a committee for a voter is given by the sum of all distances from the candidates to this voter. As a natural extension of the optimal rules for one or two agents, which select the median and closest-to-median agents, we consider a rule called MEDIAN ALTERNATION that selects k middle agents. We show that MEDIAN ALTERNATION provides a distortion close to 1 when k is small compared to n and approaches 2 as k goes to n. Despite its simplicity, the analysis of this rule poses significant challenges. In short, we reduce any metric to another with only two locations by showing the existence of a non-improving direction of movement for each agent, and then compute the worst-case distortion for this class.

Table 1: Summary of our and previous bounds on the distortion that voting rules can achieve in different settings. Values before and after the semicolon represent lower and upper bounds, respectively. Gray entries correspond to the previously studied setting with disjoint voters and candidates, either under a general metric [CSV] or under the line metric [BKSS]. The upper bounds for utilitarian additive and egalitarian additive social costs marked with $(*)$ correspond to the cases of even $n-k$ and even k, respectively; the slightly different bounds for the odd cases are stated in the corresponding theorems. The upper bound for utilitarian q-cost marked with $(**)$ in the text is valid for $q \geq \frac{k}{2}+1$ (slightly stronger than $q > \frac{k}{2}$). BKSS: Babashah et al. [8]; CSV: Caragiannis et al. [15].

	additive	***q*-cost**	
		$q \leq \frac{k}{2}$	$\frac{k}{2} < q \leq k$
utilitarian	$1+\sqrt{1+\frac{2}{k}}$; $\frac{7}{3}+\frac{4}{k}\left(\sqrt{2}-\frac{4}{3}\right)$ [BKSS]	∞ [CSV]	3; 3 [CSV]
	1.0914; $\frac{2}{k}\left(n-2\sqrt{n(n-k)}\right)^{(*)}$ [T. 3.1]	∞ [T. 3.3]	$2-\frac{k-q}{4q-k-3}^{(**)}$; 3 [T. 3.3, CSV]
	additive	***q*-cost**	
		$q \leq \frac{k}{3}$	$\frac{k}{3} < q \leq k$
egalitarian	·; 2 [BKSS]	∞ [CSV]	3; 3 [CSV]
	$\frac{3}{2}-\frac{1}{k}$; $\frac{3}{2}-\frac{1}{2(k-1)}^{(*)}$ [T. 4.3]	∞ [T. 4.4]	2; 2 [T. 4.5]

We complement the upper bound described above with a lower bound of 1.0914 for any $k \geq 3$. This implies that, even in this seemingly simple case, we cannot always select the optimal committee.

Utilitarian q-Cost. In Section 3.2, we consider utilitarian q-cost, where the cost of a committee for an agent is given by the agent's distance to their qth closest candidate in the committee. In Theorem 3.3, we show that no voting rule can provide a constant distortion when $q \leq \frac{k}{2}$, implying that this known impossibility from the setting with disjoint voters and candidates and a general metric space [15] remains in place in our restricted setting. To prove this bound, we partition all but q agents into $\left\lfloor \frac{k}{q} \right\rfloor \geq 2$ sets and consider two metrics that differ in the position of the remaining q agents: relatively close to the other agents in one metric; very far in the other. Intuitively, selecting these q agents leads to an unbounded distortion in the former case but is necessary for a bounded distortion in the latter. For $q > \frac{k}{2}$, the existence of rules with distortion 3 follows from a general result by Caragiannis et al. [15]. We provide a lower bound that varies between $\frac{3}{2}$ and 2 as q varies between $\frac{k}{2}+1$ and k, by considering three different metrics consistent with the same rankings and showing that, in one of them, there are q agents in one extreme that cannot be consistently selected. We take a closer look at the case with $k = q = 2$, where a best-possible distortion of 2 can be achieved by selecting the median agents when n is even. For odd n, we show that a rule selecting a *couple* of agents—a pair of agents who prefer each other over all other agents—among the five middle agents achieves an improved distortion of $\frac{4}{3}$, which is again best-possible. The Favorite Couple rule leverages two key principles: (1) selecting agents close to the median so as to balance overall distances from agents on each side of the median, and (2) selecting consecutive agents with a small distance between them since this distance is part of the cost of all agents. This intuition of selecting consecutive agents that are as close to each other as possible while also being close to the median in principle holds for larger k, but determining how tightly a group of k agents is clustered based solely on ordinal rankings remains a challenge.

Egalitarian Additive Cost. In Section 4, we turn our attention to the egalitarian social cost, where we focus on the maximum cost of a committee for a voter. We consider the simple k-Extremes rule, which selects half of the committee from each extreme. On an intuitive level, this constitutes a natural rule in this setting as it avoids that extreme voters are excessively disadvantaged. For the additive setting, we show in Section 4.1 that k-Extremes achieves an optimal distortion up to $O\left(\frac{1}{k}\right)$ terms. In particular, the optimal distortion of 1 is attained for $k = 2$, and distortions of $\frac{3}{2} - \frac{1}{2(k-1)}$ and $\frac{3}{2} - \frac{1}{k(k-1)}$ are achieved for even and odd $k \geq 3$, respectively, almost matching a lower bound of $\frac{3}{2} - \frac{1}{k}$. The worst-case instances involve $k+1$ agents in one extreme, a single agent in the other extreme, and k agents in the middle, which are selected in the optimal committee but cannot be detected by any rule when considering two symmetric distance metrics.

