

Approximating Electoral Control Problems

Extended Abstract

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ABSTRACT

Much research in electoral control—one of the most studied forms of electoral attacks, in which an entity alters the structure of an election to yield a preferred outcome—has focused on giving decision-complexity results. Approximability on the other hand has received little attention in electoral control, despite its prevalence in the study of other forms of electoral attacks, such as manipulation and bribery. Early work established preliminary results with respect to popular voting rules such as plurality, approval, and Condorcet. In this work, we completely determine for each of the “standard” control problems under the aforementioned voting rules whether they are approximable (for weighted and unweighted votes).

KEYWORDS

Approximation Algorithms; Electoral Control; Inapproximability; Covering Integer Programs; Minimum k -Union

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1 INTRODUCTION

During the electoral process, individuals or groups may seek to influence the outcome in their favor, e.g., to make a particular candidate win/lose. The setting wherein the electoral chair—the entity conducting the election—alters the structure of the election to yield a preferred outcome is known as *electoral control*, which is one of the major forms of electoral manipulative attacks (along with manipulation and bribery) within computational social choice. The standard control actions are adding/deleting/partitioning candidates/voters.

Much of the work on control has produced decision-complexity results because if a control problem is NP-hard, then it is “hard to solve” and so this is classically viewed as “protection against” (or “resistance to”) the attack (see [20]). A wide body of work emerged

giving fixed-parameter-tractability results, which has shaken this interpretation by showing that for small numbers of voters or candidates, some NP-hard control problems become tractable (see [1]). Moreover, in practice a chair could prefer to compute actual solutions to problems, and that line of work has been explored too [6, 7].

In this paper, we turn our focus to the optimization versions of the standard electoral control problems, where the chair seeks to minimize some property of the solution to the control problem, e.g., the number of candidates to delete. To our surprise, while approximations for manipulation and bribery are well-studied (see [4, 10, 12, 17, 18, 21, 22], to list a few), the same cannot be said for control. In particular, Brelford [3] initiated that study by providing preliminary results and by outlining difficulties one would likely face in designing approximation algorithms (or proving the lack thereof) for control. Eventually, Faliszewski et al. [11] and Brederek et al. [2] contributed additional results. Thus the amount of work that has been done in 19 years has been modest.

Our work complements the view that NP-hardness does not necessarily equate to “protection” as many NP-hard problems admit efficient approximations (for example, the Vertex Cover problem admits a 2-approximation [9]) and in practice, a suboptimal solution may be perfectly acceptable. So there’s a very real need (if one wishes to treat “resistance” as a form of protection) to argue that approximation algorithms do not undermine the resistance captured by the NP-hardness of the various electoral control problems.

2 PRELIMINARIES

Poly-APX refers to the class of optimization problems that admit $n^{O(1)}$ -approximations, and Log-APX refers to the class of optimization problems that admit $O(\log n)$ -approximations.

An *election* is a pair of two finite sets: a candidate set C and a collection of votes V , where each vote is implicitly associated to a voter. In this work, each vote is either a linear ordering of the candidates or an approval ballot (i.e., a subset of approved candidates). In a weighted election, each vote v is assigned a positive integer weight $w(v)$, which can be interpreted as $w(v)$ copies of v .

A voting rule \mathcal{E} is defined as a function that maps each election (C, V) to a subset of C , representing the set of winning candidates. This work focuses on the following three voting rules.

Plurality: Each vote is a linear order. Under plurality, each candidate receives one point for each vote that ranks them first, and a plurality winner is a candidate with maximal points.



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Table 1: Approximation complexities of the NP-hard standard control problems under plurality, Condorcet, and approval. Our (new) results are boldfaced. Nonboldfaced results are from Brelsford [3]. A “-” indicates that the decision problem is in P.

