# Stronger Approximation Guarantees for Non-Monotone $\gamma$-Weakly DR-Submodular Maximization

Hareshkumar Jadav
IIT Indore
Indore, India
hareshjadavcse@gmail.com

Ranveer Singh
IIT Indore
Indore, India
ranveer@iiti.ac.in

Vaneet Aggarwal
Purdue University
West Lafayette, USA
vaneet@purdue.edu

## ABSTRACT

Maximizing submodular objectives under constraints is a fundamental problem in machine learning and optimization. We study the maximization of a nonnegative, non-monotone $\gamma$-weakly DR-submodular function over a down-closed convex body. Our main result is an approximation algorithm whose guarantee depends smoothly on $\gamma$; in particular, when $\gamma = 1$ (the DR-submodular case) our bound recovers the 0.401 approximation factor, while for $\gamma < 1$ the guarantee degrades gracefully and, it improves upon previously reported bounds for $\gamma$-weakly DR-submodular maximization under the same constraints. Our approach combines a Frank–Wolfe–guided continuous-greedy framework with a $\gamma$-aware double-greedy step, yielding a simple yet effective procedure for handling non-monotonicity. This results in state-of-the-art guarantees for non-monotone $\gamma$-weakly DR-submodular maximization over down-closed convex bodies.

## KEYWORDS

Combinatorial Optimization, Weakly DR Submodular Function, Approximation Algorithm

**ACM Reference Format:**
Hareshkumar Jadav, Ranveer Singh, and Vaneet Aggarwal. 2026. Stronger Approximation Guarantees for Non-Monotone $\gamma$-Weakly DR-Submodular Maximization. In *Proc. of the 25th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2026), Paphos, Cyprus, May 25 – 29, 2026*, IFAAMAS, 3 pages. https://doi.org/10.65109/GAIZ8613

## 1 INTRODUCTION

Submodular maximization under various constraints is a central problem in optimization and theoretical computer science. Foundational work established much of the modern toolkit and sparked a rich line of research [5, 8, 13–15]. Informally, a set function is submodular if its marginal gains decrease as the chosen set grows (a discrete "diminishing returns" property). Many classic combinatorial tasks can be cast as maximizing a submodular objective, including *Max-Cut*, the *assignment problem*, *facility location*, and *Max-Bisection* [1, 4, 6, 7, 10, 12].

On the continuous side, *diminishing-returns* (DR) submodularity is a continuous analogue of submodularity, where marginal gains along each coordinate decrease as the current point increases (a diminishing-returns property in $\mathbb{R}^n$). We provide the formal definition in the preliminaries. In this work, we consider a generalized framework *weakly* DR submodularity which relaxes diminishing returns by a factor $\gamma \in (0, 1]$: roughly, marginal gains still decrease, but only up to a controlled multiplicative slack $\gamma$. Recent work provides unified algorithms and approximation guarantees for this framework [9, 16–19]. These developments motivate our focus on continuous DR and weakly DR objectives over down-closed convex bodies.

### 1.1 Our Contribution

In this paper, we study *non-monotone* $\gamma$-weakly DR-submodular objectives over a down-closed convex body $P \subseteq [0, 1]^n$, for $0 < \gamma \leq 1$. In this regime, the canonical approximation envelope is $\kappa(\gamma) = \gamma e^{-\gamma}$ (which recovers $e^{-1}$ at $\gamma = 1$) [2, 20]. In [2], this guarantee is achieved with time complexity $O(1/\varepsilon)$, where $\varepsilon > 0$ is an accuracy parameter, whereas [20] attains the same guarantee with running time $O(1/\varepsilon^3)$. In contrast, our algorithm achieves a $\Phi_\gamma$-approximation in time $\mathrm{Poly}(n, \delta^{-1})$. Recently, Buchbinder and Feldman introduced a technique that yields a 0.401-approximation for the (fully) DR-submodular case [3]; their algorithm also runs in time $\mathrm{Poly}(n, \delta^{-1})$.