Egalitarian q-Cost. In Section 4.2, we show that k-Extremes attains a distortion of 2 for q-cost as long as $q > \frac{k}{3}$. To do so, we prove that the social cost of the set selected by this rule is at most the distance from the agent closest to the center to their nearest extreme, and bound the social cost of the optimal set from below by half of this distance. We provide a matching lower bound by revisiting the instance used for the additive case. Finally, we show that no constant distortion is possible when $q \leq \frac{k}{3}$, again implying that the general impossibility result of Caragiannis et al. [15] still holds in our setting. In the worst-case instances, we partition the agents into $\left\lfloor \frac{k}{q} \right\rfloor$ sets and consider two symmetric distance metrics where all but one set are placed at a unit distance from one another and two sets in one extreme are at the same location. We show that no rule can pick q agents from each location.

1.2 Further Related Work

Distortion of voting rules was first introduced by Procaccia and Rosenschein [48]. Since then, extensive research has been conducted to establish lower and upper bounds on the distortion of different rules under various scenarios, both within the metric and non-metric frameworks. For a comprehensive survey, we refer to Anshelevich et al. [3].

Single-Winner Voting. In the non-metric framework, Caragiannis and Procaccia [14] showed that the distortion of any voting rule is at least $\Omega(m^2)$ and that simple rules such as Plurality achieve a distortion of at most $O(m^2)$, where m is the number of candidates. In the metric framework, Anshelevich et al. [2] established a general lower bound of 3 on the distortion of any deterministic voting rule. They also analyzed the distortion of common voting rules such as Majority, Borda, and Copeland, showing that the latter achieves the lowest distortion of 5 among them. Goel et al. [36] disproved a conjecture by Anshelevich et al. regarding a better-than-5 distortion of the Ranked Pairs rule and introduced the notion of *fairness ratio* of a rule, which captures the egalitarian social cost as a special case. These results were later improved by Munagala and Wang [47], who extended the analysis to uncovered set rules and reduced the upper bound to 4.236. Gkatzelis et al. [34] closed the gap by improving this bound to 3, and showed the validity of this bound in terms of fairness ratio and thus egalitarian social cost. Randomized voting rules have also been extensively explored in the metric framework [26, 50]. The best-known upper bound for a randomized voting rule was recently obtained by Charikar et al. [19], who showed that a carefully designed randomization over existing and novel voting rules achieves a distortion of at most 2.753. As for lower bounds, Charikar and Ramakrishnan [18] disproved a conjecture by Goel et al. [36] regarding the existence of a randomized voting rule with distortion 2, by constructing instances whose distortion approaches 2.113 as the number of candidates grows.

Multi-Winner Voting. In the study of metric distortion for multi-winner voting, various objective functions have been proposed to capture the cost incurred by each voter for the elected committee [23, 27]. A foundational result by Goel et al. [35] showed that, for the additive cost function, iterating a single-winner voting rule with distortion δ for k rounds produces a k-winner committee with the same distortion. Chen et al. [20] studied the 1-cost objective in the metric framework when each voter casts a vote for a single candidate. They proposed a deterministic rule with a tight distortion of 3 and a randomized rule with a distortion of $3 - \frac{2}{m}$. More generally, Caragiannis et al. [15] introduced the q-cost objective, where a voter's cost for a committee is determined by the distance to their qth closest member. They showed that the distortion is unbounded for $q \leq \frac{k}{3}$ and linear in n for $\frac{k}{3} < q \leq \frac{k}{2}$. For $q > \frac{k}{2}$, they presented a non-polynomial voting rule that achieves a distortion of 3 and a polynomial rule with a distortion of 9. They discussed how these upper bounds for $q > \frac{k}{2}$ and the unbounded distortion for $q \leq \frac{k}{3}$ carry over to egalitarian social cost, but interestingly showed that a constant distortion is possible for this objective when $\frac{k}{3} < q \leq \frac{k}{2}$. Kizilkaya and Kempe [41] later proposed a polynomial-time rule with a distortion of 3. Recently, Babashah et al. [8] studied the distortion of multi-winner elections with additive cost on the line, devising a rule with a distortion of roughly $\frac{7}{3}$. Caragiannis et al. [13] studied distortion in multi-winner voting for the non-metric framework, defining a voter's utility for a committee as the highest utility derived from any of its members. They proposed a rule achieving a distortion of $1 + \frac{m(m-k)}{k}$ for deterministic committee selection when selecting k out of m candidates.

Restricted Voting Settings. A specialized setting in metric voting studies single-peaked and 1-Euclidean preferences, where both voters and alternatives are embedded on the real line [9, 29, 30, 32, 45, 46, 53]. In particular, the work of Fotakis et al. [31] investigated the distortion of deterministic algorithms for k-committee selection on the line under the 1-cost objective, leveraging additional distance queries.

Mechanism Design in Committee Selection. Several recent studies have explored alternative models for committee selection. The concept of stable committees and stable lotteries has been considered in various settings, focusing on fairness and individual incentives [10, 39]. An active area of research in the last years has focused on impartial mechanisms, where agents approve a subset of other agents and the voting rule must incentivize truthful reports while selecting well-evaluated agents [1, 11, 12, 16, 17, 28, 37, 42, 52]. Finally, another line of work investigates distortion when agents have known locations, enabling mechanisms to explicitly consider distances in selection [5, 40, 49].

2 PRELIMINARIES

We let $\mathbb{N}$ denote the strictly positive integers and, for $n \in \mathbb{N}$, we write $[n] = \{1, \dots, n\}$ for the first n. A *linear order* $\succ$ on a set S is a complete, transitive, and antisymmetric binary relation on S; we denote the set of all linear orders on $[n]$ by $\mathcal{L}(n)$.

