

Selfish Routing Games with Priority Lanes

Extended Abstract

Yang Li
Northwestern Polytechnical
University
Xi'an, China
liyang123@mail.nwpu.edu.cn

Alexander Skopalik
University of Twente
Enschede, The Netherlands
a.skopalik@utwente.nl

Marc Uetz
University of Twente
Enschede, The Netherlands
m.uetz@utwente.nl

ABSTRACT

We study selfish routing games where users can choose between regular and priority service for each network edge on their chosen path. Priority users pay an additional fee, but in turn they may travel the edge prior to non-priority users, hence experiencing potentially less congestion. For this model, we establish existence of equilibria for linear latency functions and prove uniqueness of edge latencies, despite potentially different strategic choices in equilibrium. Our main contribution demonstrates that marginal cost pricing achieves system optimality: When priority fees equal marginal externality costs, the equilibrium flow coincides with the socially optimal flow, hence the price of anarchy equals 1. This voluntary priority mechanism therefore provides an incentive-compatible alternative to mandatory congestion pricing, whilst achieving the same result. We also discuss the limitations of a uniform pricing scheme. for the priority option.

KEYWORDS

Routing games, congestion, priority pricing, price of anarchy

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1 INTRODUCTION

Congestion is a fundamental challenge in transportation, communication, and service networks. Classical approaches to congestion management rely on capacity expansion or mandatory tolling. In practice, however, many systems increasingly employ *service differentiation*: examples include express toll lanes on highways, priority boarding at airports, and premium bandwidth allocation in communication networks. In these systems, users may voluntarily pay for faster service, while others accept longer delays.

From a theoretical perspective, congestion has long been studied through nonatomic selfish routing games. In such games, users independently select routes that minimize their individual latency, leading to equilibria that can be inefficient. Canonical examples include Pigou's network [9] and Braess' paradox [2]. The inefficiency

of equilibria is captured by the Price of Anarchy (PoA) [7, 10], which equals $4/3$ for linear latency functions [10].

A well-known theoretical solution is marginal-cost (Pigouvian) pricing, which charges users exactly the congestion externality they impose on others and induces socially optimal flows [1, 5]. Despite its strong efficiency guarantees, this approach requires mandatory and edge-specific tolls, which are often politically controversial or operationally infeasible [3]. This gap between theory and practice motivates the study of alternative, voluntary mechanisms.

We analyze routing games with *priority lanes* as a voluntary alternative to congestion taxation. Each edge offers two service classes: a regular lane and a paid priority lane. Priority users traverse edges before regular users and experience less congestion, while regular users are delayed by priority traffic. Users strategically choose both routes and service classes. We ask whether such voluntary mechanisms can achieve strong efficiency guarantees.

Our main result answers this question affirmatively for linear latency functions: when priority fees equal marginal congestion externalities, every equilibrium coincides with the socially optimal flow, yielding $\text{PoA} = 1$. We further analyze equilibrium structure and show that, although equilibria may be non-unique in terms of service-class choices, aggregate flows and welfare outcomes are uniquely determined. Finally, we demonstrate that uniform pricing schemes are fundamentally insufficient. Proofs have been omitted from this extended abstract and can be found in the full paper [8].

2 MODEL AND EQUILIBRIUM


We consider a standard nonatomic routing game on a directed graph $G = (V, E)$ with a finite set of commodities (s_i, t_i) and demands r_i . Let f_e denote the total flow on edge e . Each edge has a linear latency function $\hat{\ell}_e(x) = a_e x + b_e$, where $a_e, b_e \geq 0$. Users experience the average latency

$$\ell_e(f_e) = \frac{1}{f_e} \int_0^{f_e} \hat{\ell}_e(z) dz,$$

which corresponds to sequential service along the edge.

Priority lanes. We extend this model by allowing two service classes on each edge: regular (R) and priority (V). Priority users pay a fee $\omega_e \geq 0$ and are served before regular users. Let \tilde{f}_{eV} and \tilde{f}_{eR} denote priority and regular flows, respectively, with $f_e^t := \tilde{f}_{eV} + \tilde{f}_{eR}$. For linear latency functions, the perceived costs are

$$\tilde{\ell}_{eV}(\tilde{f}_{eV}) = \frac{1}{\tilde{f}_{eV}} \int_0^{\tilde{f}_{eV}} \hat{\ell}_e(z) dz + \omega_e = \frac{1}{2} a_e \tilde{f}_{eV} + b_e + \omega_e,$$

