Stability in Distance Preservation Games on Graphs

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Argyrios Deligkas
Royal Holloway, University of London
Egham, United Kingdom
argyrios.deligkas@rhul.ac.uk

Eduard Eiben
Royal Holloway, University of London
Egham, United Kingdom
eduard.eiben@rhul.ac.uk

Tiger-Lily Goldsmith
Royal Holloway, University of London
Egham, United Kingdom
tiger-lily.goldsmith@rhul.ac.uk

Dušan Knop
Czech Technical University in Prague
Prague, Czech Republic
dusan.knop@fit.cvut.cz

Šimon Schierreich
AGH University of Krakow
Krakow, Poland
schiesim@fit.cvut.cz

ABSTRACT

We introduce a new class of network allocation games called *graphical distance preservation games*. Here, we are given a graph, called a topology, and a set of agents that need to be allocated to its vertices. Moreover, every agent has an ideal (and possibly different) distance in which to be from some of the other agents. Given an allocation of agents, each one of them suffers a cost that is the sum of the differences from the ideal distance for each agent in their subset. The goal is to decide whether there is a stable allocation of the agents, i.e., no agent would like to deviate from their location. Specifically, we consider three different stability notions: envy-freeness, swap stability, and jump stability. We perform a comprehensive study of the (parameterized) complexity of the problem in three different dimensions: the topology of the graph, the number of agents, and the structure of preferences of the agents.

KEYWORDS

Matching, Network Allocation, Stability, Parameterized Complexity

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1 INTRODUCTION

It is the time of year to organize the annual banquet of your organization, and your task is to allocate the seats for the attendees. Of course, you could choose the seats in an arbitrary way, but you know that this is not the smartest idea. You are aware that a) particular people should ideally be seated at specific distances – not too close, but not too far either, so they can keep an eye on each other – and b) the allocation should be "*stable*", e.g., chatty-Bob cannot find an empty seat close to grumpy-Joe, or complaining-Jack will not envy cool-Bill for his seat, or adventurous-Alice and poetical-Ada will not both benefit from swapping their seats, as they like their current seats more.

Situations like the above occur in many scenarios, ranging from office and house allocations to the positioning of supervisors and employees on assembly lines. In each one of these scenarios, there is an underlying network whose nodes correspond to available positions, and a set of agents, each with their own preferred distance for the subset of agents they are interested in. For example, a supervisor wants to be able to reach each of their supervisees within at most three minutes, while each supervisee wants to be next to their friends, but at a distance of fifty meters from their supervisor. The goal, as argued in our initial example, is to place the agents on the nodes of the graph such that the allocation is stable.

Motivated by similar problems, very recently, Aziz et al. [2] introduced an elegant model, called *distance preservation games*, in order to formally study them. In their model, there was a set of agents, each of whom had an ideal distance for a subset of the remaining agents. The goal was to place the agents on the real unit-interval such that an objective associated with the costs of the agents was met; the cost of an agent is the sum of the differences from their ideal distance for every agent in their subset. They have studied social cost, i.e., the sum of agents' costs, and *jump stable* placements, i.e., no agent would like to jump to a different position of the interval. Nevertheless, there exist many scenarios, like some of the ones above, that cannot be captured by this model, due to its domain: we can have a more complex space than the interval, or it may not be physically possible to place an agent anywhere we want.

1.1 Our Contribution

Our contribution is threefold. First, we introduce *graphical distance preservation games* by augmenting the model of Aziz et al. [2]. Now, instead of the line segment, we have a *topology*, i.e., a graph, where we have to allocate the agents on its vertices. Then, we perform a comprehensive study of the (parameterized) complexity of finding envy-free allocations on this type of game. Finally, we initiate the study of jump stable and swap stable allocations.

Under envy-free allocations, we first observe that while they are guaranteed to exist when there are only two agents (Prop. 3.1), this is no longer true for three agents or more (Prop. 3.2). In fact, we prove that the problem is NP-complete even when the agents have symmetric preferences, i.e., for every pair of agents i, j, the

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ideal distance of i from j matches the ideal distance of j from i, and furthermore every ideal distance is equal to 1 (Thm. 3.4). Hence, in order to hope for tractability we have to impose some constraints on the instances. We do this in three different dimensions. Additionally, we observe that there exist games that admit swap stable solutions, but no envy-free ones (Prop. 3.3).

The first restriction we consider is on the structure of the topology, G. We begin by proving that the problem can be solved in polynomial time when the underlying graph is a clique or a star (Thm. 3.5). Unfortunately, this is possibly the best we can hope for under this restriction – at least for "standard" structural parameters – as we prove that the problem becomes NP-complete if G has vertex cover number 2 (Thm. 3.6), or if G is a tree of depth 2 and agents have symmetric preferences (Thm. 3.7), or if it is a path (Thm. 3.8). These results strongly indicate that we need a different restriction in order to get tractability.

