

# The Impossibility of Strategyproof Rank Aggregation

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## ABSTRACT

In rank aggregation, the goal is to combine multiple input rankings into a single output ranking. In this paper, we analyze rank aggregation methods, so-called social welfare functions (SWFs), with respect to strategyproofness, which requires that no agent can misreport his ranking to obtain an output ranking that is closer to his true ranking in terms of the Kemeny distance. As our main result, we show that no anonymous SWF satisfies unanimity and strategyproofness when there are at least four alternatives. This result is proven by SAT solving, a computer-aided theorem proving technique, and verified by Isabelle, a highly trustworthy interactive proof assistant. Further, we prove by hand that strategyproofness is incompatible with majority consistency, a variant of Condorcet-consistency for SWFs. Lastly, we show that all SWFs in two natural classes have a large incentive ratio and are thus highly manipulable.

## KEYWORDS

Rank aggregation; Social welfare functions; Social choice theory; Strategyproofness; Computer-aided theorem proving

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## 1 INTRODUCTION

An important problem for multi-agent systems is rank aggregation: multiple input rankings need to be aggregated into a single output ranking. For instance, this task arises when a hiring committee is asked to produce a ranking of the applicants based on the preferences of the committee members [25, 55], when aggregating the outputs of multiple ranking algorithms in ensemble learning [61, 67], or when recommender systems infer an output ranking based on the preferences of multiple users [1, 66]. Moreover, rank aggregation is applied in computational biology [54, 60], engineering [43], and meta-search [38, 68]. Motivated by these numerous applications, we will investigate rank aggregation through the lens of social choice theory. In this field, rank aggregation is formalized via *social welfare functions (SWFs)*, which map every profile of (complete and strict) input rankings to a single output ranking.

Specifically, we are interested in the question of whether there are SWFs that incentivize voters to report their rankings truthfully—a property that is commonly known as *strategyproofness*. We believe

strategyproofness to be important for many applications of rank aggregation: without it, voters may try to game the mechanism to obtain a better outcome from their individual perspective. For instance, if an SWF that violates strategyproofness is used to aggregate the preferences of a hiring committee, a committee member may misreport his preferences to ensure that his preferred candidates are more likely to get the job. Similarly, in recommender systems, a user may try to manipulate the output ranking so that the final recommendations are closer to his preferences. Lastly, even in ensemble learning, strategyproofness may be desirable as it offers resistance against malicious behavior from individual algorithms.

However, while both SWFs and strategyproofness are generally well understood [see, e.g., 3, 8, 75], the study of strategyproof SWFs has only recently gained attention [4–6, 14]. One possible reason for this is that it is challenging to define strategyproofness for SWFs because it is unclear how voters compare different output rankings. For instance, if a voter’s true ranking is  $a \succ b \succ c$ , does he prefer the ranking  $b \succ a \succ c$  or the ranking  $c \succ a \succ b$ ? Following the recent literature [e.g., 4, 6, 58], we will address this issue by using the Kemeny distance to define the voters’ preferences over rankings. This distance counts the number of pairs of alternatives on which two rankings disagree, and we suppose that voters prefer rankings that have a smaller Kemeny distance to their true ranking. Less formally, this means that voters want the output ranking to align as closely as possible with their true ranking. Lastly, (*Kemeny*-) *strategyproofness* requires that, by misreporting their true ranking, voters cannot obtain an output ranking that is closer to their true ranking than the one that is chosen when voting honestly.

It is known that appealing SWFs, such as the Kemeny rule, satisfy strategyproofness when there are  $m \leq 3$  alternatives, but all known SWFs fail this property if  $m \geq 4$  [4, 6]. The central question of this paper is thus whether strategyproofness allows for the design of desirable SWFs or whether an impossibility theorem similar to the Gibbard-Satterthwaite theorem [47, 70] holds for rank aggregation.

**Contribution.** As our main result, we show that no reasonable SWF satisfies strategyproofness, thereby establishing an analogue of the Gibbard-Satterthwaite theorem for SWFs. In more detail, we prove that no SWF simultaneously satisfies anonymity, unanimity, and strategyproofness when there are  $m \geq 5$  alternatives and an even number of voters  $n$ , or when there are  $m = 4$  alternatives and  $n$  is a multiple of 4 (Theorem 2). We note that anonymity and unanimity are very basic properties—anonymity requires that all voters are treated equally and unanimity that the output ranking ranks one alternative  $x$  ahead of another alternative  $y$  if all voters prefer  $x$  to  $y$ . Hence, our result shows that no SWF that seems acceptable in practice can satisfy strategyproofness.

The proof of our main theorem is obtained via SAT solving, a computer-aided theorem proving technique. Specifically, we encode the problem of deciding whether an SWF satisfies anonymity,



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Negative results	Positive results
<ul style="list-style-type: none"> <li>⊖ No majority consistent and strategyproof SWF if <math>m \geq 4</math> (Thm. 1)</li> <li>⊖ No unanimous, anonymous, and strategyproof SWF if <math>m \geq 4</math> (Thm. 2)</li> <li>⊖ The Kemeny rule, all distance scoring rules, and all positional scoring rules have an incentive ratio of at least <math>\binom{m}{2} - m</math> (Thm. 3)</li> </ul>	<ul style="list-style-type: none"> <li>⊕ The Kemeny rule is strategyproof, unanimous, anonymous, and majority consistent if <math>m \leq 3</math> [4, 6]</li> <li>⊕ There are non-dictatorial, unanimous, and strategyproof SWFs [6]</li> <li>⊕ The Kemeny rule is betweenness strategyproof for all <math>m</math> [14]</li> </ul>

**Table 1: Summary of our results and comparison to related work. All negative results have been proven in this paper. While there have been impossibility results for strategyproof SWFs before our work [e.g., 6, 15], Theorem 2 supersedes all of them.**

unanimity, and strategyproofness in a logical formula and show with the help of a computer that this formula is unsatisfiable when there are  $m = 5$  alternatives and  $n = 2$  voters, and when there are  $m = 4$  alternatives and  $n = 4$  voters. This proves two base cases for our impossibility theorem, which we then lift to our final statement by applying inductive arguments. Following standard practices, we also extract a proof of one of our base cases in a human-readable format. However, since this proof spans over 20 pages, we additionally verify our main theorem with Isabelle [63], a highly trustworthy computer program designed to verify mathematical proofs.

Further, we manually prove that no strategyproof SWF satisfies a form of Condorcet-consistency we call majority consistency (Theorem 1). To introduce this axiom, we define the majority relation of a profile as the binary relation that prefers an alternative  $x$  to another alternative  $y$  if a majority of the voters prefers  $x$  to  $y$ . Then, majority consistency requires that, when the majority relation corresponds to a ranking, the SWF needs to choose this ranking. Hence, our theorem can be seen as a counterpart to the impossibility of strategyproof and Condorcet-consistent social choice functions (which return single alternatives instead of rankings) [e.g., 21, 44, 46].

Lastly, we analyze the incentive ratios of several SWFs to measure how manipulable they are. Roughly, the incentive ratio of an SWF quantifies the worst-case ratio between the utility of a voter when manipulating and when voting honestly. This notion has been successfully applied for private goods settings [e.g., 29, 59, 77] to show that manipulable rules still limit the manipulation gain of agents, which may suffice to disincentivize strategic behavior in practice. Unfortunately, all SWFs that we consider have a high incentive ratio. Specifically, we show that the incentive ratio of the Kemeny rule and all distance scoring rules (where voters assign scores to the rankings depending on their Kemeny distance to the ranking and the ranking with minimal total score is chosen) is roughly  $\binom{m}{2}$  when there are  $m$  alternatives. Further, we prove that positional scoring rules have an unbounded incentive ratio.