Control by	Plurality		Condorcet		Approval	
	Construct.	Destruct.	Construct.	Destruct.	Construct.	Destruct.
Adding Candidates	Inapprox.	Inapprox.	-	-	-	-
Deleting Candidates	Poly-APX/$\Omega(m^{1/4})$	Inapprox.	-	-	-	-
Partition of Candidates	TE: Inapprox. TP: Inapprox.	TE: Inapprox. TP: Inapprox.	-	-	TE: - TP: -	TE: - TP: -
Run-off Partition of Candidates	TE: Inapprox. TP: Inapprox.	TE: Inapprox. TP: Inapprox.	-	-	TE: - TP: -	TE: - TP: -
Adding Voters	-	-	Inapprox.	-	Log-APX-Complete	-
Deleting Voters	-	-	Inapprox.	-	Log-APX-Complete	-
Partition of Voters	TE: - TP: Inapprox.	TE: - TP: Inapprox.	Inapprox.	-	TE: Inapprox. TP: Inapprox.	TE: - TP: -

Approval: Each vote is an approval ballot. Each candidate receives one point for each vote that approves them, and an approval winner is a candidate with maximal points.

Condorcet: Each vote is a linear order. A candidate c is a winner under Condorcet if and only if they are a Condorcet winner, i.e., c defeats every candidate $d \neq c$ in a head-to-head contest.

Due to space constraints, we only define three (optimization-based) control problems. Full definitions can be found in [5]. Let \mathcal{E} be a voting rule.

In the constructive control by **deleting candidates** problem under \mathcal{E} , we are given an election (C, V) and a distinguished candidate $p \in C$. We seek the smallest $D \subseteq C - \{p\}$, such that $\{p\} = \mathcal{E}(C - D, V)$.

In the constructive control by **deleting voters** problem under \mathcal{E} , we are given an election (C, V) and a distinguished candidate $p \in C$. We seek the smallest $V' \subseteq V$, such that $\{p\} = \mathcal{E}(C, V - V')$.

In the constructive control by **adding voters** problem under \mathcal{E} , we are given an election (C, V) , a distinguished candidate $p \in C$, and a collection of unregistered voters W . We seek the smallest $W' \subseteq W$, such that $\{p\} = \mathcal{E}(C, V \cup W')$.

3 RESULTS

All the results in this section hold for both weighted and unweighted votes. We summarize our results in Table 1. We first reduce set covering problems to voter-control problems under approval, and we model the same problems using covering integer programs, which admit an $O(\ln m)$ -approximation [19].

THEOREM 3.1. *Under approval, constructive control by adding and deleting voters are each Log-APX-complete.*

In studying approximations for plurality, we prove the following result about “voiced” voting rules. (A voting rule is voiced if in every one-candidate election, it elects that candidate. All voting rules in this work are voiced.)

LEMMA 3.2. *For each voiced voting rule \mathcal{E} , constructive control by deleting candidates under \mathcal{E} admits an $O(m)$ -approximation, and thus is in Poly-APX.*

For the same problem under plurality, we give a fine-grained conditional lower bound in $\Omega(m^{1/4})$, via a “strict” reduction from the Minimum k -Union problem (see [5] for more details).

THEOREM 3.3. *Assuming the Dense versus Random Conjecture [8], the approximation ratio of any algorithm for constructive control by deleting candidates under plurality is in $\Omega(m^{1/4})$.*

As to hardness results, we first show that partition-based control problems are inapproximable in general; our argument is not only simple, but also successfully bypasses the difficulties anticipated by Brelsford [3] by showing that finding even *one* solution is NP-hard.

THEOREM 3.4. *For each partition-based decision control problem \mathcal{T} , if \mathcal{T} is NP-complete, then the optimization version of \mathcal{T} is not approximable, unless $P = NP$.*

Using a similar approach, we prove the inapproximability of the remaining NP-hard control problems under consideration.

THEOREM 3.5. *The following are all inapproximable: destructive control by deleting candidates under plurality and constructive control by adding/deleting voters under Condorcet.*

4 FUTURE DIRECTIONS

The natural first step is to extend this study to other voting rules or other control actions. Moreover one could, for each of our partition problems whose decision problem is in P, study the optimization version of that problem; in general the P membership of a partition-based control problem does not necessarily imply a polynomial-time computable optimization counterpart [14]. Another direction is to refine the lower bound in Theorem 3.3. In fact, it would be interesting to find control problems that are “equivalent” to MkU . Also, Lemma 3.2 provides a sufficient (but not necessary) condition, so a characterization would be interesting from a purely technical standpoint. Perhaps more ambitious would be to develop dichotomy theorems for approximability in electoral control, aiming to classify when control problems are approximable. Prior work doing decision complexity has established dichotomy theorems for manipulation and bribery [10, 13, 16], and some results exist for control [11, 15].

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