Time-Varyingness in Auction Breaks Revenue Equivalence

Extended Abstract*

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ABSTRACT

The revenue equivalence theorem states that equilibrium revenue is the same across different auction mechanisms, such as first- and second-price ones. However, the environment in the real-world auctions varies over time and can prevent bidders from reaching such an equilibrium. While second-price auctions allow bidders to automatically maintain equilibrium through truthful bidding, first-price auctions require bidders to track moving equilibria through continuous learning. We demonstrate that this tracking lag breaks revenue equivalence. Which of the first- and second-price auctions yields higher revenue depends on the correlation between the basis value (the standard price to bid) and the value interval (the width of possible values). This study uncovers a novel phenomenon that can be triggered by time-varying environments in real-world auctions.

KEYWORDS

Auction, Revenue Equivalence, Learning in Games, Time-Varying

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1 INTRODUCTION

The revenue equivalence theorem is a powerful yet counterintuitive theoretical result in auction theory [16, 21, 23]. It shows that in equilibrium, the seller's revenue is independent of the auction mechanism, typically first- and second-price auctions. This theorem holds when the value of an item follows a symmetric, independent, and private distribution among bidders. It has been reported that revenue equivalence is broken down, for example, when the value distribution is asymmetric [4, 10, 12, 20] or interdependent [15]. Revenue equivalence is an important issue for designing advantageous auction mechanisms for sellers or bidders. As a related event, most supply-side platforms (sellers) have switched their mechanism from second- to first-price [1, 5, 9, 19].

Although revenue equivalence is discussed in equilibrium, real-world auctions typically do not reach there. One primary factor is a time-varying environment that continually shifts the equilibrium. Indeed, daily and weekly cycles equipped with randomness, possibly caused by the rhythm of human life, are observed in bidding from empirical auction data [9, 24, 25]. The resistance to such a time-varying environment differs between the first- and second-price auctions. In a second-price auction, they only need to bid their perceived value (called truthful bidding), leading to the equilibrium payment. Hence, they can automatically maintain the equilibrium (see the gray lines in Fig. 1-A). In a first-price auction, however, they can only track the equilibrium by learning at best (see the orange lines) because bidders should bid the equilibrium payment. Therefore, even if the distribution of an item value is symmetric, independent, and private, its time-varyingness might break revenue equivalence. A fundamental question to understand real-world auctions with a time-varying environment is;

Which of the first- and second-price auctions are preferred by bidders (or sellers)?

This study answers this question by extending the symmetric Bayesian Nash equilibrium to symmetric non-equilibrium dynamics, in which bidders learn unknown true parameters of a time-varying value distribution. As a theoretical result, we characterize a uniform distribution by key parameters (see Fig. 1-B), i.e., the basis value (the standard price to bid) and the value interval (the width of possible values), and prove that the correlation between these parameters breaks revenue equivalence. In particular, the positive correlation results in larger payoffs for bidders in first-price auctions, and the inverse is also true (see Fig. 1-C). We also experimentally demonstrate that this result generalizes to other distributions, such as log-normal ones.

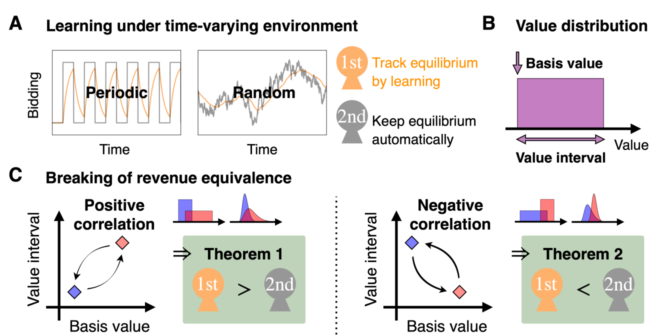

Figure 1: Overview of learning in time-varying auctions.

2 TIME-VARYING AUCTION

Classical Auction: Auction theory considers $n \in \mathbb{N}$ bidders, labeled as $i \in \{1, \cdots, n\}$, in general. An item is offered to the bidders

*The full version is available at https://arxiv.org/abs/2410.12306.