*This paper aims to close the gap between weak and full DR.* We develop $\gamma$-aware algorithms and analyses that (i) recover the best-known DR constant at $\gamma = 1$ [3] and (ii) strictly improve upon the baseline $\kappa(\gamma) = \gamma e^{-\gamma}$ throughout the weakly DR regime [2, 20] (see Figure 1).

## 2 PRELIMINARIES AND NOTATION

We use boldface letters (e.g., $\mathbf{x}, \mathbf{y}$) to denote vectors in $\mathbb{R}^n$, and write $\mathbf{x} = (x_1, \ldots, x_n)$. For $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, we use the componentwise partial order: $\mathbf{x} \leq \mathbf{y}$ if and only if $x_i \leq y_i$ for all $i$ (and $\mathbf{x} < \mathbf{y}$ if and only if $x_i < y_i$ for all $i$). The *join* and *meet* are defined as

$$\mathbf{x} \vee \mathbf{y} := (\max\{x_i, y_i\})_{i=1}^n, \qquad \mathbf{x} \wedge \mathbf{y} := (\min\{x_i, y_i\})_{i=1}^n.$$

We also use the elementwise (Hadamard) product $\mathbf{x} \odot \mathbf{y} \in \mathbb{R}^n$, defined by $(\mathbf{x} \odot \mathbf{y})_i := x_i\, y_i$ for each $i \in [n]$, and the standard inner product $\langle \mathbf{x}, \mathbf{y} \rangle := \sum_{i=1}^n x_i y_i$. For vectors with entries in $[0, 1]$, the coordinatewise *probabilistic sum* is

$$\mathbf{x} \oplus \mathbf{y} \;:=\; \mathbf{1} - \big(\mathbf{1} - \mathbf{x}\big) \odot \big(\mathbf{1} - \mathbf{y}\big). \tag{1}$$

A convex polytope $P \subseteq [0, 1]^n$ is *down-closed* if $\mathbf{y} \in P$ implies $\mathbf{x} \in P$ for every $\mathbf{x} \in \mathbb{R}^n$ with $\mathbf{0} \leq \mathbf{x} \leq \mathbf{y}$. We say that $P$ is *solvable* if linear optimization over $P$ can be performed in polynomial time. The (Euclidean) *diameter* of a set $P \subseteq \mathbb{R}^n$ is

$$D := \sup\big\{ \|\mathbf{x} - \mathbf{y}\|_2 : \mathbf{x}, \mathbf{y} \in P \big\}.$$

Full version of this paper is available at: https://arxiv.org/abs/2601.00611



*Proc. of the 25th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2026), C. Amato, L. Dennis, V. Mascardi, J. Thangarajah (eds.), May 25 – 29, 2026, Paphos, Cyprus.* © 2026 International Foundation for Autonomous Agents and Multiagent Systems (www.ifaamas.org). https://doi.org/10.65109/GAIZ8613

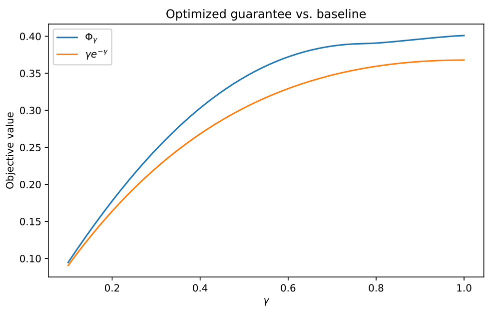


**Figure 1: Approximation guarantee versus the weakly-DR parameter. The horizontal axis is the weakly-DR parameter $\gamma \in (0, 1]$, and the vertical axis is the approximation factor. We plot our optimized guarantee $\Phi_\gamma$ (blue curve) alongside the non-monotone weakly-DR baseline $\kappa(\gamma) = \gamma e^{-\gamma}$ (orange curve). Over the entire regime $\gamma \in (0, 1)$, $\Phi_\gamma$ strictly exceeds $\kappa(\gamma)$, and at $\gamma = 1$ (full DR) our curve reaches $0.401$, matching the current best bound.**

