Complexity and Behaviour in Strategic Models of Elections

Doctoral Consortium

Colin Cleveland
King's College London
London, United Kingdom
colin.cleveland@kcl.ac.uk

ABSTRACT

This PhD studies the algorithmic and behavioural foundations of candidate positioning and strategic voting in elections, at the intersection of algorithmic game theory and computational social choice. It unifies three research strands: multi-dimensional spatial elections, prospect-theoretic voting, and strategic behaviour in primary elections. Across these settings, the thesis examines how rational and boundedly rational agents make strategic decisions under institutional and cognitive constraints, and how these decisions give rise to computational problems of optimisation, equilibrium, and complexity. The work shows how behavioural assumptions and multi-stage electoral structures fundamentally alter equilibrium properties and computational tractability. Future work develops primary elections as multi-stage games with incomplete information and learning-based dynamics.

KEYWORDS

Computational Social Choice, Algorithmic Game Theory, Spatial Voting, Prospect Theory, Primary elections

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1 INTRODUCTION & MOTIVATION

In most algorithmic game-theoretic models, elections, containing voting behavior on the electorate's side and campaigning on the candidates' side, are treated as strategic interactions in which agents aim to maximize their individual utility, for example by increasing the likelihood that a preferred candidate wins.

However, this stylized narrative requires refinement to better reflect real-world electoral behaviors. From the candidates' perspective, political decision-making often involves balancing positions across multiple issues[6, 9], which can render the identification of an "ideal" policy profile infeasible. Strategic considerations may therefore extend beyond simple utility maximization toward managing trade-offs and constraints across dimensions.

Similarly, voters may not always behave as fully rational utility maximizers[11]. Voting is frequently perceived as an act of candidate support rather than as a purely strategic choice, and psychological factors can influence how voters evaluate alternatives. As a result, voters may not compute or follow the game-theoretically optimal strategy suggested by classical models.

Beyond individual behavior, elections are often prolonged processes rather than single-shot events. Strategic dynamics arise both from institutional multi-stage structures, such as iterative voting [7, 8] and primary elections followed by a general election, and from informational uncertainty faced by voters [10]. These factors enrich the strategic environment faced by both voters and candidates, while also introducing additional computational challenges in analyzing equilibria and optimal strategies.

These reality-driven phenomena motivate my investigation of elections along three complementary research directions in my PhD:

- **Spatial Voting** [2]: I study the computational complexity of candidate positioning in multi-dimensional spatial voting models. In particular, this work analyzes the hardness of identifying optimal policy combinations when voters' distributions in the policy space are correlated.
- **Prospect-Theoretic Voting** (Under review): This line of work examines candidate positioning when voters evaluate alternatives through reference-dependent preferences inspired by prospect theory. The focus is on how psychological effects such as relative utility comparisons alter optimal candidate strategies.
- **Primary Elections** [3]: I investigate strategic voting behavior in primary elections and analyze how strategic incentives in early-stage elections shape outcomes in subsequent general elections.

In the remainder of this extended abstract, I provide an overview of my doctoral research results to date and outline my planned directions for future research.

2 RESULTS OBTAINED

Across my doctoral research, I study elections as strategic and computational systems in which voter behavior is shaped by institutional structure and behavioral constraints. While candidates are often modeled as strategic players, a unifying theme of my work is understanding how voters respond to different electoral environments, and how these responses shape candidates' incentives and the computational properties of elections.

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2.1 Optimal Candidate Positioning in Multi-Issue Elections

My work on multi-issue elections studies strategic candidate positioning in high-dimensional policy spaces. I adopt the classical spatial voting framework, representing voters and candidates as points in $\mathbb{R}^d$, where each voter supports the closest candidate under an ℓ_p-norm. Although widely used in political economy, the algorithmic implications of this model in multi-dimensional settings remain poorly understood.

I show that even in a highly restricted setting—where a single new candidate competes against one fixed opponent—computing an optimal policy position is NP-hard, revealing a fundamental computational barrier to strategic positioning. When the number of issues is constant, however, the problem becomes tractable. I develop an $O(n^{d+1})$ hyperplane-enumeration algorithm for fixed d, and a specialized $O(n \log n)$ radial-sweep algorithm for the two-dimensional case, both of which compute optimal positions exactly.

Beyond exact optimization, I derive approximation guarantees for the general multi-candidate setting and show that the same geometric techniques extend to positional scoring rules such as k-approval and Borda. Together, these results clarify the algorithmic landscape of multi-dimensional spatial elections and provide concrete tools for analyzing campaign strategies across multiple issues.

2.2 Prospect-Theoretic Voting: Strategic Implications and Computational Limits

In a second line of work, I study how behavioral decision-making affects strategic voting and candidate behavior. Motivated by empirical evidence from psychology, I model voter abstention through reference-dependent preferences inspired by prospect theory [5].

I consider a one-dimensional Hotelling–Downs [4] setting in which voters evaluate candidates relative to a reference point, interpreted as an incumbent position. Voters participate only when the perceived regret from abstaining exceeds a personal participation cost. This mechanism generates generally non-convex voting regions, breaking the classical *Median Voter* prediction even under linear value functions.

Within this framework, I analyze candidate behavior under two objectives: maximizing vote share and maximizing victory margin. Under both objectives, candidates may rationally adopt extreme positions despite a symmetric electorate. From a computational perspective, I show that computing an optimal candidate position is NP-complete when voter preferences are given by an explicit cumulative distribution function, and #P-hard when only probability density functions are available.