Our second dimension of restriction is on the number of agents. First, we prove that the problem remains NP-complete even when the size of the graph is significantly larger than the number of agents (Thm. 3.9) and we complement this negative result with an XP algorithm parameterized by the number of agents (Thm. 3.10). Hence, the problem is efficiently-solvable for any constant number of agents. Whether this XP algorithm can be improved to FPT or there is a complementary W[1]-hardness remains open. On the positive side though, if we consider as parameters the number of agents and structural parameters of the topology, G, we can design a number of fixed parameter algorithms. These structural parameters include: the vertex cover number of G (Thm. 3.11), which complements the negative result of Theorem 3.6; the *neighborhood diversity* of G (Thm. 3.12); the *modular width* of G (Thm. 3.14), which to the best of our knowledge is the first time that it is used within computational social choice; the *diameter* of G (Thm. 3.16), which complements Theorem 3.7 that proves hardness for topologies with diameter 4; the distance to clique (Cor. 3.13).

The third dimension restricts the *preferences* of the agents. We view the preferences as a directed graph D_{pref}, where an arc from i to j denotes that agent i is interested in their distance from agent j. We prove that the problem is polynomial-time solvable if D_{pref} is either an in-star (Thm. 3.17), or an out-star (Thm. 3.18). Interestingly, the in-star case – which at first glance seems like an "easy" case – requires a technical, tedious algorithm that is based on dynamic programming. Our last result for this dimension considers the case where only few agents (out of the many) have preferences, i.e., there are only a few vertices in D_{pref} with non-zero degree. Then, we prove that the problem is W[1]-hard parameterized by non-zero degree vertices and we complement this by an XP algorithm (Thm. 3.19).

Our last set of results considers jump and swap stability. First, we prove that stable allocations fail to exist even on paths with two agents for jump stability (Prop. 4.1), and three agents for swap stability (Prop. 4.2). We complement this by showing two positive results. Under symmetric preferences, both jump and swap stable allocations do always exist and, in addition, they can be computed in polynomial time (Thm. 4.3). We note here that Aziz et al. [2], using similar arguments, proved in their setting that although jump stable allocations exist for the unit interval, it is PLS-complete to find one; it is the graph structure that allows for tractability. Finally, if D_{pref} is acyclic, then again both jump and swap stable allocations always exist and can be computed in polynomial time (Thm. 4.4). This result follows the same lines as Aziz et al. [2], who proved a similar result for jump stable allocations in their setting.

1.2 Related Work

In recent years, finding stable and fair allocations of agents to a topology has emerged as an important research direction in algorithmic game theory and computational social choice literature, capturing various real-life problems.

One of the best-known models in this line of research is the so-called *Schelling games on graphs* [1, 10, 11, 16, 18, 20, 27, 37, 41], which turned out to be an important tool in the study of segregation. Here, we are given a topology and a set of agents partitioned into k colors, and the goal is to allocate these agents to the topology in a stable way. Similarly to our work, the notions of stability studied are swap and jump stability. The crucial difference from our model is that the preferences are inherent and the agents are interested only in the number of agents of their own color in their direct neighborhood (resp. in the diversity [46] or variety [45] of their neighborhood).

In the *seat arrangement* problems [4, 9, 13, 17, 52], we are given a social network over the set of agent and the so-called seating graph, and the goal is to allocate the agents to vertices of the seating graph so that the allocation satisfies some desirable stability notion; importantly, envy-freeness, swap stability, and jump stability. However, similarly to Schelling games, agents are only interested in their friends (according to the social network) allocated to their direct neighborhood in the seating graph.

In the *refugee housing* problem [38, 43, 47, 48], we have a topology, a set of inhabitants, and a set of refugees. The inhabitants are already occupying some vertices (or houses) of the topology, and the goal is to allocate the refugees to the empty houses so that the preferences of both the inhabitants and the refugees are respected. Importantly, the preferences in refugee housing are not based on the distances; the agents are interested only in the agents (of the opposite type) allocated to their direct neighborhood, and positions of some agents (the inhabitants) are already fixed in the topology.

In all three previously discussed models, agents were only interested in agents allocated to their direct neighborhoods. This is not the case in the *topological distance games (TDGs)* [15, 22], where we are also given a topology and a set of agents that must be allocated to the topology. In the allocation, the utility of an agent depends on both the agent's inherent utilities for other agents and its distance from them on the topology. That is, in TDGs, agents are interested in the whole allocation, not only their neighbors. However, they cannot express ideal distances to other agents as in our model.