Election. An instance of a committee election, or simply an *election* is described by the triple $\mathcal{E} = (A, k, \succ)$, where:

- $A = [n]$ is the set of agents,
- $k \in \mathbb{N}$ is the number of agents to be selected for the committee, and
- $\succ = (\succ_1, \succ_2, \dots, \succ_n) \in \mathcal{L}^n(n)$ comprises the agents' preference profiles, where $\succ_a \in \mathcal{L}(n)$ is a linear order on $[n]$ for every $a \in [n]$.

We let $\binom{A}{k} = \{S \subseteq A \mid |S| = k\}$ denote the feasible committees for a given election; i.e., the set of all subsets of A of size k.

Line metric. A *distance metric* on A is a function $d\colon A \times A \to \mathbb{R}_+$ satisfying (i) $d(a,a) = 0$, (ii) $d(a,b) = d(b,a)$ for every $a, b \in A$, and (iii) $d(a,c) \le d(a,b) + d(b,c)$ for every $a, b, c \in A$. In this paper, we focus on the line metric: We associate each agent $a \in A$ with a position $x_a \in (-\infty, \infty)$, and the metric d is defined by $d(a,b) = |x_a - x_b|$ for every $a, b \in A$. A metric d is said to be *consistent* with a ranking profile $\succ \in \mathcal{L}^n(n)$, denoted as $d \rhd \succ$, if for every triple of agents $a, b, c \in A$, the condition $d(a,b) < d(a,c)$ implies $b \succ_a c$.[1] Since d is fully defined by the position vector $x \in (-\infty, \infty)^A$, we often refer directly to this vector being consistent with a ranking profile $\succ \in \mathcal{L}^n(n)$ and denote it by $x \rhd \succ$. Likewise, we often exchange d by x in the definitions that follow. Finally, for a fixed election $\mathcal{E} = (A, k, \succ)$, consistent vector of locations $x \in (-\infty, \infty)^n$, and interval $I = (y, z)$ with $y < z$, we let $A(I) = \{a \in A \mid x_a \in I\}$ denote the agents with locations in I. When I is a single point $\bar{x}$, we write $A(\bar{x})$ for the agents located at this point.

Social cost. For a certain set of agents A, a committee size $k \in \mathbb{N}$, and a *candidate-aggregation function* $h\colon \mathbb{R}_+^k \to \mathbb{R}_+$, the *cost* of $S \in \binom{A}{k}$ for agent $a \in A$ is simply $\mathrm{SC}(S, a; d) = h((d(a,b))_{b \in S})$. For a set of agents A, a committee size $k \in \mathbb{N}$, and a *voter-aggregation function* $g\colon \mathbb{R}_+^n \to \mathbb{R}$, the *social cost* of $S \in \binom{A}{k}$ is $\mathrm{SC}(S, A; d) = g((\mathrm{SC}(S, a; d))_{a \in A})$. In this paper, we study a handful of candidate- and voter-aggregation functions. In terms of the voter-aggregation function $g\colon \mathbb{R}^n \to \mathbb{R}_+$, we focus on the *utilitarian social cost*, given by $g(y) = \sum_{i \in [n]} y_i$, and the *egalitarian social cost*, given by $g(y) = \max\{y_i \mid i \in [n]\}$. In terms of the candidate-aggregation function $h\colon \mathbb{R}_+^k \to \mathbb{R}_+$, we focus on the *additive social cost*, given by $h(y) = \sum_{i \in [k]} y_i$, and the *q-cost*, given by $h(y) = \tilde{y}_q$, where $\tilde{y}$ is the vector with the entries of y sorted in increasing order. Thus, for example, the *1-cost* is given by $h(y) = \min\{y_i \mid i \in [k]\}$; and the *k-cost* is given by $h(y) = \max\{y_i \mid i \in [k]\}$.

Voting rules and distortion. For $n, k \in \mathbb{N}$ with $n \ge k$, an (n,k)-*voting rule* is a function f that takes a preference profile $\succ \in \mathcal{L}^n(n)$ and returns a subset $S \in \binom{[n]}{k}$, to which we often refer as a *committee*. For an election $\mathcal{E} = ([n], k, \succ)$ and a metric d, the *distortion* $\mathrm{dist}(S, \mathcal{E}; d)$ *of* $S \subseteq A$ *under* d is the ratio between the social cost of the committee and the minimum social cost of any committee; i.e.,

$$\mathrm{dist}(S, \mathcal{E}; d) = \frac{\mathrm{SC}(S, A; d)}{\min_{S' \in \binom{A}{k}} \mathrm{SC}(S', A; d)}.$$

For an election $\mathcal{E} = (A, k, \succ)$, the *distortion* $\mathrm{dist}(S, \mathcal{E})$ *of a committee* $S \subseteq A$ is then defined as the worst-case distortion over all metrics consistent with the ranking profile $\succ$; i.e.,

$$\mathrm{dist}(S, \mathcal{E}) = \sup_{d \rhd \succ} \mathrm{dist}(S, \mathcal{E}; d).$$

Finally, for an (n,k)-voting rule f, the distortion of f is defined as the worst-case distortion of its output across all possible elections; i.e.,

$$\mathrm{dist}(f) = \sup_{\succ \in \mathcal{L}^n(n)} \mathrm{dist}(f(\succ), ([n], k, \succ)).$$

Throughout the paper, we study the distortion that voting rules can achieve under different social costs.