A nonnegative function $F : [0, 1]^n \to \mathbb{R}_{\geq 0}$ is *$\gamma$-weakly DR-submodular* if, for all $\mathbf{x}, \mathbf{y} \in [0, 1]^n$ with $\mathbf{x} \leq \mathbf{y}$, any $i \in [n]$, and any $c > 0$ such that $\mathbf{x} + c\,\mathbf{e}_i,\ \mathbf{y} + c\,\mathbf{e}_i \in [0, 1]^n$,

$$F(\mathbf{x} + c\,\mathbf{e}_i) - F(\mathbf{x}) \;\geq\; \gamma\big(F(\mathbf{y} + c\,\mathbf{e}_i) - F(\mathbf{y})\big). \tag{2}$$

When $F$ is differentiable, this condition is equivalent to $\nabla F(\mathbf{x}) \geq \gamma\,\nabla F(\mathbf{y})$ for all $\mathbf{x} \leq \mathbf{y}$, where the inequality is understood componentwise. A differentiable function $F : P \to \mathbb{R}$ is *$L$-smooth* if, for all $\mathbf{x}, \mathbf{y} \in P$, it satisfies

$$\|\nabla F(\mathbf{x}) - \nabla F(\mathbf{y})\|_2 \leq L\,\|\mathbf{x} - \mathbf{y}\|_2. \tag{3}$$

## 3 MAIN RESULTS

We propose a recursive algorithm for maximizing a non-monotone $\gamma$-weakly DR-submodular objective over a down-closed convex body. At each recursive call, the procedure invokes two subroutines: the $\gamma$-Frank–Wolfe Guided Measured Continuous Greedy ($\gamma$-FWG) algorithm and a $\gamma$-aware double-greedy algorithm. For completeness, full algorithmic details are provided in the arXiv version of our paper [11]; here we focus on the resulting performance guarantees.

We first state the guarantee for the $\gamma$-FWG subroutine. This algorithm will take $\mathrm{Poly}(n, \delta^{-1})$ time. Moreover, $\gamma$-FWG satisfies the following approximation guarantee for its output.

Theorem 3.1. *$\gamma$-FWG takes as input a nonnegative, $L$-smooth, $\gamma$–weakly DR-submodular function $F : [0, 1]^n \to \mathbb{R}_{\geq 0}$, a meta-solvable down-closed convex body $P \subseteq [0, 1]^n$ of diameter $D$, a vector $\mathbf{z} \in P$, and parameters $t_s \in (0, 1)$, $\varepsilon \in (0, 1/2)$, and $\delta \in (0, 1)$. Given this input, $\gamma$-FWG outputs a vector $\mathbf{y} \in P$ and vectors $\mathbf{x}(1), \ldots, \mathbf{x}(m) \in P$ for some $m = O(\delta^{-1} + \varepsilon^{-1})$, such that at least one of the following holds.*

(1)

$$F(\mathbf{y}) \;\geq\; A_\gamma(t_s)\,F(\mathbf{o}) + B_\gamma(t_s)\,F(\mathbf{z} \odot \mathbf{o}) + C_\gamma(t_s)\,F(\mathbf{z} \oplus \mathbf{o}) - \delta L D^2. \tag{4}$$

(2) *There exists $i \in [m]$ such that*

$$F\big(\mathbf{x}(i) \oplus \mathbf{o}\big) \;\leq\; F(\mathbf{z} \oplus \mathbf{o}) \;-\; \varepsilon\,F(\mathbf{o}), \tag{5}$$

*and the point $\mathbf{x}(i)$ satisfies the $\gamma$–weakly DR local–value bound*

$$F\big(\mathbf{x}(i)\big) \geq \frac{\gamma^2\,F\big(\mathbf{x}(i) \vee \mathbf{o}\big) + F\big(\mathbf{x}(i) \wedge \mathbf{o}\big)}{1 + \gamma^2} - \frac{\delta\,\gamma}{1 + \gamma^2}\left(\max_{\mathbf{y}' \in Q(i)} F(\mathbf{y}') + \tfrac{1}{2} L D^2\right). \tag{6}$$