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every time, and its value is $v_i > 0$ for bidder i. Each bidder independently determines the bidding b_i for the item. The bidder with the highest bid wins the item. The winner's payment depends on the auction mechanism $\mathcal{M}$. In first-price auctions, the payment is the highest price ($b^{1\text{st}} = \max_j b_j = b_i$), while in second-price auctions, it is the second-highest price ($b^{2\text{nd}} = \max_{j\neq i} b_j$). Thus, the payoffs of first-price ($\mathcal{M}$ = 1st) and second-price ($\mathcal{M}$ = 2nd) auctions are

$$u_i^{\mathcal{M}}(b_i|v_i) = (v_i - b^{\mathcal{M}})\mathbb{1}[b_i = \max_j b_j]. \quad (1)$$

Each bidder's strategy is to bid b_i on the observed value v_i, i.e., the function of $b_i(v_i)$. This study considers that v_i follows the same distribution $f(v_i;\boldsymbol{\theta}^*)$ on $[0,\infty)$ independently for each i (called the setting of "symmetric, independent, and private" value [11]). Here, we assume that $f(v_i;\boldsymbol{\theta}^*)$ is characterized by finite parameters, denoted as $\boldsymbol{\theta}^* \in \mathbb{R}^d$ and that $F(v_i;\boldsymbol{\theta}^*)$, the cumulative distribution of $f(v_i;\boldsymbol{\theta}^*)$, is continuous for v_i and $\boldsymbol{\theta}^*$.

Equilibrium Bidding: Given $\boldsymbol{\theta}^*$, the symmetric Bayesian Nash equilibrium in a first-price auction is $b_i(\cdot) = b^*(\cdot)$ for all i such that

$$b^*(v) = v - \frac{1}{F(v;\boldsymbol{\theta}^*)^{n-1}}\int_0^v F(z;\boldsymbol{\theta}^*)^{n-1}\mathrm{d}z. \quad (2)$$

The equilibrium in a second-price auction is given by truthful bidding $b_i(\cdot) = b^{\mathrm{TB}}(\cdot)$ for all i such that $b^{\mathrm{TB}}(v) = v$, meaning that each one bids the value it perceives. The revenue equivalence theorem states that the expected values of $b^{1\text{st}}$ and $b^{2\text{nd}}$ (i.e., payments) are the same.

Time-Varying Value Distribution: This study considers that the parameters of the value distribution vary over time, i.e., $\boldsymbol{\theta}^*(t)$ for $t \in [0,T]$. Because truthful bidding $b^{\mathrm{TB}}(v)$ is independent of $\boldsymbol{\theta}^*$, bidders in second-price auctions can maintain their equilibrium. On the other hand, the equilibrium bidding in first-price auctions $b^*(v)$ obviously depends on $\boldsymbol{\theta}^*$. Thus, suppose that bidders estimate the unobservable true parameter $\boldsymbol{\theta}^*(t)$ as $\boldsymbol{\theta}(t)$ and alternatively bid

$$b(v;\boldsymbol{\theta}(t)) = v - \frac{1}{F(v;\boldsymbol{\theta}(t))^{n-1}}\int_0^v F(z;\boldsymbol{\theta}(t))^{n-1}\mathrm{d}z.$$

Learning Dynamics in First-Price Auction: Let $w_{\boldsymbol{\theta}^*}(\boldsymbol{\theta}',\boldsymbol{\theta})$ denote the expected payoff of a focal bidder who uses the bidding $b(v;\boldsymbol{\theta}')$ while all the others use $b(v;\boldsymbol{\theta})$ under the true parameter $\boldsymbol{\theta}^*$. This expected payoff is described as

$$w_{\boldsymbol{\theta}^*}(\boldsymbol{\theta}',\boldsymbol{\theta}) = \int_0^\infty (v - b(v;\boldsymbol{\theta}'))f(v;\boldsymbol{\theta}^*)F(v';\boldsymbol{\theta}^*)^{n-1}\mathrm{d}v.$$

Here, we defined $v' = v'(v,\boldsymbol{\theta},\boldsymbol{\theta}')$ such that $b(v';\boldsymbol{\theta}) = b(v;\boldsymbol{\theta}')$. We extend the symmetric Bayesian Nash equilibrium to non-equilibrium learning dynamics as

$$\dot{\boldsymbol{\theta}}(t) = \left.\frac{\partial w_{\boldsymbol{\theta}^*(t)}(\boldsymbol{\theta}',\boldsymbol{\theta}(t))}{\partial \boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}'=\boldsymbol{\theta}(t)}.$$

Our continuous-time approach has often been taken in learning in games [2, 3, 6–8, 13, 14, 17, 18, 22].