**Related Work.** Both SWFs and strategyproofness have been studied for decades, and we refer to the book by Arrow et al. [3] and the survey of Barberà [8] for introductions to these topics. In more detail, SWFs are studied since Arrow’s foundational work of social choice theory [2]. To date, there is a large range of SWFs, including the Kemeny rule [52, 53], various types of scoring rules [e.g., 12, 31, 57, 73, 79], and Condorcet-style rules [e.g., 32, 71, 72]. These SWFs are primarily studied with respect to consistency notions such as population consistency or independence axioms. This line of work resulted in influential characterizations of, e.g., the Kemeny rule [26, 80, 81] or the Borda rule [64, 78].

Similarly, strategyproofness in voting has attracted significant attention, although the results in this area are more negative. In particular, Gibbard and Satterthwaite [47, 70] have shown that no reasonable deterministic single-winner voting rule is strategyproof. Motivated by this result, numerous works have aimed to circumvent this impossibility theorem, for instance by allowing randomized or set-valued outcomes [e.g., 18, 22, 48, 51] or by restricting the feasible input rankings [e.g., 24, 28, 62]. Except for domain restrictions, these approaches have mostly led to strengthened impossibility theorems. Our paper can also be interpreted in this line of work: since rankings contain more information than a single winner, one may attempt to escape the Gibbard-Satterthwaite theorem by studying SWFs and a suitable strategyproofness notion. As our results show, this approach does not work when using Kemeny-strategyproofness.

More directly related, there are several works that study Kemeny-strategyproofness for SWFs. To our knowledge, Bossert and Storcken [15] were the first to study this condition. Specifically, these authors show that group Kemeny-strategyproofness leads to an impossibility when requiring a technical auxiliary property called weak extrema independence. Moreover, Athanoglou [4] and Athanoglou et al. [6] prove that, when  $m \leq 3$ , the Kemeny rule (with suitable tie-breaking) and other SWFs are Kemeny-strategyproof, but these positive results break when  $m \geq 4$ . Further, Athanoglou et al. [6] show that no anonymous SWF satisfies Kemeny-strategyproofness and a technical property called preference selection. This demanding condition requires an SWF to always return a ranking that is present in the input profile.

Furthermore, Bossert and Sprumont [14] introduce an alternative strategyproofness notion for SWFs called betweenness strategyproofness. This condition requires that, by manipulating, voters cannot obtain a ranking that lies on a single-crossing sequence of rankings from the manipulator’s true ranking to the output ranking chosen when voting truthfully. Betweenness strategyproofness is weaker than Kemeny-strategyproofness and Bossert and Sprumont [14] show that, e.g., the Kemeny rule always satisfies it. This condition was further analyzed by Sato [69], Harless [50], and Athanoglou [5], the latter two of whom use it to characterize so-called status-quo rules. More strategyproofness notions for SWFs have been studied by Bonkougou [13] and Dindar et al. [37].

Lastly, rank aggregation can be seen as a special case of judgment aggregation, where we need to aggregate the voters’ preferences over logical formulas. In this setting, strategyproofness was studied before [e.g., 35, 36, 76] and Dietrich and List [35] have shown an analogue to the Gibbard-Satterthwaite theorem. However, their strategyproofness notion is much stronger than Kemeny-strategyproofness, thus making their results incomparable to ours.

## 2 PRELIMINARIES

Let  $A = \{a, b, c, \dots\}$  be a set of  $m$  alternatives and  $N = \{1, \dots, n\}$  be a set of  $n$  voters. Every voter  $i \in N$  reports a *ranking*  $\succ_i$  over the alternatives to indicate his preferences. Formally, a ranking  $\succ_i$  is a transitive, antisymmetric, and complete binary relation on  $A$ . The set of all rankings is denoted by  $\mathcal{R}$ . A (*ranking*) *profile*  $R = (\succ_1, \dots, \succ_n)$  is the collection of the rankings of all voters in  $N$ , and the set of all profiles is  $\mathcal{R}^N$ . We will write rankings as sequences of alternatives and indicate the voter submitting a ranking directly before it. For example,  $3 : abc$  means that voter 3 prefers  $a$  to  $b$  to  $c$ .

The object of study of this paper are *social welfare functions* (SWFs) which map every ranking profile to a single output ranking. More formally, a social welfare function is a function  $f$  of the type  $\mathcal{R}^N \rightarrow \mathcal{R}$ . To clearly distinguish between input and output rankings, we will denote the former by  $\succ$  and the latter by  $\triangleright$ .

### 2.1 Classes of SWFs

We will next introduce several natural classes of SWFs. Since all of the following rules may return multiple winning rankings, we assume that such ties are broken based on an external ranking  $>$  over the alternatives that is lexicographically extended to rankings. Specifically, given two rankings  $\triangleright_1 = x_1 \dots x_m$  and  $\triangleright_2 = y_1 \dots y_m$ , it holds that  $\triangleright_1 > \triangleright_2$  if and only if there is  $\ell \in \{1, \dots, m\}$  such that  $x_\ell > y_\ell$  and  $x_i = y_i$  for all  $i \in \{1, \dots, \ell - 1\}$ . To fully specify our SWFs, we always choose the most preferred ranking with respect to  $>$  that is winning for the considered SWF. We note, however, that all our results are independent of this tie-breaking convention.

**Kemeny rule.** The Kemeny rule was first suggested by Kemeny [52] and is maybe the most prominent method in rank aggregation. To introduce this rule, we define the *Kemeny distance* (which is also known as swap distance or Kendall-tau distance) between two rankings  $\succ$  and  $\triangleright$  by  $\Delta(\succ, \triangleright) = |\{(x, y) \in A^2 : x \succ y \wedge y \triangleright x\}|$ . Less formally,  $\Delta(\succ, \triangleright)$  is the number of pairs of alternatives on which  $\succ$  and  $\triangleright$  disagree. The *Kemeny rule* chooses the (lexicographically most preferred) ranking that minimizes the total Kemeny distance to the input rankings, i.e.,  $f_{\text{Kemeny}}(R) = \arg \min_{\triangleright \in \mathcal{R}} \sum_{i \in N} \Delta(\succ_i, \triangleright)$ .

**Distance scoring rules.** In distance scoring rules, every voter  $i$  assigns a score to every ranking  $\triangleright$  depending on the Kemeny distance between his input ranking  $\succ_i$  and  $\triangleright$ , and we choose the ranking with the minimal total score. More formally, these rules are defined based on *distance scoring functions*  $s : \{0, \dots, \binom{m}{2}\} \rightarrow \mathbb{R}$ , and a voter with a Kemeny distance of  $x$  to a ranking assigns a score of  $s(x)$  to this ranking. Throughout the paper, we will require of distance scoring functions  $s$  that  $s(x) > s(x-1)$  for all  $x \in \{1, \dots, \binom{m}{2}\}$ , and  $s(x) - s(x-1) \geq s(x-1) - s(x-2)$  for all  $x \in \{2, \dots, \binom{m}{2}\}$ . These conditions formalize that rankings that are further away from a voter's ranking get a higher score and that  $s$  is convex. Finally, a *distance scoring rule*  $f$  is defined by a distance scoring function  $s$  and chooses the (lexicographically most preferred) ranking  $\triangleright$  that minimizes  $\sum_{i \in N} s(\Delta(\succ_i, \triangleright))$ . For example, the Kemeny rule is defined by the distance scoring function  $s(x) = x$ , and the Squared Kemeny rule of Lederer et al. [58] by  $s(x) = x^2$ .

**Positional scoring rules.** Another prominent class of SWFs are positional scoring rules. For these rules, the voters assign points to the alternatives depending on their positions in the input ranking,

and the output ranking orders the alternatives in decreasing order of their total score. To formalize this, we define the *rank* of an alternative  $x$  in a ranking  $\succ$  by  $r(\succ, x) = 1 + |\{y \in A \setminus \{x\} : y \succ x\}|$ . Then, positional scoring rules are defined by *positional scoring functions*  $p : \{1, \dots, m\} \rightarrow \mathbb{R}$  and a voter who places an alternative at rank  $k$  assigns a score of  $p(k)$  to this alternative. We will require of positional scoring functions that  $p(1) \geq p(2) \geq \dots \geq p(m)$  and  $p(1) > p(m)$ , i.e., voters give more points to higher-ranked alternatives and do not assign the same score to all alternatives. Finally, an SWF  $f$  is a *positional scoring rule* if there is a positional scoring function  $p$  such that  $f$  returns for every profile  $R$  the (lexicographically most preferred) ranking  $\triangleright$  such that  $x \triangleright y$  implies  $\sum_{i \in N} p(r(\succ_i, x)) \geq \sum_{i \in N} p(r(\succ_i, y))$ . For instance, the Borda rule is induced by the positional scoring function  $p(x) = m - x$ .