2.1 Computing the Order From an Election

An essential property in line metric settings is the ability to determine the order of agents based on their preferences. This result has been established in prior work. Specifically, Elkind and Faliszewski [22] and Babashah et al. [8] demonstrate that if voters' preference lists are pairwise distinct, it is possible to uniquely determine the ordering on the line, along with the ordering of the non-Pareto-dominated alternatives. While their setting differentiates between voters and alternatives, this result naturally extends to our context, where agents serve as both voters and candidates. In our setting, this follows directly from a simpler observation: for any three agents, their relative order on the line can be reconstructed from their preference rankings. We state this result as a lemma, which serves as a foundation for many results in this paper as it guarantees that the order of agents in any election can be uniquely identified.

Lemma 2.1 (Elkind and Faliszewski [22], Babashah et al. [8]). *For every election $\mathcal{E} = ([n], k, \succ)$, we can compute a permutation $\pi\colon [n] \to [n]$ of the agents such that, for any consistent position vector $x \in (-\infty, \infty)^n$ with $x \rhd \succ$, we have either $x_{\pi(1)} \le x_{\pi(2)} \le \cdots \le x_{\pi(n)}$ or $x_{\pi(n)} \le x_{\pi(n-1)} \le \cdots \le x_{\pi(1)}$.*

We briefly highlight some structural features that distinguish the peer selection setting from the general setting. First, unlike

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[1] Note that this definition allows for agent-dependent tie-breaking; i.e., when $d(a,b) = d(a,c)$ agent a can rank either $b \succ_a c$ or $c \succ_a b$, independently of other agents. This assumption makes the problem in principle harder, so that our upper bounds on the distortion remain valid if a common tie-breaking rule is employed, and it allows us to construct simpler examples for lower bounds. It is not hard to see that the same lower bounds can be obtained without the assumption: Whenever a metric has ties, distances can be perturbed by a small enough constant ε so that there are no longer ties and the distortion does not improve.

the general voting setup with disjoint voters and candidates, the peer selection problem provides us with not only the order of the candidates but also the exact order of the voters. Second, this order can be efficiently reconstructed: we can identify extreme agents by observing who consistently appears at the bottom of others' rankings. Once such an agent is identified, their ranking determines the full order of agents along the line. Third, the mutual evaluations are more restricted, so we gain more information about relative preferences, such as how many voters prefer one candidate over another, than in the general voting setting. These features inform the design of improved mechanisms in peer selection and present new algorithmic and structural challenges.

For simplicity, whenever we fix an election throughout the paper, we will assume w.l.o.g., that the agents are already ordered, i.e., that the permutation π stated in the lemma is the identity. Hence, we denote the ordered agents by $1, \dots, n$ and informally refer to this order as *from left to right*.

3 UTILITARIAN SOCIAL COST

Using Lemma 2.1, we know that the order of agents can be fully determined from the preference profile $\succ$. This allows us to identify the *median agent*, who minimizes the total distance to all other agents. Hence, when selecting a single agent ($k = 1$) under the utilitarian objective, the median agent is optimal. This observation holds regardless of whether the individual cost is defined additively or via the q-cost, since both notions coincide when $k = 1$.

For larger committee sizes ($k > 1$), it becomes necessary to define how a voter's distances to the selected agents are aggregated. In this section, we study two aggregation rules: one that considers the sum of all distances to selected agents in Section 3.1, and one that considers the distance to the qth closest selected agent in Section 3.2.

3.1 Utilitarian Additive Cost

In this section, we focus on the *utilitarian additive* objective for committee selection. This objective aims to minimize the *utilitarian additive social cost*, which is defined as the total distance from all agents to the selected committee. Formally, the *utilitarian additive social cost* of a committee $S' \in \binom{A}{k}$ is given by

$$\mathrm{SC}(S', A; d) = \sum_{a \in A} \sum_{b \in S'} d(a, b).$$

The cost of each agent $a \in A$ is the sum of their distances to all members of the selected committee S', and the overall social cost is the sum of these individual costs across all agents in A.

It is straightforward to verify that the optimal committee can be directly identified from the preference profile when the committee size is both $k = 1$ and $k = 2$. For $k = 1$, the optimal committee consists of the median agent, as discussed above. For $k = 2$, the optimal committee consists of the two median agents if n is even, or the median agent together with the agent closest to them, if n is odd. In both cases, the optimal agents can be determined solely from $\succ$, without knowledge of the underlying metric, yielding a distortion of 1.

We now introduce our voting rule for selecting a committee of size $k \geq 2$, called the Median Alternation rule. This rule can be viewed as a generalization of the *Polar Comparison* rule proposed by Babashah et al. [8] for multi-winner elections. In contrast to the Polar Comparison rule, the Median Alternation rule (i) applies to all committee sizes k, (ii) is simpler since it does not require case distinctions or pairwise comparisons, and (iii) achieves improved distortion guarantees under peer selection.

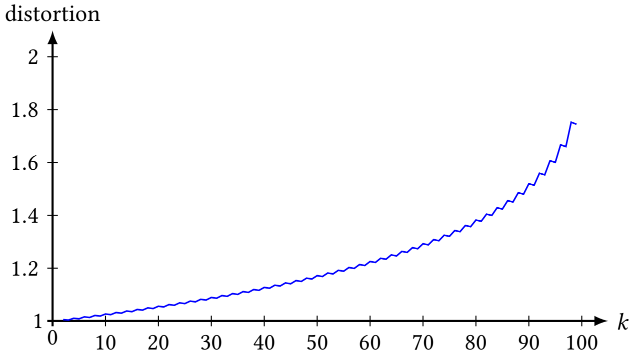


Figure 1: Distortion of Median Alternation stated in Theorem 3.1 for $n = 100$ and $k \in \{2, \dots, 99\}$.