When the electorate is represented as a finite set of n voters, I provide a more refined algorithmic analysis. I develop an $\tilde{O}(n)$ geometric sweep algorithm for computing best responses in two-candidate elections, outline a recursive boundary-tracking method for the multi-candidate case, and present an $\tilde{O}(n^4)$ algorithm for computing an equilibrium. These results illustrate how behavioral realism introduces both strategic richness and computational hardness.

2.3 The Complexity of Strategic Behavior in Primary Elections

A third component of my research examines the strategic and computational implications of primary elections. Unlike direct voting systems, primaries introduce a multi-stage decision process in which voters first influence party nominations before a general election determines the final outcome. This structure allows voters to act strategically in early stages, for instance by supporting compromise candidates who are more competitive in the general election, or by attempting to influence opposing parties' primaries to affect downstream outcomes.

Building on and contrasting with the welfare-based analysis of Borodin et al. [1], I focus on the game-theoretic properties of primary elections. I formalize a model of primaries under first-past-the-post voting with fixed tie-breaking rules and analyze voters' strategic behavior across stages. I show that determining whether a pure Nash equilibrium exists is $\Sigma_2^{\mathbf{P}}$-complete, that computing a best response is NP-complete, and that deciding the existence of subgame-perfect equilibria in sequential primary elections is PSPACE-complete.

These results demonstrate that introducing primary elections fundamentally increases the computational difficulty of strategic reasoning, positioning multi-stage electoral systems as a rich and challenging domain within computational social choice.

3 ONGOING AND FUTURE WORK

Building on my existing work, I plan to further develop my study of primary elections, which remain relatively underexplored compared to other electoral models in computational social choice. I view primary elections as instances of *Multi-Stage Games with Selection and External Decisions* (SEED), and aim to integrate this framework with established models such as spatial voting and prospect-theoretic preferences. This perspective enables a unified computational analysis of strategic behavior by both voters and candidates across multiple electoral stages.

Objectives and Research Questions

My future research will address the following theoretical questions:

(1) **Voter Decision-Making:** How should voters optimally vote in primary elections under uncertainty about candidate viability and downstream general election outcomes?
(2) **Coordination Effects:** How does the presence of coordinated voting blocs, such as factions or interest groups, affect equilibrium existence and structure compared to atomistic voting?
(3) **Candidate Positioning:** How does the two-stage structure of primary elections influence candidate ideology and positioning in spatial models such as Hotelling–Downs?

In addition to analytical results, I plan to complement this work with a computational approach. Specifically, I propose using multi-agent reinforcement learning to simulate boundedly rational agents in SEED environments. These simulations will serve both to validate theoretical predictions and to explore strategic dynamics in settings where analytical equilibrium analysis becomes intractable.

REFERENCES

[1] Allan Borodin, Omer Lev, Nisarg Shah, and Tyrone Strangway. 2024. Primarily about Primaries. *Artificial Intelligence* 329 (2024), 104095.

[2] Colin Cleveland, Bart de Keijzer, and Maria Polukarov. 2025. *Optimal Candidate Positioning in Multi-Issue Elections*. IOS Press.

[3] Colin Cleveland, Bart de Keijzer, and Maria Polukarov. 2026. The Complexity of Strategic Behavior in Primary Elections. In *The 25th International Conference on Autonomous Agents and Multi-Agent Systems*. https://openreview.net/forum?id=p5mqMKXGYQ

[4] Anthony Downs. 1957. *An Economic Theory of Democracy*. Harper & Row New York.

[5] Daniel Kahneman and Amos Tversky. 1979. Prospect Theory: An Analysis of Decision under Risk. *Econometrica* 47, 2 (1979), 263–291.

[6] Richard D. McKelvey and Richard E. Wendell. 1976. Voting Equilibria in Multidimensional Choice Spaces. *Mathematics of Operations Research* 1, 2 (1976), 144–158.

[7] Reshef Meir, Maria Polukarov, Jeffrey Rosenschein, and Nicholas Jennings. 2010. Convergence to Equilibria in Plurality Voting. *Proceedings of the AAAI Conference on Artificial Intelligence* 24, 1 (Jul. 2010), 823–828. https://doi.org/10.1609/aaai.v24i1.7624

[8] Svetlana Obraztsova, Evangelos Markakis, Maria Polukarov, Zinovi Rabinovich, and Nicholas R. Jennings. 2015. On the convergence of iterative voting: how restrictive should restricted dynamics be?. In *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence* (Austin, Texas) *(AAAI'15)*. AAAI Press, 601 Pennsylvania Ave, NW Suite 900 Washington, DC 20004 USA, 993–999.

[9] G. Bingham Powell and G. Bingham Powell Jr. 2000. *Elections as Instruments of Democracy: Majoritarian and Proportional Visions*. Yale University Press.

[10] Annemieke Reijngoud and Ulle Endriss. 2012. Voter response to iterated poll information. In *Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems - Volume 2* (Valencia, Spain) *(AAMAS '12)*. International Foundation for Autonomous Agents and Multiagent Systems, Richland, SC, 635–644.

[11] William H. Riker and Peter C. Ordeshook. 1968. A Theory of the Calculus of Voting. *The American Political Science Review* 62, 1 (1968), 25–42. http://www.jstor.org/stable/1953324