### 2.2 Strategyproofness

The central axiom in our analysis is strategyproofness, which requires that voters cannot benefit by lying about their true ranking. Following the literature [e.g., 4, 6, 15], we will define this axiom by assuming that the voters' preferences over rankings are induced by the Kemeny distance  $\Delta(\succ, \triangleright) = |\{(x, y) \in A^2 : x \succ y \wedge y \triangleright x\}|$ : a voter with ranking  $\succ_i$  prefers a ranking  $\triangleright$  to another ranking  $\triangleright'$  if  $\Delta(\succ_i, \triangleright) < \Delta(\succ_i, \triangleright')$ . This formalizes that voters prefer rankings that are more similar to their true ranking. Based on this assumption, we can define strategyproofness as usual by requiring that voters cannot obtain a more preferred ranking by voting strategically.

**Definition 1** (Strategyproofness). An SWF  $f$  is *strategyproof* if  $\Delta(\succ_i, f(R)) \leq \Delta(\succ_i, f(R'))$  for all voters  $i \in N$  and profiles  $R, R' \in \mathcal{R}^N$  such that  $\succ_j = \succ'_j$  for all  $j \in N \setminus \{i\}$ .

This definition of strategyproofness is motivated by the fact that the Kemeny distance is by far the most common distance over rankings in rank aggregation [e.g., 25, 38, 54, 61]. In particular, many papers propose to minimize the total Kemeny distance to find good output rankings, which implicitly assumes that voters prefer rankings that have a closer Kemeny distance to their input ranking. Moreover, the Kemeny distance is also theoretically well-understood and allows for appealing characterizations [27, 34, 52]. Nevertheless, we acknowledge that one can define alternative strategyproofness notions by, e.g., using different distances on rankings or even approaches that are not based on any distance measure, which may lead to different results.

To further illustrate strategyproofness for SWFs, we will next discuss an example showing that the Kemeny rule is manipulable.

**Example 1.** Consider the following profile  $R$  with  $m = 4$  alternatives  $A = \{a, b, c, d\}$  and  $n = 5$  voters  $N = \{1, \dots, 5\}$ .

$$R: \quad 1: abcd \quad 2: cdab \quad 3: dbac \quad 4: bcda \quad 5: adcb$$

For this profile, the Kemeny rule chooses the ranking  $\triangleright = dabc$ , which has a total Kemeny distance of  $\sum_{i \in N} \Delta(\succ_i, \triangleright) = 3 + 3 + 1 + 4 + 2 = 13$ . By contrast, if voter 4 misreports his true ranking by swapping  $b$  and  $c$ , i.e., if  $\succ'_4 = cbda$ , the Kemeny rule chooses the ranking  $\triangleright' = cdab$ . Further, it holds that  $\Delta(\succ_4, \triangleright) = \Delta(bcda, dabc) = 4 > 3 = \Delta(bcda, cdab) = \Delta(\succ_4, \triangleright')$ . This shows that voter 4 prefers the ranking  $\triangleright'$  to the ranking  $\triangleright$  selected for  $R$ , so the Kemeny rule fails strategyproofness when  $m = 4$  and  $n = 5$ .

### 2.3 Further Axioms

Additionally to strategyproofness, we will consider three further axioms, namely anonymity, unanimity, and majority consistency.

**Anonymity.** Intuitively, anonymity requires that the identities of voters should not matter for the outcome. Formally, we say an SWF  $f$  is *anonymous* if  $f(R) = f(\pi(R))$  for all profiles  $R \in \mathcal{R}^N$  and permutations  $\pi : N \rightarrow N$ . Here,  $R' = \pi(R)$  is the profile given by  $\succ'_i = \succ_{\pi(i)}$  for all  $i \in N$ . When assuming anonymity, we may interpret profiles as multisets of rankings because we only need to know how often each ranking is reported to compute the outcome.

**Unanimity.** Unanimity is a minimal efficiency notion which requires that if all voters unanimously prefer one alternative  $x$  to another alternative  $y$ , then the output ranking should also rank  $x$  ahead of  $y$ . More formally, an SWF  $f$  is *unanimous* if, for all profiles  $R \in \mathcal{R}^N$  and alternatives  $x, y \in A$ , it holds for the output ranking  $\triangleright = f(R)$  that  $x \triangleright y$  if  $x \succ_i y$  for all voters  $i \in N$ .

**Majority consistency.** One of the dominant notions in social choice theory is the Condorcet principle: if an alternative is favored to another alternative by a majority of the voters, then the former is often seen as more desirable than the latter. To formalize this idea, we define the *majority relation*  $\succeq_R$  of a profile  $R$  by  $x \succeq_R y$  if and only if  $|\{i \in N : x \succ_i y\}| \geq |\{i \in N : y \succ_i x\}|$ , i.e.,  $x \succeq_R y$  if and only if a majority of voters prefer  $x$  to  $y$ . Then, an SWF  $f$  is *majority consistent* if it returns the majority relation whenever this relation is a ranking, i.e.,  $f(R) = \succeq_R$  for all profiles  $R$  such that the majority relation  $\succeq_R$  is transitive and antisymmetric. We note that the majority relation does not necessarily form a ranking, and majority consistency permits any outcome in such situations.

### 3 MAJORITY CONSISTENCY AND STRATEGYPROOFNESS

As our first result, we will show that no majority consistent SWF is strategyproof if there are sufficiently many voters and at least four alternatives. While we include this theorem primarily to showcase a proof of an impossibility theorem based on strategyproofness, it is also one of the strongest impossibility results in rank aggregation.

**Theorem 1.** *No strategyproof SWF satisfies majority consistency if  $m \geq 4$ ,  $n \geq 9$ , and  $n \notin \{10, 12, 14, 16\}$ .*

**PROOF.** We will first study the case that  $m = 4$  and  $n = 9$  and later on generalize the result to larger values of  $n$  and  $m$ . Hence, assume for contradiction that there is a strategyproof and majority consistent SWF  $f$  for 4 alternatives and 9 voters. We will focus on the following two profiles  $\bar{R}$  and  $\hat{R}$  to derive a contradiction.

$\bar{R}$	1: <i>cdba</i>	2: <i>badc</i>	3: <i>dbac</i>	4: <i>cbad</i>	5: <i>adcb</i>
	6: <i>cabd</i>	7: <i>dcba</i>	8: <i>dabc</i>	9: <i>abcd</i>	
$\hat{R}$	1: <i>cdba</i>	2: <i>badc</i>	3: <i>dbca</i>	4: <i>cbad</i>	5: <i>adcb</i>
	6: <i>cabd</i>	7: <i>dcba</i>	8: <i>dabc</i>	9: <i>abcd</i>	

We will show that  $f$  has to choose  $\bar{\triangleright} = adcb$  for  $\bar{R}$  and  $\hat{\triangleright} = dcba$  for  $\hat{R}$ . Since the profiles  $\bar{R}$  and  $\hat{R}$  only differ in the ranking of voter 3,  $f$  is manipulable because  $\Delta(\bar{\triangleright}_3, \hat{\triangleright}) = 3 > 2 = \Delta(\bar{\triangleright}_3, \bar{\triangleright})$ , i.e., voter 3 prefers  $\hat{\triangleright}$  to  $\bar{\triangleright}$ . It remains to show that  $f$  indeed needs to choose *adcb* and *dcba* for  $\bar{R}$  and  $\hat{R}$ , respectively.