Voting Rule 1 (Median Alternation). *Compute the order of the agents $1, \dots, n$ on the line and let $m = \lceil \frac{n+1}{2} \rceil$ denote the (upper) median agent. If $n - k$ is even, return $S = \left\{ \frac{n-k}{2} + 1,\ \frac{n-k}{2} + 2,\ \dots,\ \frac{n+k}{2} \right\}$. If $n - k$ odd and $\lceil \frac{n-k}{2} \rceil \succ_m \lceil \frac{n+k}{2} \rceil$, return $S = \left\{ \lceil \frac{n-k}{2} \rceil, \lceil \frac{n-k}{2} \rceil + 1, \dots, \lceil \frac{n+k}{2} \rceil - 1 \right\}$. Otherwise, return $S = \left\{ \lceil \frac{n-k}{2} \rceil + 1, \lceil \frac{n-k}{2} \rceil + 2, \dots, \lceil \frac{n+k}{2} \rceil \right\}$.*

On an intuitive level, the rule can be understood as constructed by going through the rank list of the median(s) agent(s), selecting agents in the order reported by them but alternating between those to their left and to their right. This ensures a balanced representation of agents on both sides.

For larger committees, a key ingredient in our analysis is that there always exists an optimal committee consisting of consecutive agents. We formalize this in the full version. We now present our main result in terms of utilitarian additive social cost, regarding the distortion guaranteed by Median Alternation.

Theorem 3.1. *For every $n, k \in \mathbb{N}$ with $n \geq k \geq 2$, under the utilitarian additive social cost the distortion of Median Alternation is at most*

$$\frac{1}{k} \left(2n + \chi - 2\sqrt{n(n - k + \chi)} \right),$$

where $\chi = 1$ if $n - k$ is odd and $\chi = 0$ otherwise.

The bound ranges between 1 and 2: it is closer to 1 when k is small relative to n, and approaches 2 as k approaches n. Figure 1 illustrates the bound for $n = 100$ and k between 2 and $n - 1$.

We now summarize the main ideas involved in the proof of Theorem 3.1. The key ingredient is a reduction from any metric to another one where all agents are in one of two locations and the distortion of our rule has not decreased. To prove this reduction, we use the linearity of the social cost to show that an agent (or set of agents at the same location) can always be moved in one direction

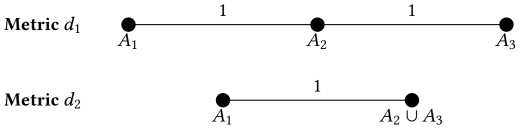


Figure 2: Metrics considered in the proof of Theorem 3.2. In this and all similar figures throughout the paper, the (sets of) agents are represented by circles, with the identity of the agents or sets below them, and the distances between them are written on top of the corresponding line segments. All figures consider indistinguishable metrics for a certain preference profile of the agents and thus any voting rule must select the same subsets for any of these metrics.

such that the distortion does not improve. By iteratively moving agents in their non-improving direction, we can reach a metric where all agents lie in one of the original extreme positions and the distortion has not improved. We finally compute the worst-case number of agents on each of these points for the distortion of the Median Alternation rule to conclude.

In terms of lower bounds, we prove that for any $k \geq 3$, there exists n such that any (n, k)-voting rule has distortion at least 1.0914. To do so, for any fixed k we consider n agents partitioned into sets A_1, A_2, A_3, with $|A_3| \leq |A_1| \leq k$ and $|A_2| + |A_3| = k$. We consider two metrics depicted in Figure 2, both with the agents in the same set located at the same point and $x_a \leq x_b \leq x_c$ for any $a \in A_1, b \in A_2$, and $c \in A_3$. In one metric, A_2 is equidistant to A_1 and A_3; in the other, A_2 and A_3 are located at the same point. Intuitively, a rule performs badly in the first instance if it selects many agents from A_3 and in the second instance otherwise. In the full version, we formalize this distinction through a threshold, obtain bounds parameterized on this threshold for each of the corresponding cases, and obtain the overall bound by optimally picking the threshold and the number of agents in each set that maximizes the minimum distortion between the two cases.

Theorem 3.2. *For every $k \in \mathbb{N}$ with $k \geq 3$, there exists $n \in \mathbb{N}$ with $n \geq k$ such that, for every (n, k)-voting rule f, $dist(f) \geq 1.0914$ for utilitarian additive social cost.*

3.2 Utilitarian q-Cost

In this section, we study the distortion of voting rules in the context of utilitarian q-cost: For agents A, committee size k, committee $S' \in \binom{A}{k}$, and metric d, if $\tilde{d}(a) \in \mathbb{R}_+^S$ contains the values $\{d(a, s) \mid s \in S'\}$ in increasing order then the social cost of S' is $\text{SC}(S', A; d) = \sum_{a \in A} \tilde{d}(a)_q$.

