Stackelberg Equilibria of Blotto Games

Extended Abstract

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ABSTRACT

The Colonel Blotto game, introduced by Borel in 1921, models the allocation of a fixed budget of troops across multiple battlefields and is widely used in applications such as elections, innovation contests, advertising, and sports. While Nash equilibria in Blotto games can be computed in polynomial time despite the exponential strategy space, much less is known about (pure) Stackelberg equilibria. The best known result is a polynomial-time 2-approximation due to Behnezhad et al. (SODA'18). We improve this by giving a polynomial-time algorithm that computes Stackelberg strategies with approximation factor $1+\epsilon$ for any constant $\epsilon > 0$.

KEYWORDS

Equilibrium, Stackelberg, Blotto, Zero-sum

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1 INTRODUCTION

Nash and Stackelberg equilibria are fundamental solution concepts in game theory. A Nash equilibrium is a (possibly mixed) strategy profile in which no player can benefit from unilateral deviation. A Stackelberg equilibrium models a leader–follower interaction, where the leader commits to a strategy anticipating the follower's best response. In constant-sum games such as Colonel Blotto, mixed Nash and mixed Stackelberg equilibria coincide, but pure Stackelberg equilibria form a distinct and less understood class. [1]

Computing equilibria is a central challenge in algorithmic game theory. While Nash equilibria are guaranteed to exist, computing them is PPAD-complete even for two-player games [5, 7, 8], and remains hard to approximate [6]. Stackelberg equilibria are tractable when strategy spaces are polynomially bounded, but many important games have exponentially large strategy spaces, where computing either equilibrium concept remains difficult.

A canonical example is the Colonel Blotto game, introduced by Borel in 1921 [4]. Two players allocate a fixed number of troops across multiple battlefields, winning a battlefield by committing more troops than the opponent. Blotto has been widely used to model competition in politics, sports, and advertising [2, 9–12]. Although equilibria in Blotto have been extensively studied, a polynomial-time algorithm for computing a Nash equilibrium was obtained only recently by Ahmadinejad *et al.* [1]. Since Blotto is constant-sum, this also yields a mixed Stackelberg equilibrium, but pure Stackelberg equilibria remain poorly understood.

Behnezhad *et al.* [3] presented a polynomial-time algorithm that computes a 2-approximation of the Stackelberg equilibrium. In this paper, we improve this result and give a polynomial-time algorithm that achieves a $(1+\epsilon)$-approximation for any $\epsilon > 0$. As in [1, 3], polynomial time means polynomial in the number of troops, battlefields, and maximum battlefield weight.

Theorem 1.1 (informal). *For any $\epsilon > 0$, one can find a $(1+\epsilon)$-approximation of the Stackelberg equilibrium of Colonel Blotto in polynomial time.*

A limitation of the algorithm of Behnezhad *et al.* [3] is that, while it approximates the leader's utility, it provides no guarantee on the follower's utility. For instance, if the leader's optimal utility is 100 and the follower's is 1, their algorithm ensures a leader utility of at least 50, but the follower's utility could be as high as 51, far above the optimal. We improve on this by computing a Stackelberg strategy that overestimates the follower's utility by at most a factor of $1+\epsilon$ for any constant ϵ, in polynomial time.

Theorem 1.2 (informal). *For any $\epsilon > 0$, one can compute in polynomial time a pure Colonel Blotto strategy whose follower utility is at most $1+\epsilon$ times that under the optimal Stackelberg strategy.*

Due to space constraints, we outline the proof of Theorem 1.1 here and defer full details to the extended version of the paper.

2 PRELIMINARIES

Colonel Blotto is a two-player constant-sum game, with player A as the *leader* and player B as the *follower*. The game is played over k battlefields $b_1, \ldots, b_k$, each with weight w_i satisfying $w_1 \geq \cdots \geq w_k$. Player A has n troops and player B has m troops. A strategy of player A is a vector $x = \langle x_1, \ldots, x_k \rangle$ of non-negative integers summing to n; similarly, player B's strategy is $y = \langle y_1, \ldots, y_k \rangle$ summing to m. On battlefield b_i, the player with more troops wins and gains payoff w_i; ties favor player B (our results extend if ties favor A). Players' utilities are the sum of battlefield payoffs. Strategies may use fewer than the total troops without affecting results.