**Claim 1:**  $f(\bar{R}) = adcb$ . For proving this claim, we consider the following five profiles  $\bar{R}^1, \dots, \bar{R}^5$ , all of which differ from  $\bar{R}$  in the ranking of a single voter. This ranking is highlighted in blue.

$\bar{R}^1$	1: <i>cdab</i>	2: <i>badc</i>	3: <i>dbac</i>	4: <i>cbad</i>	5: <i>adcb</i>
	6: <i>cabd</i>	7: <i>dcba</i>	8: <i>dabc</i>	9: <i>abcd</i>	
$\bar{R}^2$	1: <i>cdba</i>	2: <i>abcd</i>	3: <i>dbac</i>	4: <i>cbad</i>	5: <i>adcb</i>
	6: <i>cabd</i>	7: <i>dcba</i>	8: <i>dabc</i>	9: <i>abcd</i>	
$\bar{R}^3$	1: <i>cdba</i>	2: <i>badc</i>	3: <i>dabc</i>	4: <i>cbad</i>	5: <i>adcb</i>
	6: <i>cabd</i>	7: <i>dcba</i>	8: <i>dabc</i>	9: <i>abcd</i>	
$\bar{R}^4$	1: <i>cdba</i>	2: <i>badc</i>	3: <i>dbac</i>	4: <i>cabd</i>	5: <i>adcb</i>
	6: <i>cabd</i>	7: <i>dcba</i>	8: <i>dabc</i>	9: <i>abcd</i>	
$\bar{R}^5$	1: <i>cdba</i>	2: <i>badc</i>	3: <i>dbac</i>	4: <i>cbad</i>	5: <i>dabc</i>
	6: <i>cabd</i>	7: <i>dcba</i>	8: <i>dabc</i>	9: <i>abcd</i>	

In all five profiles, the majority relation is transitive and anti-symmetric and therefore a ranking. In more detail, in  $\bar{R}^1$  to  $\bar{R}^4$ , all of which arise from  $\bar{R}$  by swapping  $a$  and  $b$  in the ranking of a single voter, the majority relation corresponds to the ranking *adcb*. Further, in  $\bar{R}^5$ , the majority relation is given by *dbac*. Hence, majority consistency requires that  $f(\bar{R}^1) = \dots = f(\bar{R}^4) = adcb$  and  $f(\bar{R}^5) = dbac$ . Consequently, strategyproofness implies the following constraints for the ranking  $\bar{\triangleright}$  chosen for  $\bar{R}$ .

- (1) Strategyproofness from  $\bar{R}$  to  $\bar{R}^1$  requires that  $\Delta(cdba, \bar{\triangleright}) \leq \Delta(cdba, f(\bar{R}^1)) = \Delta(cdba, adcb) = 4$ .
- (2) Strategyproofness from  $\bar{R}$  to  $\bar{R}^2$  requires that  $\Delta(badc, \bar{\triangleright}) \leq \Delta(badc, f(\bar{R}^2)) = \Delta(badc, adcb) = 3$ .
- (3) Strategyproofness from  $\bar{R}$  to  $\bar{R}^3$  requires that  $\Delta(dbac, \bar{\triangleright}) \leq \Delta(dbac, f(\bar{R}^3)) = \Delta(dbac, adcb) = 3$ .
- (4) Strategyproofness from  $\bar{R}$  to  $\bar{R}^4$  requires that  $\Delta(cbad, \bar{\triangleright}) \leq \Delta(cbad, f(\bar{R}^4)) = \Delta(cbad, adcb) = 4$ .
- (5) Strategyproofness from  $\bar{R}$  to  $\bar{R}^5$  requires that  $\Delta(adcb, \bar{\triangleright}) \leq \Delta(adcb, f(\bar{R}^5)) = \Delta(adcb, dbac) = 3$ .

We claim that only the ranking *adcb* satisfies these constraints. To prove this, we will consider several cases. First,  $d$  cannot be bottom-ranked in  $\bar{\triangleright}$  because  $\Delta(dbac, \bar{\triangleright}) \geq 4$  for all rankings  $\bar{\triangleright}$  that rank  $d$  last and are not equal to *bacd*. Hence, all these rankings fail Condition (3). Further, it holds that  $\Delta(adcb, bacd) = 4$ , so the ranking *bacd* fails Condition (5). Next,  $\bar{\triangleright}$  cannot bottom-rank  $a$ : every ranking  $\bar{\triangleright}$  that places  $a$  last and is not equal to *dcb* violates Condition (5) since  $\Delta(adcb, \bar{\triangleright}) \geq 4$ , and the ranking *dcb* fails Condition (2) since  $\Delta(badc, dcb) = 4$ . Thirdly, we show that alternative  $c$  cannot be bottom-ranked by  $\bar{\triangleright}$ . For this, we observe that Condition (1) implies that  $\bar{\triangleright} \notin \{abcd, badc, adbc\}$  because all these rankings have a swap distance of at least 5 to *cdba*. Further, Condition (4) requires that  $\bar{\triangleright} \notin \{dabc, dbac\}$  because these rankings have a swap distance of at least 5 to *cbad*. Lastly, Condition (5) shows that  $\bar{\triangleright} \neq bdac$  because  $\Delta(adcb, bdac) = 4$ . We now conclude that  $c$  is not bottom-ranked by  $\bar{\triangleright}$ . Because all other options have been ruled out,  $b$  must be bottom-ranked by  $\bar{\triangleright}$ . This means that  $\Delta(badc, \bar{\triangleright}) \geq 3$ . Moreover, this inequality needs to be tight due to Condition (2), which is only true if  $f(\bar{R}) = \bar{\triangleright} = adcb$ .

**Claim 2:**  $f(\hat{R}) = dcba$ . We will next show that  $f(\hat{R}) = dcba$ , for which we consider the following five profiles. All of these profiles only differ in the highlighted ranking from  $\hat{R}$ .

$\widehat{R}^1$	1: <i>cdba</i>	2: <i>badc</i>	3: <i>dbca</i>	4: <i>cbda</i>	5: <i>adcb</i>
	6: <i>cadb</i>	7: <i>dcba</i>	8: <i>dabc</i>	9: <i>abcd</i>	
$\widehat{R}^2$	1: <i>cdba</i>	2: <i>badc</i>	3: <i>dbca</i>	4: <i>cbad</i>	5: <i>dacb</i>
	6: <i>cadb</i>	7: <i>dcba</i>	8: <i>dabc</i>	9: <i>abcd</i>	
$\widehat{R}^3$	1: <i>cdba</i>	2: <i>badc</i>	3: <i>dbca</i>	4: <i>cbad</i>	5: <i>adcb</i>
	6: <i>cdab</i>	7: <i>dcba</i>	8: <i>dabc</i>	9: <i>abcd</i>	
$\widehat{R}^4$	1: <i>cdba</i>	2: <i>bdac</i>	3: <i>dbca</i>	4: <i>cbad</i>	5: <i>adcb</i>
	6: <i>cadb</i>	7: <i>dcba</i>	8: <i>dabc</i>	9: <i>abcd</i>	
$\widehat{R}^5$	1: <i>cdba</i>	2: <i>badc</i>	3: <i>dbca</i>	4: <i>cbad</i>	5: <i>adcb</i>
	6: <i>cadb</i>	7: <i>cdab</i>	8: <i>dabc</i>	9: <i>abcd</i>	

The profiles  $\widehat{R}^1, \dots, \widehat{R}^4$  only differ from  $\widehat{R}$  in the fact that a voter swapped  $d$  and  $a$ . Consequently, the majority relation of these profiles corresponds to the ranking  $dcb a$ . By contrast, in  $\widehat{R}^5$ , the majority relation is given by the ranking  $cad b$ . Hence, majority consistency requires that  $f(\widehat{R}^1) = \dots = f(\widehat{R}^4) = dcb a$  and  $f(\widehat{R}^5) = cad b$ . In turn, strategyproofness between  $\widehat{R}$  and our five profiles requires the following constraints for the ranking  $\widehat{\triangleright}$  chosen for  $\widehat{R}$ .