Similarly to the setting with disjoint voters and candidates, the distortion of voting rules heavily depends on the value of q. The key intuition is that when q is large, the q-cost objective emphasizes the lower-ranked positions in each agent's ranking, which leads to a higher optimal cost. This makes the problem easier, as the selected committee can better approximate the worst-case distances. In contrast, when q is small, the objective focuses on the top few distances, so there may exist very good committees that are difficult to identify with only ordinal information, leading to greater potential distortion. Indeed, a result by Caragiannis et al. [15] implies the existence of (n, k)-voting rules with distortion 3 for q-cost whenever $q > \frac{k}{2}$, since their result holds in the setting with disjoint voters and candidates and general distance metrics. We complement this result with a lower bound that ranges from $\frac{3}{2}$ to 2 as q varies between $\lceil \frac{k}{2} \rceil + 1$ and k. For $q \leq \frac{k}{2}$, Caragiannis et al. [15] showed that no rule provides a bounded distortion; we prove that this still holds in our setting.

Theorem 3.3. *For every $k \in \mathbb{N}$ with $k \geq 2$ and $q \in \mathbb{N}$ with $q \leq \frac{k}{2}$, there exists $n \in \mathbb{N}$ with $n \geq k$ such that, for every (n, k)-voting rule f, $\text{dist}(f)$ is unbounded for utilitarian q-cost. For every $k \in \mathbb{N}$ with $k \geq 3$ and $q \in \mathbb{N}$ with $\frac{k}{2} + 1 \leq q \leq k$, there exists $n \in \mathbb{N}$ with $n \geq k$ such that, for every (n, k)-voting rule f, $\text{dist}(f)$ is at least $2 - \frac{k-q}{4q-k-3}$ for utilitarian q-cost.*

These lower bounds are proven in ??. To prove the bound for $q \leq \frac{k}{2}$, we partition all but q agents into $\lfloor \frac{k}{q} \rfloor \geq 2$ sets and consider two metrics that differ in the position of the remaining q agents: relatively close to the other agents in one metric; very far in the other. Selecting these q agents leads to an unbounded distortion in the former case but is necessary for a bounded distortion in the latter. These metrics are illustrated in Figure 3. To prove the bound for $q > \frac{k}{2}$, we consider three different metrics consistent with the same rankings and show that, in one of them, q agents in one extreme cannot be selected. This bound increases in q, varying between $\frac{3}{2} + \frac{3}{2(k+1)}$ for $q = \frac{k}{2} + 1$ and 2 for $q = k$. Part (a) of ?? in ?? illustrates the bounds and part (b) the metrics we use to prove them.

In the remainder of this section, we study the case where $q = k = 2$ in further detail and achieve the best-possible distortions of $\frac{4}{3}$ and 2 for odd and even n, respectively, through natural voting rules that are able to leverage the different objectives involved in the problem. In this setting, the social cost of a committee S' for an agent a is determined by the distance to the *farthest* agent in S': For agents A and $S' \in \binom{A}{2}$, the social cost is $\text{SC}(S', A; d) = \sum_{a \in A} \max_{s \in S'} d(a, s)$. Intuitively, one aims to select agents that are both close to each other and close to the median agent(s). In particular, the optimal committee always consists of consecutive agents: For non-consecutive agents, replacing the most extreme agent with another closer to the median cannot decrease the social cost.

A visual aid for computing the social cost of a committee is what we call *stair diagrams*, illustrated in Figure 4. The area below both *staircases* is a cost that every committee of size $k = 2$ must incur. A specific committee $\{s_1, s_2\}$ must incur, in addition, a cost equal to the area of the rectangle whose basis is the line segment between both selected candidates and whose height is n (and potentially an additional area to reach this point from the median). In the full version, we state and prove a useful lemma to bound the social cost of any committee from below.

We now establish tight distortion bounds of $\frac{4}{3}$ and 2 for odd and even values of n, respectively. For $n = 3$, the optimal set corresponds to the median agent and the agent that the median prefers among the others, implying a simple rule with distortion 1. For $n \geq 5$ odd, we introduce a voting rule called Favorite Couple. For an election $\mathcal{E} = (A, k, \succ)$, we say that $a, b \in A$ are a *couple* if they rank each other above all other agents; i.e., if $b \succ_a c$ and $a \succ_b c$ for every $c \in A \setminus \{a, b\}$. Favorite Couple selects the closest couple to the median when restricting to the five middle agents.

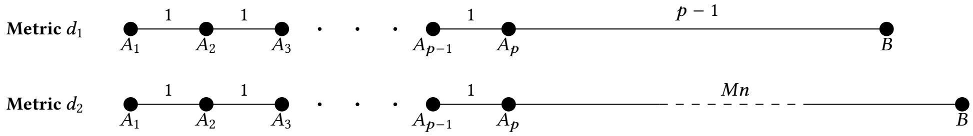
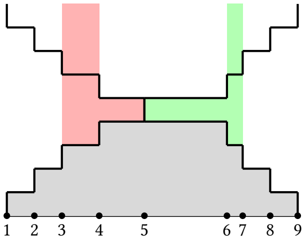
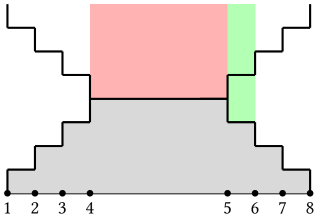


Figure 3: Metrics considered in the proof of Theorem 3.3 for the case with $q \leq \frac{k}{2}$. The set B contains q agents; the remaining agents are distributed evenly among the sets $A_1, \ldots, A_p$, where $p = \left\lfloor \frac{k}{q} \right\rfloor$.



(a) Stair diagram for $n = 9$. The red area corresponds to the committee $\{3, 4\}$; the green area to $\{6, 7\}$.



(b) Stair diagram for $n = 8$. The red area corresponds to the committee $\{4, 5\}$; the green area to $\{5, 6\}$.