*i.e., $\mathbf{x}(i)$ is an approximate local maximum with respect to $\mathbf{o}$ under the $\gamma$–weakly DR guarantee and the Frank–Wolfe certificate over $Q(i)$.*

*Here the $\gamma$–dependent coefficients are*

$$A_\gamma(t_s) := -\frac{e^{\gamma t_s - \gamma}}{1 - \gamma} + \frac{e^{-\gamma^2}}{\gamma(1 - \gamma)}\Big(e^{\gamma^2 t_s} - (1 - \gamma)\Big) - O(\varepsilon) \tag{7a}$$

$$B_\gamma(t_s) := \frac{e^{-\gamma} - e^{\gamma t_s - \gamma}}{\gamma} \tag{7b}$$

$$\begin{aligned} C_\gamma(t_s) :=\ & \frac{e^{\gamma^2 t_s} - 1}{\gamma(1 - \gamma)}\Big(e^{-\gamma(1 - t_s) - \gamma^2 t_s} - e^{-\gamma^2}\Big) \\ &+ \frac{e^{-\gamma(1 - t_s)}}{\gamma}\left[\Big(e^{-\gamma t_s} - 1\Big) + \frac{e^{-\gamma^2 t_s} - e^{-\gamma t_s}}{1 - \gamma}\right] \\ &+ e^{-\gamma(1 - t_s) - \gamma^2 t_s}\Bigg[\frac{\gamma^2}{1 - \gamma}(1 - t_s)\,e^{\gamma(1 - \gamma)(1 - t_s)} \\ &\qquad + \frac{\gamma}{(1 - \gamma)^2}\Big(1 - e^{\gamma(1 - \gamma)(1 - t_s)}\Big)\Bigg]. \end{aligned} \tag{7c}$$

Our recursive procedure is designed so that, along the recursion tree, at least one call is *successful* in the sense of meeting the condition of Theorem 3.1(1); for that call, the $\gamma$-FWG subroutine provides the corresponding certified performance guarantee. In the same successful call, we also run the $\gamma$-aware double-greedy subroutine, which yields the guarantee stated in following lemma.

Lemma 3.2. *Let $F : [0, 1]^n \to \mathbb{R}_{\geq 0}$ be nonnegative, $L$-smooth, and $\gamma$-weakly DR-submodular for some $\gamma \in (0, 1]$. Let $\mathbf{z}^* \in [0, 1]^n$ be the incumbent vector provided to the recursive call, Then*

$$F(\mathbf{z}'^*) \geq \max_{r \geq 0} \frac{\Big(2\gamma^{3/2} r + \frac{\gamma}{1+\gamma^2} r^2\Big) F(\mathbf{z}^* \odot \mathbf{o}) + \frac{\gamma^2}{1+\gamma^2} r^2 F(\mathbf{z}^* \oplus \mathbf{o})}{r^2 + 2\gamma^{3/2} r + 1} - O(\varepsilon) F(\mathbf{o}) - O(\delta L D^2). \tag{8}$$

We therefore obtain two certified lower bounds for any successful call. Since the algorithm returns the better of these two values, *any* convex combination of the two bounds is also a valid lower bound on the algorithm's output. Let $\alpha \in [0, 1]$ denote the mixing parameter. By rewriting both bounds in a common form and combining them, we obtain our final approximation guarantee $\Phi_\gamma$, shown as the blue curve in Figure 1. Additional details are included in the arXiv version of our paper [11].


## ACKNOWLEDGMENTS

The first author would like to thank Mr. Jaydip Parmar for helpful discussions during this project. The second author is supported by ANRF/ARGM/2025/002341/TS.

# REFERENCES


[1] Per Austrin, Siavosh Benabbas, and Konstantinos Georgiou. 2016. Better balance by being biased: A 0.8776-approximation for max bisection. *ACM Transactions on Algorithms (TALG)* 13, 1 (2016), 1–27.

[2] An Bian, Kfir Y. Levy, Andreas Krause, and Joachim M. Buhmann. 2017. Continuous DR-submodular Maximization: Structure and Algorithms. In *Advances in Neural Information Processing Systems (NeurIPS)*.