- (1) Strategyproofness from  $\widehat{R}$  to  $\widehat{R}^1$  requires that  $\Delta(cbad, \widehat{\triangleright}) \leq \Delta(cbad, f(\widehat{R}^1)) = \Delta(cbad, dcb a) = 3$ .
- (2) Strategyproofness from  $\widehat{R}$  to  $\widehat{R}^2$  requires that  $\Delta(adcb, \widehat{\triangleright}) \leq \Delta(adcb, f(\widehat{R}^2)) = \Delta(adcb, dcb a) = 3$ .
- (3) Strategyproofness from  $\widehat{R}$  to  $\widehat{R}^3$  requires that  $\Delta(cadb, \widehat{\triangleright}) \leq \Delta(cadb, f(\widehat{R}^3)) = \Delta(cadb, dcb a) = 3$ .
- (4) Strategyproofness from  $\widehat{R}$  to  $\widehat{R}^4$  requires that  $\Delta(badc, \widehat{\triangleright}) \leq \Delta(badc, f(\widehat{R}^4)) = \Delta(badc, dcb a) = 4$ .
- (5) Strategyproofness from  $\widehat{R}$  to  $\widehat{R}^5$  requires that  $\Delta(dcba, \widehat{\triangleright}) \leq \Delta(dcba, f(\widehat{R}^5)) = \Delta(dcba, cad b) = 3$ .

Analogously to the last claim, these constraints entail that  $\widehat{\triangleright} = dcb a$ . To see this, we first note that Conditions (2) and (5) show that  $d$  cannot be bottom-ranked by  $\widehat{\triangleright}$ . In more detail, every ranking  $\triangleright$  other than  $cbad$  that bottom-ranks  $d$  violates Condition (5) since  $\Delta(dcb a, \triangleright) \geq 4$ . On the other hand, the ranking  $cbad$  violates Condition (2) as  $\Delta(adcb, cbad) = 4$ . Next, Conditions (2) and (3) show that  $c$  cannot be bottom-ranked by  $\widehat{\triangleright}$ : the only ranking  $\triangleright$  that bottom-ranks  $c$  and satisfies that  $\Delta(cbad, \triangleright) \leq 3$  (Condition (2)) is  $badc$ , but this ranking fails Condition (3) since  $\Delta(badc, cadb) = 5$ . Thirdly,  $\widehat{\triangleright}$  cannot bottom-rank  $b$ : Condition (4) rules out that  $\widehat{\triangleright} \in \{cdab, dcab, cadb\}$  since all these rankings have a distance of at least 5 to  $badc$ , Condition (1) shows that  $\widehat{\triangleright} \notin \{adcb, dacb\}$  since these rankings have a Kemeny distance of at least 4 to  $cbad$ , and Condition (5) shows that  $\widehat{\triangleright} \neq acdb$  since  $\Delta(dcba, acdb) = 4$ . Hence, we conclude that  $a$  must be bottom-ranked in  $\widehat{\triangleright}$ . In turn, we infer from Condition (2) that  $\widehat{\triangleright}$  must be  $dcb a$  since every other ranking that bottom-ranks  $a$  satisfies that  $\Delta(adcb, \triangleright) \geq 4$ .

**Extension to larger values of  $m$  and  $n$ .** Lastly, we explain how to generalize our result to larger numbers of voters  $n$  and alternatives  $m$ . First, to increase  $m$ , we can add new alternatives in the same order at the bottom of the rankings of all voters. After this extension, the majority relation is still transitive and antisymmetric for all profiles  $\widehat{R}^i$  and  $\widehat{R}^j$  with  $i \in \{1, \dots, 5\}$ . Using majority consistency, the inequalities (1) to (5) thus remain intact for both cases. Finally, these inequalities still imply that we need to choose rankings for  $\widehat{R}$  and  $\widehat{R}$  that permit a manipulation for voter 3.

To extend our construction to larger numbers of voters, we apply two different techniques. Firstly, we can generalize our impossibility to every odd  $n > 9$  by adding pairs of voters with inverse rankings. These voters cancel each other out with respect to the majority relation and therefore do not affect our analysis. Secondly, to extend our impossibility to an even number of voters, we can double all voters in our profiles. While this requires intermediate profiles  $\tilde{R}$  to go from, e.g.,  $\widehat{R}$  to  $\widehat{R}^1$ , we can still infer the same inequalities by chaining the strategyproofness conditions. For instance, for  $\widehat{R}^1$ , strategyproofness implies that  $\Delta(cdba, f(\widehat{R})) \leq \Delta(cdba, f(\tilde{R}^1)) \leq \Delta(cdba, f(\widehat{R}^1))$ , where  $\tilde{R}^1$  denotes the intermediate profile. Hence, our analysis remains intact after this extension. Lastly, for any even  $n > 18$ , we can again add pairs of voters with inverse rankings.  $\square$

**Remark 1.** When  $m \leq 3$ , Theorem 1 ceases to hold as the Kemeny rule (with suitable tie-breaking) is strategyproof and majority consistent in this case [6, 81]. Moreover, under mild additional conditions, namely anonymity, cancellation (i.e., adding pairs of voters with inverse rankings does not affect the outcome), and a weak form of neutrality, it can be shown that strategyproofness and majority consistency require to choose a ranking that minimizes the total Kemeny distance to the input rankings when there are 3 alternatives. We refer to the extended version of this paper for further details [41, Appendix A].

**Remark 2.** We did not minimize the number of voters for Theorem 1 as we aimed for a simple proof. However, with the help of a computer, we showed that this impossibility already holds when there are  $m = 4$  alternatives and  $n \in \{3, 4\}$  voters. Based on our inductive arguments for  $n$ , it thus follows that no majority consistent SWF is strategyproof if there are  $m = 4$  alternatives and  $n \geq 3$  voters. By contrast, our inductive argument for  $m$  is specific to the profiles in Theorem 1, so it is unclear whether the computer proof extends to more alternatives. Further, we verified the correctness of the computer proof and our human-readable proof by Isabelle/HOL, a highly trustworthy interactive theorem prover [39].

## 4 STRATEGYPROOFNESS AND UNANIMITY

We will now turn to our main theorem: there is no anonymous SWF that satisfies strategyproofness and unanimity if there are  $m \geq 5$  alternatives and an even number of voters  $n$ , or when there are  $m = 4$  alternatives and the number of voters  $n$  is a multiple of 4. Put differently, this result shows that every reasonable SWF is manipulable and it can thus be seen as an analog of the Gibbard-Satterthwaite theorem for rank aggregation.

**Theorem 2.** *No anonymous SWF satisfies strategyproofness and unanimity if  $m \geq 5$  and  $n$  is even, or  $m = 4$  and  $n$  is a multiple of 4.*

We note that we have shown Theorem 2 based on a computer-aided theorem proving technique called SAT solving. In the context of social choice theory, such computer-aided techniques have been pioneered by Tang and Lin [74] and have since then been used to show a large number of results [e.g., 16, 17, 19, 22, 33, 42, 45, 65]. We refer to the survey of Geist and Peters [46] for an introduction to these techniques. In the following three sections, we outline how we apply SAT solving to obtain Theorem 2 (cf. Sections 4.1 and 4.2) and how we verified our result (cf. Section 4.3).

**Remark 3.** All axioms of Theorem 2 are necessary for the impossibility. Specifically, constant SWFs, which always return a fixed ranking, satisfy strategyproofness and anonymity but violate unanimity. Dictatorships, which return the ranking of a fixed voter, satisfy unanimity and strategyproofness but violate anonymity. It is also not possible to weaken anonymity to non-dictatorship as Athanassoglou et al. [6] design non-dictatorial (and non-anonymous) SWFs that are strategyproof and unanimous. Thirdly, e.g., the Kemeny rule satisfies unanimity and anonymity but violates strategyproofness. Further, we cannot significantly weaken strategyproofness as Bossert and Sprumont [14] show that the Kemeny rule satisfies betweenness strategyproofness, which is only slightly weaker than our Kemeny-strategyproofness. Finally, when  $m \leq 3$ , Theorem 2 ceases to hold as the Kemeny rule is strategyproof in this case [6].