Long-Run Payoff: Each time t, the expected payoffs in first- and second-price auctions are described as

$$w^{1\text{st}}_{\boldsymbol{\theta}^*(t)}(\boldsymbol{\theta}(t)) = w_{\boldsymbol{\theta}^*(t)}(\boldsymbol{\theta}(t),\boldsymbol{\theta}(t)), \quad w^{2\text{nd}}_{\boldsymbol{\theta}^*(t)} = w^{1\text{st}}_{\boldsymbol{\theta}^*(t)}(\boldsymbol{\theta}^*(t)).$$

This study evaluates the long-run payoff of bidders, defined as

$$\bar{w}^{1\text{st}}(T) = \frac{1}{T}\int_0^T w^{1\text{st}}_{\boldsymbol{\theta}^*(t)}(\boldsymbol{\theta}(t))\mathrm{d}t, \quad \bar{w}^{2\text{nd}}(T) = \frac{1}{T}\int_0^T w^{2\text{nd}}_{\boldsymbol{\theta}^*(t)}\mathrm{d}t.$$

3 BREAKING OF REVENUE EQUIVALENCE

To capture theoretical insight, we suppose that value distributions are uniform.

Definition 1 (Uniform distribution). *Uniform distributions assume the parameters $\boldsymbol{\theta}^*(t) = (v_{\mathrm{m}}(t), v_{\mathrm{M}}(t))$ and are defined as*

$$f(v;\boldsymbol{\theta}^*(t)) = \frac{1}{v_{\mathrm{M}}(t) - v_{\mathrm{m}}(t)}\mathbb{1}[v_{\mathrm{m}}(t) \le v \le v_{\mathrm{M}}(t)].$$

We also define the estimated parameters as $\boldsymbol{\theta}(t) = (x(t), y(t))$.

For this uniform distribution, the equilibrium bidding is calculated as

$$b(v;\boldsymbol{\theta}^*(t)) = \frac{n-1}{n}(v - v_{\mathrm{m}}(t)) + v_{\mathrm{m}}(t),$$

meaning that bidders can bid optimally only by knowing $v_{\mathrm{m}}(t)$ and considering the difference between it and the perceived price $v - v_{\mathrm{m}}(t)$. Thus, $v_{\mathrm{m}}(t)$ is interpreted as the "basis value" in bidding. In addition, since bidders observes a random value between $v_{\mathrm{m}}(t)$ and $v_{\mathrm{M}}(t)$, $\Delta v(t) = v_{\mathrm{M}}(t) - v_{\mathrm{m}}(t)$ is defined as the "value interval" of uniform distribution.

When $v_{\mathrm{m}}(t)$ and $\Delta v(t)$ positively correlate, bidders prefer first-price auctions as shown in the following theorem.

Theorem 1 (Revenue inequivalence by positive correlation). *Suppose that the true parameter can take any K-states, i.e., $\boldsymbol{\theta}^*(t) \in \{\boldsymbol{\theta}^{(1)},\cdots,\boldsymbol{\theta}^{(K)}\}$ and that $v_{\mathrm{m}}^{(k)} < v_{\mathrm{m}}^{(k')} \Rightarrow \Delta v^{(k)} < \Delta v^{(k')}$ holds for all $k, k' \in \{1,\cdots,K\}$, then $\bar{w}^{1\text{st}}(\infty) > \bar{w}^{2\text{nd}}(\infty)$ holds.*

The inverse of this theorem is true (Theorem 2 in the full version). Suppose that the correlation is negative (i.e., $v_{\mathrm{m}}^{(k)} < v_{\mathrm{m}}^{(k')} \Rightarrow \Delta v^{(k)} > \Delta v^{(k')}$ in the statement), then bidders prefer second-price auctions (i.e., $\bar{w}^{1\text{st}}(\infty) < \bar{w}^{2\text{nd}}(\infty)$).

Key Insight: Our theorem provides a key insight that the correlation between the basis value and the value interval breaks revenue equivalence. This insight is not restricted to uniform distributions. For instance, in log-normal distributions, the mean and variance of the logarithmic value intuitively correspond to the basis value and the value interval, respectively. Indeed, our experiments demonstrate that the correlation between the mean and variance determines how revenue equivalence is broken (see the full version for experiments).

4 CONCLUSION

This study demonstrated that the correlation between the basis value and the value interval breaks revenue equivalence, but it employed several simplifications for its theoretical analysis. An impactful future direction is to verify the breakdown of revenue equivalence using empirical data. To make the setting more realistic, it will be important to consider heterogeneity in learning among bidders, as well as asymmetry and interdependence in their value distributions. Despite these extensions, this study uncovers a novel phenomenon that can be triggered by time-varying environments in real-world auctions.

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