Geometric State Fusion for Autonomous Agents: A Comparative Analysis of Dual Quaternion Observer and Kalman Filters

Extended Abstract

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ABSTRACT

Autonomous agents operating in dynamic 3D environments require robust pose estimation that preserves the geometric structure of rigid-body motion. Traditional filtering approaches often decouple rotation and translation, leading to kinematic inconsistencies. This paper presents a comprehensive benchmark of three geometrically-aware architectures: a Dual Quaternion Geometric Observer (GeoDQ), a manifold-aware Unscented Kalman Filter (UKF-M), and an Error-State Kalman Filter (ESKF). We evaluate these on the complete RoNIN dataset (35 trajectories). Results demonstrate that the proposed GeoDQ method significantly outperforms filtering baselines, reducing RMSE by 3.7× compared to ESKF and 6.1× compared to UKF-M. Furthermore, robustness analysis reveals that the geometric observer maintains superior stability under sparse updates. Despite its mathematical rigor, GeoDQ is the most computationally efficient, executing 10% faster than ESKF and 5× faster than UKF-M, making it ideal for resource-constrained embedded agents.

KEYWORDS

Dual Quaternions; Sensor Fusion; SE(3) Manifold; Autonomous Agents; SCLERP

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1 INTRODUCTION

Autonomous agents, from mobile robots to wearables, rely on 6-DoF pose estimation for navigation in GPS-denied environments. Standard approaches like the Extended Kalman Filter (EKF) often decouple translation and rotation, which ignores the intrinsic geometry of the Special Euclidean group $SE(3)$ and leads to linearization artifacts [1, 10]. While Visual-Inertial Odometry (VIO) has advanced the state-of-the-art [5, 9], these methods remain computationally expensive for resource-constrained embedded agents.

Recent directions exploit Lie group theory to improve robustness [2, 4]. Moreover, dual quaternions offer a more elegant alternative by representing pose as a unified entity on the $SE(3)$ manifold. This representation allows for coordinate-free uncertainty propagation and avoids the singularity issues inherent in Euler angles. While advanced filters like UKF-M provide high precision, their computational footprint is too large for embedded hardware. This work proposes a **Dual Quaternion Geometric Observer (GeoDQ)** that leverages Screw Linear Interpolation (SCLERP) [7]. We show that by treating pose as a single geometric entity, we can achieve superior accuracy and speed, fulfilling the requirements of high-frequency real-time estimation on resource-constrained agents.

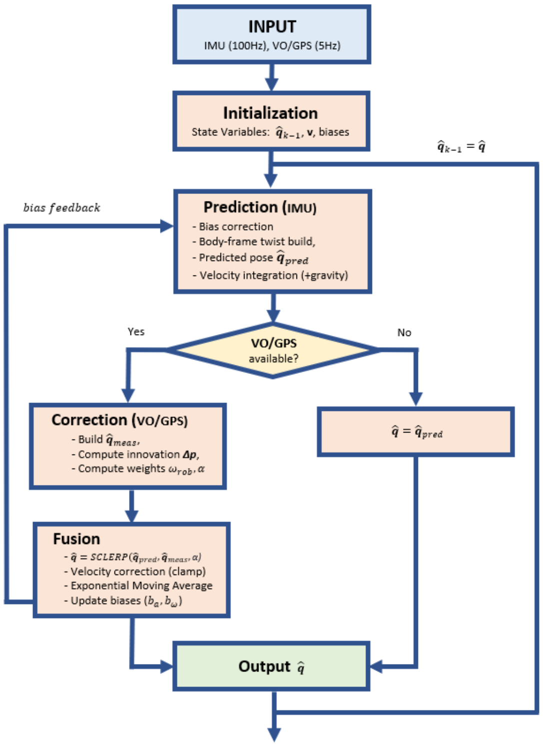

Figure 1: GeoDQ Observer Architecture: IMU prediction corrected via SCLERP fusion with bias feedback loops.

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Table 1: Benchmark Summary (35 RoNIN Trajectories)

Method	*RMSE [m]*	*Execution [ms]*
ESKF	0.141 ± 0.021	3189.2
UKF-M	0.231 ± 0.038	14624.7
GeoDQ (Ours)	**0.038 ± 0.008**	**2813.6**

2 GEOMETRIC STATE FUSION

2.1 Mathematical Framework

A dual quaternion $\hat{\mathbf{q}} = \mathbf{q}_r + \varepsilon \mathbf{q}_d$ encapsulates rotation ($\mathbf{q}_r$) and translation ($\mathbf{q}_d$) without artificial separation, avoiding gimbal lock and artificial decoupling [8]. We utilize SCLERP for the fusion step:

$$\mathrm{SCLERP}(\hat{\mathbf{q}}_1, \hat{\mathbf{q}}_2, \alpha) = \hat{\mathbf{q}}_1 \odot \exp(\alpha \log(\hat{\mathbf{q}}_1^{-1} \odot \hat{\mathbf{q}}_2)) \tag{1}$$

where α is an adaptive weight. This update rule ensures the estimated trajectory follows a geodesically optimal "screw" path on the manifold, preserving the geometric integrity of the motion.

2.2 Observer Architecture

As shown in Fig. 1, the system operates as a non-linear observer. High-frequency IMU data drives the prediction, while asynchronous Visual Odometry (VO) corrections trigger the SCLERP fusion. We utilize an integral feedback loop to continuously estimate sensor biases ($\mathbf{b}_a, \mathbf{b}_g$) and a proportional loop for velocity correction, which ensures the error dynamics remain globally stable on $SE(3)$.