**Remark 4.** A drawback of Theorem 2 is that we cannot extend this result to an odd number of voters. The primary reason for this is technical: we could not find an inductive argument that generalizes our theorem from an even number of voters to an odd one. Moreover, based on our SAT approach, we showed that there are SWFs that satisfy all axioms of Theorem 2 when  $m = 4$  and  $n \in \{3, 5\}$ , which indicates that such an argument may not exist. Similar problems are common for impossibility theorems in social choice theory [e.g., 19, 20, 23, 33, 65], as it is often challenging to generalize such results from a fixed number of voters  $n$  to arbitrary values of  $n$ . We note, however, that we can extend Theorem 2 to odd  $n$  when strengthening unanimity. Specifically, based on Theorem 2, one can show that no anonymous SWF satisfies strategyproofness and a property called near unanimity when  $m \geq 5$  and  $n \geq 3$  is odd. This latter condition requires that the output ranking puts  $x$  ahead of  $y$  whenever all but one voter prefer  $x$  to  $y$  [10, 56]. In particular, if we had an SWF that satisfies anonymity, strategyproofness, and near unanimity for odd  $n \geq 3$ , we could construct an SWF that satisfies anonymity, strategyproofness, and unanimity for  $n - 1$  voters by fixing the ranking of a single voter. We further observe that near unanimity becomes less demanding as  $n$  increases, and all common SWFs satisfy this condition when  $n$  is sufficiently larger than  $m$ .

## 4.1 SAT Solving

To show Theorem 2, we rely on SAT solving, a computer-aided theorem proving technique. The central idea of this approach is that, for fixed numbers of voters  $n$  and alternatives  $m$ , there is a large but finite number of ranking profiles and possible outcomes. For instance, when  $n = 2$  and  $m = 5$ , there are  $(5!)^2 = 14,400$  ranking profiles, for each of which one of  $5! = 120$  rankings must be chosen. Based on this observation, it is possible to write a large logical formula that is satisfiable if and only if there is an anonymous SWF that satisfies unanimity and strategyproofness for the given values of  $n$  and  $m$ . We then prove two base cases of our theorem by letting a computer program, a so-called SAT solver, show that our formula is unsatisfiable when there are  $n = 2$  voters and  $m = 5$  alternatives, and when there  $n = 4$  voters and  $m = 4$  alternatives.

In our logical formula, we follow the standard encoding of voting rules. Specifically, our formula will use variables  $x_{R,\triangleright}$  for all profiles  $R$  and rankings  $\triangleright$ , which will encode whether the ranking  $\triangleright$  is chosen for the profile  $R$ . Moreover, since we focus on anonymous SWFs, we will treat profiles as multisets of rankings. Formally, this

means that the variable  $x_{R,\triangleright}$  will state whether the ranking  $\triangleright$  is chosen for all non-anonymous profiles that can be obtained by assigning the rankings in the multiset  $R$  to the voters in  $N$ . This is possible as anonymity necessitates that we need to choose the same ranking for all such profiles. By representing profiles as multisets, we reduce the number of variables in our formula as a single multiset corresponds up to  $n!$  non-anonymous ranking profiles. Also, due to this representation, anonymity is implicitly encoded, so we do not need to add constraints for this axiom.

Next, we have to ensure that our variables  $x_{R,\triangleright}$  indeed encode an (anonymous) SWF. This necessitates us to formalize that for every profile  $R$ , exactly one ranking  $\triangleright$  is chosen. To this end, we will require for each profile  $R$  that exactly one variable  $x_{R,\triangleright}$  is true. Moreover, to further simplify our formula, we encode unanimity in this step by only introducing variables  $x_{R,\triangleright}$  for profiles  $R$  and rankings  $\triangleright$  such that  $\triangleright$  satisfies unanimity for  $R$ . To make this more formal, let  $X(R) = \{(x, y) \in A^2 : \forall i \in N : x \succ_i y\}$  denote the pairs of alternatives  $(x, y)$  such that all voters prefer  $x$  to  $y$  in  $R$ . Then,  $U(R) = \{\triangleright \in \mathcal{R} : \forall (x, y) \in X(R) : x \triangleright y\}$  is the set of rankings that satisfy unanimity for  $R$ . We will only introduce variables  $x_{R,\triangleright}$  for each profile  $R$  and ranking  $\triangleright \in U(R)$  as rankings outside of  $U(R)$  are not allowed to be chosen by unanimity. We can now enforce that our variables encode an unanimous SWF by adding the following constraints for every profile  $R$ , which respectively state that at least and at most one ranking in  $U(R)$  must be chosen for  $R$ .

$$\bigvee_{\triangleright \in U(R)} x_{R,\triangleright} \quad \text{and} \quad \bigwedge_{\triangleright, \triangleright' \in U(R) : \triangleright \neq \triangleright'} (\neg x_{R,\triangleright} \vee \neg x_{R,\triangleright'})$$

Lastly, we need to encode strategyproofness. We recall here that this axiom requires that, for every profile  $R$  and voter  $i$ , it is not possible for the voter to deviate such that the ranking chosen when lying has a smaller Kemeny distance to his truthful ranking than the one chosen when reporting his truthful ranking. Put differently, if  $R$  and  $R'$  only differ in the ranking of voter  $i$ , we cannot choose rankings  $\triangleright$  and  $\triangleright'$  for these profiles such that  $\Delta(\triangleright, \succ_i) > \Delta(\triangleright', \succ_i)$ . To make this more formal, we define by  $D(R, \succ)$  the set of profiles that can be derived from  $R$  by letting a voter with ranking  $\succ$  deviate to an arbitrary other ranking. We note that  $D(R, \succ) = \emptyset$  if no voter in  $R$  reports  $\succ$ . Further, given two rankings  $\succ$  and  $\triangleright$ , we let  $B(\succ, \triangleright) = \{\triangleright' \in \mathcal{R} : \Delta(\succ, \triangleright') < \Delta(\succ, \triangleright)\}$  denote the set of rankings that have a smaller Kemeny distance to  $\succ$  than  $\triangleright$ . Based on this notation, strategyproofness can be formalized via implications: if we choose a ranking  $\triangleright$  for  $R$ , we cannot choose a ranking  $\triangleright' \in B(\succ, \triangleright) \cap U(R')$  for all profiles  $R' \in D(R, \succ)$  and rankings  $\succ \in \mathcal{R}$ . Using our variables  $x_{R,\triangleright}$ , this results in the following constraints.

$$\bigwedge_{R \in \mathcal{R}^N} \bigwedge_{\succ \in \mathcal{R}} \bigwedge_{R' \in D(R, \succ)} \bigwedge_{\triangleright \in U(R)} \bigwedge_{\triangleright' \in B(\succ, \triangleright) \cap U(R')} (\neg x_{R,\triangleright} \vee \neg x_{R',\triangleright'})$$

We can now write a computer program that generates this logical formula for given values of  $m$  and  $n$ . For instance, for  $m = 5$  and  $n = 2$ , our program produces a formula with 227,880 variables and 59,445,060 clauses. We then hand this formula for both  $m = 5$  and  $n = 2$ , and  $m = n = 4$  to a SAT solver (e.g., Glucose [7] or Cadical [11]), which proves both formulas unsatisfiable in less than a minute. We hence derive the following result.