We focus on Stackelberg strategies where the leader commits to a pure strategy and the follower best-responds. A strategy x is an α-approximation of the optimal Stackelberg strategy if its utility against the follower's best response is at least $1/\alpha$ of the maximum utility any pure strategy of the leader can achieve.

[1] Throughout this paper, Stackelberg equilibria refer to pure strategies.

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3 $(1+\epsilon)$ STACKELBERG STRATEGY

To present our $(1+\epsilon)$-approximation algorithm, we first describe how player B's best-response strategies are computed. Let $x = \langle x_1, \ldots, x_k \rangle$ be a strategy of player A. Given A's strategy, B has only two reasonable options per battlefield b_i: match x_i troops to win w_i, or skip the battlefield. The best response selects a subset of battlefields maximizing total weight while respecting B's troop limit m, which can be computed via dynamic programming.

Directly optimizing A's strategy against this best response is difficult. Behnezhad *et al.* [3] introduce a simpler, nearly optimal greedy algorithm: battlefields are sorted by x_i/w_i, and B matches A's troops in this order until exhausting her troops. This algorithm has two key properties: first, its utility differs from the optimal response by at most w_1. Furthermore, A's best response can be computed in polynomial time.

We extend this approach to achieve a $(1+\epsilon)$-approximation. We introduce our greedy algorithm, called the *relaxed best-response algorithm*, which adds a parameter p (*pivot*) to the standard problem. For the first p battlefields ($i \le p$), it performs a brute-force search over all 2^p subsets of player A's troop allocations. For the remaining battlefields ($i > p$), it applies a greedy strategy, sorting by x_i/w_i. While the basic greedy algorithm runs in $O(k)$ time, the relaxed algorithm runs in $O(k2^p)$, exponential in p. The advantage is that, unlike the greedy algorithm whose additive error can reach w_1, the relaxed best-response algorithm's error is bounded by w_{p+1}.

Lemma 3.1. *Let y be the best response from the optimal dynamic programming algorithm, and y' the response from the relaxed algorithm. Then, the utility difference between y and y' is at most w_{p+1}.*

We now describe our $(1+\epsilon)$-approximation algorithm. If the optimal Stackelberg payoff is at least w_1/ϵ, the algorithm of [3] already gives a $1+\epsilon$ approximation. If B has at least as many troops as A, B can match all battlefields, so A's optimal utility is 0. If A has more troops than B, placing all troops on b_1 guarantees utility $\ge w_1$. Thus, we assume the optimal Stackelberg utility lies in $[w_1, w_1/\epsilon]$.

Let $\epsilon' = \epsilon/4$. We define *heavy battlefields* as those with $w_i \ge \epsilon' w_1$ and *light battlefields* as the rest. We first discretize the heavy battlefield weights: for each b_i with $w_i \ge \epsilon' w_1$, let a be the largest integer such that $\lceil (1+\epsilon')^a \rceil \le w_i$, and round w_i to $\lceil (1+\epsilon')^a \rceil$.

After discretization, each heavy battlefield's weight decreases by at most a factor of $1+\epsilon'$, so any strategy's utility drops by at most $1/(1+\epsilon')$. The ordering $w_1 \ge \cdots \ge w_k$ may no longer hold, but this does not affect our analysis, as all light battlefield weights remain bounded by $\epsilon' w_1$.

Let d be the number of unique heavy battlefield weights. By discretization, $d \le \lceil \log_{1+\epsilon'} 1/\epsilon' \rceil + 1 \le 2/\epsilon'^2$. We show that the number of reasonable ways for player A to distribute troops over heavy battlefields is bounded by $(n+1)^d$. For two battlefields b_α and b_β with equal weight and a strategy x where $x_\beta > x_\alpha + 1$, define x' by shifting one troop from b_β to b_α. Against any best response y' of B, we can construct a strategy y such that x' (versus y') achieves at least the same utility as x (versus y). The cases are:

- If y' wins neither b_α nor b_β, it gets the same utility versus x.
- If y' wins both, define y by shifting one troop from b_α to b_β: $y_\alpha = y'_\alpha - 1$, $y_\beta = y'_\beta + 1$, and $y_i = y'_i$ for all other i. Then y achieves the same utility against x.
- If y' wins exactly one, define y by $y_\alpha = \max(y'_\alpha, y'_\beta)$, $y_\beta = \min(y'_\alpha, y'_\beta)$, and $y_i = y'_i$ for all other i. This ensures x' achieves at least as much utility as x.