Finally, preferences based on distances between agents also appear in other contexts in collective decision making. One such example is *social distance games* [7, 8, 14, 29, 36], a class of *hedonic games*, where we are given a social network over a set of agents, and the goal is to partition these agents into groups called coalitions. The utility of agents is computed as a (weighted) distance to all other agents in the same coalition, and the stability requirements usually arise from coalition formation literature and are very different from ours.

2 THE MODEL

We use $\mathbb{N}$ to denote the set of all positive integers. For any $i \in \mathbb{N}$, we denote $[i] := \{1, \dots, i\}$ and set $[i]_0 = [i] \cup \{0\}$.

A *distance preservation game*[1] (DPG) is a quadruplet $\Gamma = (A, (M_a)_{a\in A}, (\mathrm{d}_a)_{a\in A}, G)$, where A is a set of *agents*, $M_a \subseteq A \setminus \{a\}$ is a (possibly empty) *relationship set* containing all "interesting agents" from the perspective of agent $a \in A$, $\mathrm{d}_a : M_a \to \mathbb{N}$ is a *distance function* assigning to each $b \in M_a$ the ideal distance agent a wants to be from b, and G is a simple, undirected, and connected graph called *topology* with $|V| \geq |A|$. Based on agents' relationship sets, we define a *preference graph* D_{pref}, which is a directed graph over A with an arc (a, b), $a, b \in A$, if and only if $b \in M_a$. If for every pair of agents $a, b \in A$ such that $b \in M_a$ it holds that also $a \in M_b$ and $\mathrm{d}_a(b) = \mathrm{d}_b(a)$, we say that the preferences are *symmetric*. Moreover, if $M_a = \emptyset$ for some agent $a \in A$, we say that a is *indifferent*.

The ultimate goal is to find an *allocation* $\pi : A \to V$, which is an injective mapping between the set of agents and the set of vertices of the topology. We use V^π to denote the set of all vertices used by the allocation π, and we say that a vertex $v \in V$ is *empty* if $v \notin V^\pi$ and *occupied* otherwise. Let π be an allocation and $a, b \in A$ be a pair of distinct agents. A vertex $v \in V^\pi$ such that $\pi(a) = v$ is called a *location* of agent a. We use $\pi^{a\leftrightarrow b}$ to denote the allocation in which agents a and b swapped their positions, and all other agents remain allocated to the same vertices. Similarly, for an agent $a \in A$ and an empty vertex $v \in V \setminus V^\pi$, we use $\pi^{a\mapsto v}$ to denote the allocation where the agent a is allocated to the vertex v and the positions of all other agents remain the same.

Naturally, not all allocations are equally good with respect to agents' relationship sets and distance functions. Therefore, we define and study several axioms capturing conditions under which, if met, the agents consider an allocation acceptable. Let $a \in A$ be an agent and π be an allocation. Then, the cost for agent a in the allocation π, denoted $\mathrm{cost}(a, \pi)$, is defined as

$$\mathrm{cost}(a, \pi) = \sum_{b \in M_a} \Big| \mathrm{d}_a(b) - \mathrm{dist}_G(\pi(a), \pi(b)) \Big|,$$

where $\mathrm{dist}_G(u, v)$ is the length of a shortest path between the vertices u and v in the topology G. Intuitively, the cost agent a suffers from agent b is equal to the difference between the distance prescribed by the distance function and the actual distance of these two agents with respect to the allocation π.

First, we are interested in *fair* allocations, where the agents are not envious of the positions of other agents. Specifically, we adapt the well-known notion of envy-freeness [26] from the fair division literature to our setting. Formally, envy-freeness in our context is defined as follows.

Definition 2.1. An allocation π is called *envy-free* (EF) if there is no pair of agents $a, b \in A$ such that

$$\mathrm{cost}(a, \pi) > \mathrm{cost}(a, \pi^{a\leftrightarrow b}) .$$

If $\mathrm{cost}(a, \pi) > \mathrm{cost}(a, \pi^{a\leftrightarrow b})$, then we say that *$a$ envies b*.

In other words, envy-freeness requires that no agent can benefit from swapping positions with another agent. Note that in the definition of EF, we could also compare $\mathrm{cost}(a, \pi)$ with the cost of a in $\pi^{a\mapsto\pi(b)}$, assuming that we removed b from the instance. However, since the position of b affects the cost of a, and therefore b can be seen as *externality* for a, we define EF with respect to swaps to be in line with similar models [5, 23, 49].