**Proposition 1.** *No anonymous SWF satisfies both strategyproofness and unanimity if  $m = 5$  and  $n = 2$  or  $m = 4$  and  $n = 4$ .*

## 4.2 Inductive Arguments

Proposition 1 shows that no anonymous SWF satisfies strategyproofness and unanimity for only two cases, namely when there are  $m = 5$  alternatives and  $n = 2$  voters or  $m = 4$  alternatives and  $n = 4$  voters. By contrast, Theorem 2 claims that the impossibility holds for a large range of combinations of  $m$  and  $n$ . To close this gap, we will next present a lemma that generalizes our impossibility theorem from fixed numbers of voters and alternatives to a large range. In combination with Proposition 1, this lemma proves Theorem 2. The full proof of Lemma 1 can be found in [41, Appendix B].

**Lemma 1.** *Assume there is no anonymous SWF that satisfies strategyproofness and unanimity for  $m$  alternatives and  $n$  voters. The following claims hold:*

- (1) *For every  $m' > m$ , there is no anonymous SWF that satisfies strategyproofness and unanimity for  $m'$  alternatives and  $n$  voters.*
- (2) *For every  $\ell \in \mathbb{N}$ , there is no anonymous SWF that satisfies strategyproofness and unanimity for  $m$  alternatives and  $\ell n$  voters.*

**PROOF SKETCH.** For both claims, we will show the contrapositive: we assume that there is an anonymous, unanimous, and strategyproof SWF for the larger numbers of alternatives and voters, and show that this implies that there is also an SWF that satisfies our properties for  $m$  alternatives and  $n$  voters. In more detail, to show Claim (1), we suppose that there is an SWF  $f$  for  $m' > m$  alternatives and  $n$  voters that satisfies our axioms. We then define an SWF  $g$  for  $m$  alternatives and  $n$  voters as follows: given a profile for these parameters, we add  $m' - m$  dummy alternatives in the same order at the bottom of the rankings of all voters, apply  $f$  to compute a ranking on these  $m'$  alternatives, and then delete the  $m' - m$  dummy alternatives from this ranking to infer the final output ranking. Since  $f$  is unanimous, the dummy alternatives must appear at the bottom of the intermediate output ranking. Based on this insight, it can be shown that  $g$  inherits strategyproofness, anonymity, and unanimity from  $f$ , which contradicts that no SWF satisfies these axioms for  $m$  alternatives and  $n$  voters.

For Claim (2), we assume that there is a unanimous, anonymous, and strategyproof SWF  $f$  for  $m$  alternatives and  $\ell n$  voters. We then define an SWF  $g$  for  $m$  alternatives and  $n$  voters as follows: we clone each voter’s ranking  $\ell$  times and then apply  $f$  to compute the output ranking. It can be verified that  $g$  is anonymous, unanimous, and strategyproof as  $f$  satisfies these conditions, which again contradicts the assumption of this lemma.  $\square$

## 4.3 Verification

Because Proposition 1 has been shown by SAT solving, it is not immediately clear how to verify the correctness of this result. We have chosen a threefold approach to address this issue.

Firstly, following prior works [e.g., 16, 19, 22, 65], we provide in the full version [41, Appendix D] a proof of Proposition 1 for the case that  $m = 5$  and  $n = 2$  in a human-readable format. This proof was obtained by analyzing minimal unsatisfiable subsets (MUSes) of the original formula, i.e., inclusion-minimal subsets of the formula that are unsatisfiable. More intuitively, such MUSes can be seen as the reason why a formula is unsatisfiable and they tend to be much smaller than the original formula. For instance, for their Theorem 2, Brandt et al. [22] ended up with a MUS that only reasons about 13

profiles, which allows to give a compact human-readable proof. Unfortunately, our MUSes, which we obtained by using the program MUSer2 [9], are much bigger: even after several optimizations, the smallest MUS we found requires roughly 200 profiles and uses intricate reasoning. Although we were able to extract a human-readable proof by using a custom computer program that translates MUSes into a readable format, the resulting proof spans over 20 pages and requires the verification of more than 3000 strategyproofness applications. As such, the proof allows readers to build confidence in the correctness of our SAT-based approach by inspecting some intermediate steps, but manually verifying the full argument would be extremely tedious. We moreover note that we cannot provide a human-readable proof for the case that  $m = n = 4$  as the computer reasons about thousands of profiles in this case.

Because of these issues, we offer two further means of verification. Firstly, we have published our code for creating the SAT formula described in Section 4.1 [40]. This enables researchers to directly check that our code correctly constructs the desired formula, which means we only need to trust the correctness of the SAT solvers. Moreover, since our implementation is rather standard, we expect researchers familiar with this type of work to be able to verify our code in less than a day.

Lastly, following more recent works [e.g., 16, 22, 33], we have verified Theorem 2—including both base cases and the inductive arguments—with the interactive theorem prover Isabelle/HOL [63]. Such interactive theorem provers are computer programs designed to verify the correctness of mathematical proofs and have a high degree of trustworthiness. Specifically, Isabelle/HOL offers a rich mathematical logic which makes it simple to formalize our setting. Thus, our Isabelle verification directly derives Proposition 1 as well as Lemma 1 from our axioms. This formal verification releases us from the need to check the intermediate steps because Isabelle verifies the correctness of each deduction step based on a small and highly trustworthy set of logical operations. Consequently, to trust our results, one only needs to trust the implementation of our axioms in Isabelle. We further note that experts in verification see such formal proofs as the “gold standard” for increasing the trustworthiness of mathematical results [49]. Our Isabelle proof development is available in the *Archive of Formal Proofs* [39].

## 5 APPROXIMATE STRATEGYPROOFNESS

As our last contribution, we analyze how manipulable particular SWFs are. Indeed, while Theorem 2 shows that all reasonable SWFs must be manipulable, it may be the case that voters can only gain a small amount of utility by lying about their true ranking. In practice, this may be enough to disincentivize voters from manipulating as casting strategic votes requires effort to, e.g., learn the rankings of the other voters and to compute a successful manipulation.

Unfortunately, all the SWFs discussed in Section 2.1 are severely manipulable. To formalize this, we will make several changes in our assumptions. Firstly, we will consider a variable electorate setting and use as many voters as necessary for our counterexamples. Secondly, we will analyze the potential utility gain rather than the decrease in cost. To this end, we define the utility of a voter  $i$  with ranking  $\succ_i$  for another ranking  $\triangleright$  by  $u(\succ_i, \triangleright) = \binom{m}{2} - \Delta(\succ_i, \triangleright) = |\{(x, y) \in A^2 : x \succ_i y \wedge x \triangleright y\}|$ . We note that  $u$  is minimal if  $\succ$

and  $\triangleright$  are inverse to each other (yielding  $u(\succ, \triangleright) = 0$ ) and maximal if  $\succ \triangleright$  (yielding  $u(\succ, \triangleright) = \binom{m}{2}$ ). When using this utility function, strategyproofness demands that voters cannot increase their utility by lying about their ranking, which is equivalent to Definition 1.

Finally, we will use the incentive ratio to measure the manipulability of SWFs. This ratio quantifies the worst-case ratio between a voters' utility when lying and when voting truthfully. Formally, the *incentive ratio* of an SWF  $f$  for  $m$  alternatives is defined by

$$\gamma_m(f) = \sup_{R, i, \succ'_i} \frac{u_i(f(\succ'_i, R_{-i}))}{u_i(f(R))},$$

where we only consider profiles on  $m$  alternatives and  $(\succ'_i, R_{-i})$  denotes the profile obtained from  $R$  by letting voter  $i$  deviate to  $\succ'_i$ . Since  $u_i(f(R))$  can be 0, we use the conventions that  $\frac{x}{0} = \infty$  for all  $x > 0$  and  $\frac{0}{0} = 1$ . We observe that  $\gamma_m(f) = 1$  if  $f$  is strategyproof for  $m$  alternatives and  $\gamma_m(f) > 1$  otherwise. Moreover, an SWF has an incentive ratio of  $\gamma_m(f) = \infty$  if a voter with utility 0 can manipulate and  $\gamma_m(f) \leq \binom{m}{2}$  otherwise. The incentive ratio has been successfully used for private good settings to show that several manipulable mechanisms are still close to strategyproof [29, 59, 77].