Algorithm 1 GeoDQ Geometric Observer

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Input: $\omega$, $\mathbf{a}$ (IMU), $\mathbf{p}_{vo}, \mathbf{q}_{vo}$ (VO), Params: $dt, \alpha_{base}, k_{ba}$
for each IMU sample do
    $\hat{\mathbf{q}} \leftarrow \hat{\mathbf{q}} \odot \mathrm{dq\_exp}((\boldsymbol{\omega} - \mathbf{b}_g)dt, \mathbf{v}_b dt)$
    $\mathbf{v} \leftarrow \mathbf{v} + (\mathbf{R}(\hat{\mathbf{q}})(\mathbf{a} - \mathbf{b}_a) + \mathbf{g})dt$
    if VO available then
        $\Delta\mathbf{p} \leftarrow \mathbf{p}_{vo} - \mathbf{p}(\hat{\mathbf{q}})$
        $\hat{\mathbf{q}} \leftarrow \mathrm{SCLERP}(\hat{\mathbf{q}}, \hat{\mathbf{q}}_{vo}, \alpha)$
        $\mathbf{b}_a \leftarrow \mathbf{b}_a - k_{ba} \cdot (\mathbf{R}(\hat{\mathbf{q}})^\top \Delta\mathbf{p})$
    end if
end for
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3 EXPERIMENTAL RESULTS

We benchmarked GeoDQ against ESKF and UKF-M on 35 trajectories from the RoNIN dataset [6] (200Hz IMU, 5Hz VO).

Accuracy & Speed: As shown in Table 1, GeoDQ achieves an RMSE of 0.038 m, significantly outperforming the filters. In terms of efficiency, the Numba-JIT optimized GeoDQ is the fastest, executing in 2.8 ms per trajectory. This performance is critical for agents with limited battery life or high-frequency control loops. We observe that while UKF-M is precise, it overshoots during high-dynamic directional changes, whereas the geometric observer maintains a smooth path (see Fig. 2).

Qualitative Tracking: Fig. 2 shows a representative sequence (‘a025_2‘). While UKF-M exhibits significant oscillations and vertical drift, GeoDQ (red dashed) maintains a smooth and precise path, closely matching the ground truth. This is critical for practical applications like AR navigation [3, 11].

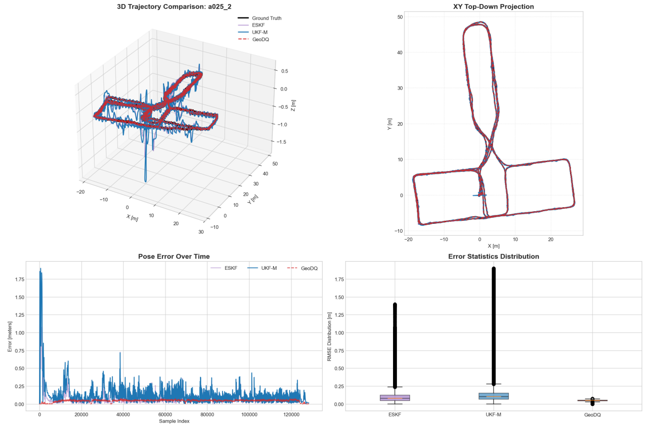

Figure 2: Tracking a025_2. GeoDQ (red dashed) shows superior stability compared to UKF-M (blue) and ESKF (purple).

Robustness: Fig. 3 shows the RMSE vs. Update Interval. Standard filters diverge as updates become sparse. GeoDQ remains stable up to a 30-sample interval (7Hz), demonstrating superior handling of asynchronous, noisy measurements. This resilience allows agents to operate safely even during temporary sensor dropouts.

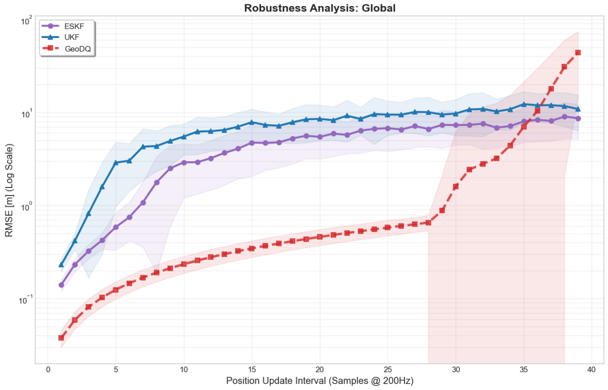

Figure 3: Robustness Analysis: GeoDQ (red) maintains sub-meter error longer than filters as update intervals increase.

4 CONCLUSION

GeoDQ provides a rigorous solution for 6-DoF fusion. By unifying pose on $SE(3)$ into a single algebraic entity, the proposed geometric observer eliminates the accuracy-speed trade-off inherent in traditional Kalman filtering frameworks. GeoDQ’s sub-millisecond execution time and drift-resistant nature make it an alternative for resource-constrained autonomous agents. By avoiding the artificial decoupling of rotation and translation, this approach ensures kinematic consistency in mass-market mobile ecosystems where computational power and battery life are at a premium. Project:

https://afanasyspb.github.io/SE3-Manifold-Lib/geodq.html

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