Figure 4: Stair diagrams for 9 and 8 agents. The common cost incurred by any committee is shown in gray; the additional cost of two specific committees is shown in red and green.

Voting Rule 2 (Favorite Couple). *For a preference profile $\succ$, compute the order from left to right $1, \ldots, n$ and let $m = \frac{n+1}{2}$ be the median agent. If $\{m-1, m\}$ or $\{m, m+1\}$ is a couple, return this couple. Else, return $\{m+1, m+2\}$ if $m+2 \succ_m m-2$, and $\{m-2, m-1\}$ otherwise.*

Theorem 3.4. *For $n \geq 5$ odd, Favorite Couple achieves a distortion of $\frac{4}{3}$ for utilitarian 2-cost. There exists $n \in \mathbb{N}$ such that, for every $(n, 2)$-voting rule f, $\operatorname{dist}(f) \geq \frac{4}{3}$ for utilitarian 2-cost.*

To establish the distortion of Favorite Couple, we address different cases depending on the set selected by this rule and the optimal set, bounding the social cost of the former from above and the optimal social cost from below. Intuitively, Favorite Couple can select a suboptimal committee (e.g. the red one instead of the green one in Figure 4.(a)), but this imposes several restrictions on the distances (such as $d(3,4) \leq \min\{d(4,5), d(6,7)\}$ and $2d(3,4) \leq d(5,7)$ in this example). These inequalities imply lower bounds on the cost incurred by any voting rule.

We remark that the restriction to a few middle agents is necessary: Choosing an arbitrary couple can lead to a distortion of up to 2. For example, if there are n agents with distances $d(a, a+1) = 1 + a\varepsilon$ for all $a \in [n-1]$ and a small $\varepsilon > 0$, the only couple is $\{1, 2\}$ with a social cost of approximately $\frac{n^2}{2}$, while the committee consisting of the median agent and any neighbor is close to $\frac{n^2}{4}$.

When n is even, the voting rule that selects the two median agents attains the best-possible distortion of 2.

Proposition 3.5. *For an even number of agents n, the voting rule that selects the two median agents achieves a distortion of 2 for utilitarian 2-cost. Moreover, there exists $n \in \mathbb{N}$ such that, for every $(n, 2)$-voting rule f, we have $\operatorname{dist}(f) \geq 2$ for utilitarian 2-cost.*

4 EGALITARIAN SOCIAL COST

In this section, we study the worst-case distortion achievable by voting rules in the context of egalitarian social cost, where the social cost of a committee corresponds to the maximum cost of this committee for some agent.

In the context of single-candidate elections, any voting rule achieves a distortion of 2, since for any election and any selected candidate, one of the extreme agents will incur a cost of at least half of its distance to the other extreme agent, and no rule can induce a social cost larger than the distance between the extreme agents. Despite its simplicity, this turns out to be the best possible distortion in this context. To see this, consider four agents and two metrics, each of them with two agents at one extreme, one agent in the opposite extreme, and one in the middle. No rule can choose the middle agent in both instances, which yields a distortion of 2. We state these facts in the following proposition.

Proposition 4.1. *For every $n \in \mathbb{N}$, any $(n, 1)$-voting rule has distortion 2 for egalitarian social cost. There exists $n \in \mathbb{N}$ such that, for every $(n, 1)$-voting rule f, $\operatorname{dist}(f) \geq 2$ for egalitarian social cost.*

In what follows, we focus on the case where $k \geq 2$ agents are selected.

4.1 Egalitarian Additive Cost

In this section, we focus on egalitarian additive social cost: For agents A, committee size k, and metric d, the social cost of committee $S' \in \binom{A}{k}$ is $\operatorname{SC}(S', A; d) = \max\left\{\sum_{s \in S'} d(a, s) \mid a \in A\right\}$.

When $k = 2$ candidates are to be selected, a simple rule selecting both extreme candidates achieves the best-possible distortion of 1. The intuition is that, for any committee, (1) the cost of the committee is maximized for one of the extreme agents, and (2) the sum of the costs of the committee for both extreme agents is fixed (and equal to two times the distance between them). Thus, selecting both extreme agents ensures they incur the same cost and minimizes the maximum cost between them. For larger k, the intuition regarding the cost of any committee being maximized for the extreme agents remains true. We state this property in the following lemma for later use.

Lemma 4.2. *For every set of agents $A = [n]$, committee size k, committee $S' \in \binom{A}{k}$, and distance metric d, it holds that $SC(S', A; d) = \max\{SC(S', 1; d), SC(S', n; d)\}$.*

Since for any set of agents A, committee size k, committee $S' \in \binom{A}{k}$, and distance metric d we have

$$\mathrm{SC}(S', 1; d) + \mathrm{SC}(S', n; d) = \sum_{a \in S'} (d(1, a) + d(a, n)) = kd(1, n), \quad (1)$$

the previous lemma implies that the optimal committee will be the set that balances the cost for the extreme agents as much as possible. Thus, it is natural to generalize the rule that selects both extreme agents to larger committees, by selecting roughly $\frac{k}{2}$ agents from each extreme.