If an agent a envies an agent b, then the corresponding allocation is not considered acceptable regardless of the impact of the swap on the cost of agent b. In certain scenarios, agents are *altruistic* and care about the costs for other agents. Therefore, we define a weaker notion of stability based on the swapping of two agents.

Definition 2.2. An allocation π is called *swap stable* if there is no pair of distinct agents $a, b \in A$ such that

$$\mathrm{cost}(a, \pi) > \mathrm{cost}(a, \pi^{a\leftrightarrow b}) \quad \text{and} \quad \mathrm{cost}(b, \pi) > \mathrm{cost}(b, \pi^{b\leftrightarrow a}) .$$

If a pair of agents a, b can improve their cost by swapping positions, we say that a and b admit *swap deviation*.

Not surprisingly, there is a certain relation between envy-freeness and swap stability.

Proposition 2.3. *If an allocation π is envy-free, then it is also swap stable.*

Proof. Let π be an envy-free allocation. For the sake of contradiction, let $a, b \in A$ be a pair of distinct agents that admit a swap deviation, that is, we have $\mathrm{cost}(a, \pi) > \mathrm{cost}(a, \pi^{a\leftrightarrow b})$ and $\mathrm{cost}(b, \pi) > \mathrm{cost}(b, \pi^{b\leftrightarrow a})$. Clearly, agent a envies b and vice versa, which contradicts that π is envy-free. □

In the opposite direction, not all swap stable allocations are envy-free. We give a formal argument for this claim later in Theorem 3.3.

Observe that in EF (or swap stable) allocations, agents are envious of the positions *of other agents*. In particular, an allocation is considered fair even though there is an agent $a \in A$ and an *empty* vertex $v \in V$ such that $\mathrm{cost}(a, \pi) > \mathrm{cost}(a, \pi^{a\mapsto v})$. One can argue that even though such an allocation is formally fair, selfish agents would still have an incentive to move to a better location. Motivated by this, we define *jump stability*.

Definition 2.4. An allocation π is called *jump stable* if there is no agent $a \in A$ and an empty vertex $v \in V$ such that

$$\mathrm{cost}(a, \pi) > \mathrm{cost}(a, \pi^{a\mapsto v}) .$$

If an agent a can improve its cost by jumping to an empty vertex, then we say that a admits a *jump deviation*.

Naturally, we can combine envy-freeness with jump stability. However, as we show in the following proposition, then the problem reduces to a simple decision of the existence of envy-free allocations for a carefully constructed instance with no empty vertex.

Proposition 2.5. *A distance preservation game $\Gamma = (A, (M_a)_{a\in A}, (\mathrm{d}_a)_{a\in A}, G)$ admits an envy-free and jump stable allocation π if and only if the distance preservation game $\Gamma' = (A', (M'_a)_{a\in A'}, (\mathrm{d}'_a)_{a\in A'}, G)$, where $A' = A \cup \{x_i \mid i \in [|V| - |A|]\}$ and $M'_a = M_a$ and $\mathrm{d}'_a = \mathrm{d}_a$ whenever $a \in A$ and $M'_a = \mathrm{d}'_a = \emptyset$, otherwise, admits an envy-free allocation π'.*

To illustrate our model and the studied stability notions, we conclude this section with a running example.

[1]From now on, we drop 'graphical' from the name of our model, as it is shorter and there is no possibility of confusion.

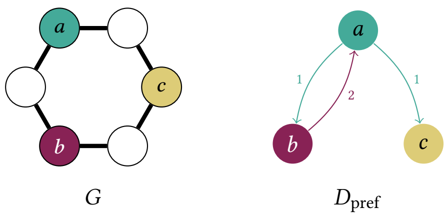

Figure 1: An example of a topology with allocated agents (on the left) and a preference graph (on the right). Observe that $M_a = \{b, c\}$, $M_b = \{a\}$, and $M_c = \emptyset$, that is, agent c is indifferent. The preferences are clearly not symmetric.

Example 2.6. Let us have an instance with three agents a, b, and c, and let the topology G, the preference graph D_{pref}, and the allocation π be as depicted on Figure 1. The cost of agent b is zero, as b requires to be at distance two from a and this is satisfied. The cost for agent c is always zero, since he is indifferent. The cost of agent a, on the other hand, is two, as she requires to be at distance one from both remaining agents, but her distance from both of them is two.

Regarding stability of π, clearly no agent b or c can participate in a swap, as they are already on their best position. Hence, π is swap stable. Similarly, agent a is not envious of the position of any of the other two agents, as if she swaps with one of them, the distances are preserved. Finally, allocation π is not jump stable, as agent a can jump to the common neighbor v of b and c and decrease her cost to zero. In response, agent b will jump so she is distance 2 from agent a. It is easy to see that π' is jump stable.