[3] Niv Buchbinder and Moran Feldman. 2024. Constrained submodular maximization via new bounds for DR-submodular functions. In *Proceedings of the 56th Annual ACM Symposium on Theory of Computing*.

[4] Chandra Chekuri and Sanjeev Khanna. 2005. A polynomial time approximation scheme for the multiple knapsack problem. *SIAM J. Comput.* 35, 3 (2005), 713–728.

[5] Michele Conforti and Gérard Cornuéjols. 1984. Submodular set functions, matroids and the greedy algorithm: tight worst-case bounds and some generalizations of the Rado-Edmonds theorem. *Discrete applied mathematics* 7, 3 (1984), 251–274.

[6] Gerard Cornuejols, Marshall Fisher, and George L Nemhauser. 1977. On the uncapacitated location problem. In *Annals of Discrete Mathematics*. Vol. 1. Elsevier, 163–177.

[7] Uriel Feige and Jan Vondrák. 2006. Approximation algorithms for allocation problems: Improving the factor of 1-1/e. In *2006 47th Annual IEEE Symposium on Foundations of Computer Science (FOCS'06)*. IEEE, 667–676.

[8] Marshall L Fisher, George L Nemhauser, and Laurence A Wolsey. 2009. An analysis of approximations for maximizing submodular set functions—II. In *Polyhedral Combinatorics: Dedicated to the memory of DR Fulkerson*. Springer, 73–87.

[9] Hamed Hassani, Mahdi Soltanolkotabi, and Amin Karbasi. 2017. Gradient methods for submodular maximization. *Advances in Neural Information Processing Systems* 30 (2017).

[10] Johan Håstad. 2001. Some optimal inapproximability results. *Journal of the ACM (JACM)* 48, 4 (2001), 798–859.

[11] Hareshkumar Jadav, Ranveer Singh, and Vaneet Aggarwal. 2026. Stronger Approximation Guarantees for Non-Monotone {\gamma}-Weakly DR-Submodular Maximization. *arXiv preprint arXiv:2601.00611* (2026).

[12] Richard M Karp. 2009. Reducibility among combinatorial problems. In *50 Years of Integer Programming 1958-2008: from the Early Years to the State-of-the-Art*. Springer, 219–241.

[13] Bernhard Korte and Dirk Hausmann. 1978. An analysis of the greedy heuristic for independence systems. In *Annals of Discrete Mathematics*. Vol. 2. Elsevier, 65–74.

[14] George L Nemhauser and Laurence A Wolsey. 1978. Best algorithms for approximating the maximum of a submodular set function. *Mathematics of operations research* 3, 3 (1978), 177–188.

[15] George L Nemhauser, Laurence A Wolsey, and Marshall L Fisher. 1978. An analysis of approximations for maximizing submodular set functions—I. *Mathematical programming* 14, 1 (1978), 265–294.

[16] Guanyu Nie, Vaneet Aggarwal, and Christopher Quinn. 2024. Gradient methods for online DR-submodular maximization with stochastic long-term constraints. *Advances in Neural Information Processing Systems* 37 (2024), 20510–20539.

[17] Mohammad Pedramfar and Vaneet Aggarwal. 2024. From linear to linearizable optimization: A novel framework with applications to stationary and non-stationary dr-submodular optimization. *Advances in Neural Information Processing Systems* 37 (2024), 37626–37664.

[18] Mohammad Pedramfar, Yididiya Y. Nadew, Christopher John Quinn, and Vaneet Aggarwal. 2024. Unified Projection-Free Algorithms for Adversarial DR-Submodular Optimization. In *The Twelfth International Conference on Learning Representations*.

[19] Mohammad Pedramfar, Christopher Quinn, and Vaneet Aggarwal. 2023. A unified approach for maximizing continuous DR-submodular functions. *Advances in Neural Information Processing Systems* 36 (2023), 61103–61114.

[20] Mohammad Pedramfar, Christopher John Quinn, and Vaneet Aggarwal. 2024. A Unified Approach for Maximizing Continuous $\gamma$-weakly DR-submodular Functions. https://optimization-online.org/?p=25915.