We next prove that all SWFs in Section 2.1 have a large incentive ratio, thereby demonstrating that they are highly manipulable. Specifically, we show that, while distance scoring rules (including the Kemeny rule) cannot be manipulated by voters with utility 0, voters with utility 1 can gain almost their maximal utility by lying. Further, we prove that the incentive ratio of positional scoring rules is unbounded as they are manipulable by voters with utility 0.

**Theorem 3.** *The following statements are true:*

- (1) For all  $m \geq 4$ , the incentive ratio of the Kemeny rule  $f_{\text{Kemeny}}$  satisfies  $\binom{m}{2} - m \leq \gamma_m(f_{\text{Kemeny}}) \leq \binom{m}{2}$
- (2) For all  $m \geq 3$ , the incentive ratio of every distance scoring rule  $f_{\text{dist}}$  other than  $f_{\text{Kemeny}}$  satisfies  $\binom{m}{2} - 1 \leq \gamma_m(f_{\text{dist}}) \leq \binom{m}{2}$ .
- (3) For all  $m \geq 3$ , the incentive ratio of every positional scoring rule  $f_{\text{pos}}$  is  $\gamma_m(f_{\text{pos}}) = \infty$ .

**PROOF SKETCH.** We will only prove here that  $\gamma_m(f) \leq \binom{m}{2}$  for all distance scoring rules  $f$ . The proofs of our lower bounds can be found in [41, Appendix C]. Equivalently, this upper bound means that voters with utility 0 cannot manipulate distance scoring rules. To prove this claim, we fix such a rule  $f$  and its distance scoring function  $s$ , a profile  $R$ , a voter  $i$  with ranking  $\succ_i$ , and let  $\triangleright = f(R)$ . Moreover, we suppose that  $u(\succ_i, \triangleright) = 0$ , which means that  $\succ_i$  is inverse to  $\triangleright$  and  $\Delta(\succ_i, \triangleright) = \binom{m}{2}$ . Lastly, let  $R'$  denote a profile derived from  $R$  by assigning an arbitrary ranking  $\succ'_i \neq \succ_i$  to voter  $i$ .

As the first step, we consider another ranking  $\triangleright' \in \mathcal{R} \setminus \{\triangleright\}$  and show that  $s(\Delta(\succ_i, \triangleright)) - s(\Delta(\succ'_i, \triangleright)) \geq s(\Delta(\succ_i, \triangleright')) - s(\Delta(\succ'_i, \triangleright'))$ . To this end, we observe that  $\Delta(\succ_i, \triangleright) - \Delta(\succ'_i, \triangleright) = \Delta(\succ_i, \succ'_i)$  since  $\triangleright$  and  $\succ_i$  are inverse to each other. Next, let  $z = \Delta(\succ_i, \triangleright') - \Delta(\succ'_i, \triangleright')$  and note that  $z \leq \Delta(\succ_i, \succ'_i)$  since  $\Delta$  is a metric. If  $z \leq 0$ , our inequality holds since  $s$  is non-decreasing,  $\Delta(\succ_i, \triangleright) \geq \Delta(\succ'_i, \triangleright)$ ,  $\Delta(\succ_i, \triangleright') \leq \Delta(\succ'_i, \triangleright')$ . In particular, these insights imply that  $s(\Delta(\succ_i, \triangleright)) - s(\Delta(\succ'_i, \triangleright)) \geq 0 \geq s(\Delta(\succ_i, \triangleright')) - s(\Delta(\succ'_i, \triangleright'))$ . Next, suppose that  $z > 0$ . In this case, we recall that, by definition,  $s(x) - s(x-1) \geq s(x-1) - s(x-2)$  for all  $x \in \{2, \dots, \binom{m}{2}\}$ , which implies that  $s(\binom{m}{2}) - s(\binom{m}{2} - z) \geq s(\Delta(\succ_i, \triangleright')) - s(\Delta(\succ'_i, \triangleright') - z)$ . Since  $s$  is non-decreasing,  $\Delta(\succ_i, \triangleright) = \binom{m}{2}$ , and  $z \leq \Delta(\succ_i, \succ'_i)$ , we

again infer our target inequality as

$$\begin{aligned} s(\Delta(\succ_i, \triangleright)) - s(\Delta(\succ'_i, \triangleright)) &= s(\Delta(\succ_i, \triangleright)) - s(\Delta(\succ_i, \triangleright) - \Delta(\succ_i, \succ'_i)) \\ &\geq s(\Delta(\succ_i, \triangleright)) - s(\Delta(\succ_i, \triangleright) - z) \\ &\geq s(\Delta(\succ_i, \triangleright')) - s(\Delta(\succ_i, \triangleright') - z) \\ &= s(\Delta(\succ_i, \triangleright')) - s(\Delta(\succ'_i, \triangleright')). \end{aligned}$$

Next, it holds that  $\sum_{j \in N} s(\Delta(\succ_j, \triangleright)) \leq \sum_{j \in N} s(\Delta(\succ_j, \triangleright'))$  for all  $\triangleright' \in \mathcal{R}$  since  $f(R) = \triangleright$ . Hence, we derive for all  $\triangleright' \in \mathcal{R}$  that

$$\begin{aligned} \sum_{j \in N} s(\Delta(\succ'_j, \triangleright)) &= \sum_{j \in N \setminus \{i\}} s(\Delta(\succ_j, \triangleright)) - (s(\Delta(\succ_i, \triangleright) - \Delta(\succ'_i, \triangleright)) \\ &\leq \sum_{j \in N \setminus \{i\}} \Delta(\succ_j, \triangleright') - (s(\Delta(\succ_i, \triangleright')) - s(\Delta(\succ'_i, \triangleright'))) \\ &= \sum_{j \in N} \Delta(\succ'_j, \triangleright'). \end{aligned}$$

This proves that  $\triangleright$  minimizes the total score in  $R'$ . Further, if this inequality is tight for some ranking  $\triangleright'$ , then  $\sum_{j \in N} s(\Delta(\succ_j, \triangleright)) = \sum_{j \in N} s(\Delta(\succ_j, \triangleright'))$  and  $\triangleright$  is lexicographically preferred to  $\triangleright'$ . Hence,  $f(R') = \triangleright$  and voters with utility 0 cannot manipulate  $f$ .  $\square$

**Remark 5.** A natural follow-up question to Theorem 3 is whether there are appealing SWFs that have a significantly better incentive ratio than our considered SWFs. Motivated by this question, we discuss the *minimal compromise rule*  $f_{\text{MC}}$  in [41, Appendix C], which has an incentive ratio of  $m - 2$  when  $m \geq 4$ . To define this rule, we denote the min score of an alternative  $x$  in a profile  $R$  by  $s_{\min}(R, x) = \min_{i \in N} m - r(\succ_i, x)$ . Then,  $f_{\text{MC}}$  orders the alternatives in decreasing order of their min scores, with ties broken lexicographically. We note that this rule was studied before by Congar and Merlin [30].

## 6 CONCLUSION

In this paper, we study social welfare functions (SWFs) with respect to (Kemeny-)strategyproofness, which requires that voters cannot obtain a ranking that is closer to their true ranking in terms of the Kemeny distance by voting strategically. As our main result, we show a sweeping impossibility theorem, demonstrating that no anonymous and unanimous SWF satisfies strategyproofness if there are  $m \geq 4$  alternatives. Moreover, we prove that no majority consistent SWF is strategyproof when  $m \geq 4$  and that many natural SWFs are severely manipulable as they have a high incentive ratio.

A natural follow-up question to our work is how we can circumvent our impossibility theorems. Possible directions to this end are the study of randomized or set-valued SWFs, a more detailed analysis of the incentive ratio of SWFs, or the study of alternative strategyproofness notions. For example, one could analyze which distances between rankings allow for appealing strategyproof SWFs.

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AAMAS '26, May 25 – 29, 2026, Paphos, Cyprus

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