Voting Rule 3 (k-Extremes). *For a preference profile $\succ$, compute the order of agents from left to right $1, \dots, n$ and return $S = \{1, \dots, \lfloor \frac{k}{2} \rfloor\} \cup \{n - \lceil \frac{k}{2} \rceil + 1, \dots, n\}$.*

Theorem 4.3. *For every $n, k \in \mathbb{N}$ with $n \geq k \geq 2$, k-Extremes has a distortion for egalitarian additive social cost of at most $\frac{3}{2} - \frac{1}{2(k-1)}$ if k is even and at most $\frac{3}{2} - \frac{1}{k(k-1)}$ if k is odd. Conversely, for every $k \in \mathbb{N}$ with $k \geq 3$ there exists $n \in \mathbb{N}$ with $n \geq k$ such that, for every (n, k)-voting rule f, $\mathrm{dist}(f) \geq \frac{3}{2} - \frac{1}{k}$ for egalitarian additive social cost.*

Note that the theorem captures the previously claimed distortion of 1 for $k = 2$ and approaches $\frac{3}{2}$ as k grows, which is best possible up to $O(\frac{1}{k})$ terms. For the distortion of k-Extremes, we assume w.l.o.g. that agent 1 (and not agent n) incurs the maximum cost. The result follows easily when agent 1 incurs a small cost; the most involved part of the proof involves bounding the social cost of any committee from below when this is not the case. As for the lower bound, our worst-case instances involve $k+1$ agents in one extreme, a single agent in the other extreme, and k agents in the middle, which are selected in the optimal committee but cannot be detected by any rule when considering two symmetric distance metrics.

4.2 Egalitarian q-cost

In this brief section, we state our results for the egalitarian q-cost objective. The social cost is now the maximum over agents of the distance from each agent to their qth closest candidate; i.e., $\mathrm{SC}(S', A; d) = \max \{\tilde{d}(a)_q \mid a \in A\}$ for agents A, committee size k, committee $S' \in \binom{A}{k}$, and distance metric d, where $\tilde{d}(a) \in \mathbb{R}_+^{S'}$ contains the values $\{d(a, s) \mid s \in S'\}$ in increasing order.

The following theorem states that no voting rule can guarantee a constant distortion for q-cost when $q \leq \frac{k}{3}$, as in the setting of disjoint voters and candidates [15]. To prove it, we partition the agents into $\lfloor \frac{k}{q} \rfloor$ sets and consider two symmetric distance metrics where all but one set are placed at a unit distance from one another and two sets in one extreme are at the same location. We show that no rule can pick q agents from each location.

Theorem 4.4. *For every $k, q \in \mathbb{N}$ with $\frac{k}{3} \geq q$, there exists $n \in \mathbb{N}$ with $n \geq k$ such that, for every (n, k)-voting rule f, $\mathrm{dist}(f)$ is unbounded for egalitarian q-cost.*

In the context of egalitarian q-cost for $q > \frac{k}{3}$, much better results are possible. The case with $q > \frac{k}{2}$ behaves similarly to the setting where a single candidate is to be selected: Any voting rule achieves a distortion of 2, and this is best possible. When $\frac{k}{3} < q \leq \frac{k}{2}$, the best-possible distortion a voting rule can achieve is again 2, but not any rule does so. We show that k-Extremes attains it. For the upper bounds, we prove that the social cost of the set selected by this rule is at most the distance from the agent closest to the center to their nearest extreme, and bound the social cost of the optimal set from below by half of this distance.

Theorem 4.5. *Let $n, k, q \in \mathbb{N}$ be such that $n \geq k \geq 2$ and $q > \frac{k}{3}$. If $q > \frac{k}{2}$, any (n, k)-voting rule has distortion 2 for egalitarian q-cost. If $q > \frac{k}{3}$, k-Extremes has distortion 2 for egalitarian q-cost. For every $k, q \in \mathbb{N}$ with $q > \frac{k}{3} \geq 1$, there exists $n \in \mathbb{N}$ with $n \geq k$ such that, for every (n, k)-voting rule f, $\mathrm{dist}(f) \geq 2$.*

5 DISCUSSION

In this work, we have introduced the study of metric distortion in committee elections where voters and candidates coincide and provided a first step towards an understanding of this setting by focusing on the line metric. Our results span a variety of social costs and include both analyses of voting rules and constructions of negative instances to provide impossibility results. Although most of our results are tight, an intriguing gap remains for utilitarian q-cost when q is greater than $\frac{k}{2}$. We believe that rules with a distortion better than the current upper bound of 3 exist and their design may benefit from the insights provided by our rule for $q = k = 2$.

The study of the distortion of voting rules in more general metric spaces constitutes an interesting direction for future work, even in the general setting. The main open question concerns the design of voting rules that achieve small distortion beyond the line metric under the utilitarian additive cost.

Another challenge in the design of elections is preventing strategic behavior. A mild assumption in the context of peer selection, adopted by the growing literature on impartial selection, is that agents' primary concern is whether they are selected themselves, and a voting rule is deemed impartial if an agent cannot affect this fact by changing their reported preferences. On the other hand, a rule is called strategyproof in the voting literature if no agent can misreport their preferences and lead to a better outcome with respect to their actual preferences. Both notions—impartiality and strategyproofness–can be readily applied to our setting, the former being a relaxed version of the latter in this case. Most of the voting rules developed in this work depend on the order of the agents and are thus strategyproof if one restricts voters' deviations to those that are consistent with this order. This constitutes a sensible way to define these axioms, as inconsistent reports could be easily detected and punished by the designer. A notable exception is the Favorite Couple rule, which does not depend exclusively on the order and is not even impartial: For instance, an agent next to the median agent could in some cases modify their ranking, reporting the median agent immediately after themselves, to create a couple and become selected. Designing impartial and strategyproof voting rules with bounded distortion for peer selection constitutes an interesting challenge for future work in the area.

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