Hence, for any reasonable strategy, troops assigned to equal-weight battlefields differ by at most 1. Once A decides the total troops β for α equal-weight battlefields, individual allocations are uniquely determined: $\beta - \alpha\lfloor \beta/\alpha \rfloor$ battlefields receive $\lfloor \beta/\alpha \rfloor + 1$ troops, and the rest receive $\lfloor \beta/\alpha \rfloor$. Since there are at most d distinct heavy weights, the total number of reasonable strategies over heavy battlefields is at most $(n+1)^d$, which is polynomial in n.

Next, we bound the number of reasonable best-responses of player B on the heavy battlefields. Since A's utility is at most w_1/ϵ, and each heavy battlefield has weight at least $\epsilon' w_1/(1+\epsilon')$ (due to discretization), any reasonable best-response can afford to lose on at most $(1+\epsilon')/(\epsilon\epsilon')$ heavy battlefields.

For a fixed allocation of A's troops on heavy battlefields, let e denote the number of reasonable responses of B. She must choose which heavy battlefields to forgo and match troops on the rest. Because B is indifferent among battlefields of equal weight and will give up those with more troops from A, she has at most $d+1$ choices per lost-battlefield option (d weights plus the option of giving up none). Hence, $e \le (d+1)^{(1+\epsilon')/(\epsilon\epsilon')}$, which depends only on ϵ, d.

So far, the number of reasonable strategies of A on heavy battlefields is $(n+1)^d$, and the number of reasonable responses of B is e (constant for fixed ϵ). Our algorithm iterates over all $(n+1)^d$ heavy allocations of A and finds the one that, combined with the optimal allocation on light battlefields, maximizes the guaranteed payoff. Fix an allocation on the heavy battlefields and set p to the last heavy battlefield. The relaxed best-response algorithm then considers all e reasonable responses of B.

For each of the e iterations, the relaxed best-response applies a greedy algorithm on the light battlefields. In each iteration, the greedy algorithm either matches all troops or stops at a battlefield b_i with j troops when B has fewer than j troops. We track the status of each iteration with a flag: **null** if all troops are matched, or a pair (b_i, j) if it gets stuck. Therefore, each iteration has at most $(n+1)k + 1$ possible statuses.

The key idea is as follows: after fixing A's allocation on heavy battlefields and the statuses of all e relaxed best-response iterations, the reaction of each iteration to any light battlefield allocation is uniquely determined. If the status is **null**, B matches all troops; otherwise, B matches only if the battlefield's troop-to-weight ratio exceeds that of the stored status, breaking ties by battlefield index.

Given this, we use dynamic programming (DP) to compute the optimal allocation for A on light battlefields: we first fix A's heavy battlefield strategy and the statuses of B across all e iterations. Then the DP finds the best strategy for A, and we verify consistency: (1) if the status is **null**, B matches all troops; (2) if the status is (b_i, j), the remaining troops of B must be less than j. This ensures that the DP solution aligns with the relaxed best-response behavior.

Theorem 1.1. *We can approximate the Stackelberg Equilibria of Blotto Games within a factor of $1+\epsilon$ in time*

$$O\left(\left(\frac{33}{\epsilon^2}\right)^{\frac{5}{\epsilon^2}} n^{\frac{32}{\epsilon^2}+\left(\frac{33}{\epsilon^2}\right)^{\frac{5}{\epsilon^2}}+2} k^{2\left(\frac{33}{\epsilon^2}\right)^{\frac{5}{\epsilon^2}}+1} m^{\left(\frac{33}{\epsilon^2}\right)^{\frac{5}{\epsilon^2}}} w_1^{\left(\frac{33}{\epsilon^2}\right)^{\frac{5}{\epsilon^2}}}\right).$$

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