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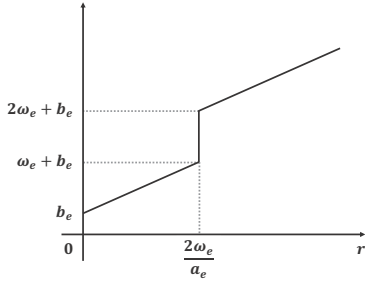


Figure 1: The perceived cost in equilibrium as a function of the total flow r on the single edge e .

$$\tilde{\ell}_e^R(\tilde{f}_e^R, \tilde{f}_e^V) = \frac{1}{\tilde{f}_e^R} \int_{\tilde{f}_e^V}^{\tilde{f}_e^V + \tilde{f}_e^R} \hat{\ell}_e(z) dz = a_e \tilde{f}_e^V + \frac{1}{2} a_e \tilde{f}_e^R + b_e.$$

That is, priority users are affected only by other priority users, while regular users experience congestion from both groups.

Users choose routes and service classes so as to minimize perceived cost. Our first result follows by either using a Kakutani fixed point argument, or can be seen to be implied by [11].

THEOREM 2.1 (EXISTENCE). *For any vector of priority prices ω , an equilibrium exists.*

Equilibrium structure and intuition. Equilibria in this model exhibit distinctive features absent from classical routing games. In particular, equilibria may be non-unique in terms of how flow is split between priority and regular service. For example, on a single edge with total flow exactly at the threshold where users are indifferent between paying for priority and remaining regular, any division of flow between service classes constitutes an equilibrium. See Figure 1 for the equilibrium latency of a network consisting of just a single edge e , a demand of r , and priority cost of ω_e .

Nevertheless, we can show that total edge flows and total latencies are unique across equilibria:

THEOREM 2.2. *If \tilde{f}, \tilde{f}' are equilibrium flows for $(G, r, \tilde{\ell}, \omega)$, and $C(f^t) = \sum_e \ell_e(f_e^t) f_e^t$ denotes the total cost of total edge flow f^t , then*

- (a) $\ell_e(f_e^t) = \ell_e(f_e^{t'})$ for all $e \in E$ and
- (b) $C(f^t) = C(f^{t'})$.

In the proof we argue that the total edge flows fulfill the variational inequality that characterizes equilibria in classical network routing games [6]. Because the cost functions in terms of total edge flows are not a function but a correspondence (e.g. in Figure 1), we need to show that the variational inequalities still hold. For this, we construct a piecewise linear, single-valued function that allows to connect the perceived costs in the extended model to the latencies.

3 OPTIMAL PRICING

While equilibrium existence and uniqueness are structural properties of the model, a central question in mechanism design is how to guide the system towards socially optimal outcomes. In classical congestion games, taxes based on marginal cost pricing are known to induce optimal flows, e.g., [4, 12]. We show that a similar principle applies in our model: if each edge charges a priority fee equal

to its marginal externality cost, then the resulting equilibrium reproduces the socially optimal flow. This generalizes the marginal cost pricing principle to our priority model, and is the main result of this paper, namely that *voluntary* upgrade options – under the right pricing – can fully internalize congestion externalities.

Let f^* be a socially optimal flow minimizing total latency in the original instance without priorities. For linear latency functions, the marginal congestion externality imposed on edge e is $f_e^* \ell'_e(f_e^*)$.

THEOREM 3.1 (OPTIMALITY). *If each edge e charges a priority fee*

$$\omega_e = f_e^* \ell'_e(f_e^*),$$

then every equilibrium of the priority-lane routing game induces total flows equal to f^ . Consequently, the price of anarchy equals 1.*

The intuition is that priority users internalize exactly the congestion they impose on regular users. Although upgrading to priority service is voluntary, equilibrium behavior replicates the outcome of mandatory marginal-cost tolling. Thus, priority lanes provide an incentive-compatible mechanism for congestion management. When priced appropriately, they can fully eliminate inefficiencies of selfishness for linear latency functions, on a voluntary basis.

4 LIMITS OF UNIFORM PRICING

While edge-specific marginal-cost pricing achieves full efficiency, it may be difficult to implement. A natural restriction is *uniform pricing*, where all edges share the same price for the priority option. In addition, one may decide to only install the priority option for a subset of the edges. This, however, does not suffice.

THEOREM 4.1. *There exist instances with linear latency functions in which any uniform priority price yields a price of anarchy arbitrarily close to 4/3, and this remains true if priority pricing is allowed to be installed on any subset of the edges.*

This highlights an intrinsic limitation of simple pricing schemes: Uniform prices cannot generally replicate the effects of marginal-cost taxation and may fail to improve worst-case outcomes.

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