3 ENVY-FREENESS

As we showed in the previous section, envy-freeness is closely related to all other stability notions assumed in this work. Thus, we begin our investigation with this stability notion.

First, we study the conditions under which envy-free allocations are guaranteed to exist. It is easy to see that if there are two agents, any allocation is EF, as if the agents swap their positions, their distance remains the same.

Proposition 3.1. *If $|A| = 2$, an envy-free allocation is guaranteed to exist and can be found in constant time.*

Proof. Let G be a topology and π be an arbitrary allocation of our agents. Since the graph is undirected, we have that $\text{dist}_G(\pi(a_1), \pi(a_2))$ is the same as $\text{dist}_G(\pi(a_2), \pi(a_1))$. Therefore, if the agents swap their positions, their utility remains the same. Consequently, π is envy-free. □

However, as we show in our next result, already with three agents, envy-free allocation may not exist.

Proposition 3.2. *For every $|A| \geq 3$, an envy-free allocation is not guaranteed to exist.*

Proof sketch. Let $|A| = 3$ and let the topology G be a cycle with four vertices. Each agent a_i, $i \in [|A|]$, has $M_i = \{a_{(i \bmod |A|)+1}\}$ and requires this agent at distance two. For the sake of contradiction, assume that there is an envy-free allocation π. First, let $\text{dist}_G(\pi(a_1), \pi(a_2)) = 1$. If a_3 is at distance two from a_2, then a_1 is envious of a_3's position. Hence, a_3 is at a distance exactly one from a_2. However, in this case, a_3 is simultaneously at distance two from a_1, which means that a_2 is envious of a_1's position. Thus, it must be the case that $\text{dist}(\pi(a_1), \pi(a_2)) = 2$. Regardless of the position of a_3, she is always at distance one from a_1 and envies agent a_2. That is, no envy-free allocation exists. □

With the previous no-instance in hand, we finally give a formal proof that there are games admitting swap stable but not envy-free allocations, as promised in Section 2.

Proposition 3.3. *There is a distance preservation game Γ that admits a swap stable allocation but no envy-free allocation.*

Proof. Recall the instance from Theorem 3.2 with $|A| = 3$. It was shown that such a game does not admit envy-free allocation. However, assume an allocation with $\pi(a_1) = v_1$, $\pi(a_3) = v_2$, and $\pi(a_2) = v_3$ (we suppose that $v_1, \ldots, v_5$ is a DFS order if we start with the vertex v_1). Then, the cost for the agent a_1 is zero, and the costs for both a_2 and a_3 are one. Therefore, a_1 participates in no swap deviation. Although the cost for a_3 in $\pi^{a_3 \leftrightarrow a_2}$ is zero, that is, decreased, the agent a_2 is in $\pi^{a_3 \leftrightarrow a_2}$ still at distance one from a_3, meaning that the swap is not beneficial to him. That is, the allocation π is swap stable. □

Since there are instances with no envy-free allocation, it is natural to ask, given a distance preservation game Γ, how hard it is to decide whether Γ admits an EF allocation. In our first hardness result, we show that this problem is NP-complete, even if the preferences of our agents are symmetric, i.e., if an agent a requires agent b to be at distance δ, then also agent b requires agent a to be at the same distance.

Theorem 3.4. *It is NP-complete to decide whether a distance preservation game Γ admits an envy-free allocation, even if the preferences are symmetric and the domain of each d_a, $a \in A$, is $\{1\}$.*

Proof sketch. We reduce from the 3-Partition problem. In this problem, we are given a multi-set $S = \{s_1, \ldots, s_{3N}\}$ of integers such that $\sum_{i \in [3N]} s_i = N \cdot B$, and the goal is to decide whether a set $\mathcal{X}$ of N pairwise disjoint 3-sized subsets of S exists so that the elements of every $X \in \mathcal{X}$ sum up to exactly B. The problem is known to be NP-complete even if $B/4 < s < B/2$ for every $s \in S$ [30].

Given an instance S of 3-Partition, we construct an equivalent distance preservation game Γ as follows. The topology G consists of a disjoint union of N cliques $C_1, \ldots, C_N$, each of size exactly B, and one apex vertex v_g connected to all other vertices. The set of agents contains exactly s_i *item agents* $a_i^1, \ldots, a_i^{s_i}$ for every $s_i \in S$, and one *guard agent* g. For every item agent a_i^j, we have $M_{a_i^j} = \{a_i^1, \ldots, a_i^{s_i}\} \setminus \{a_i^j\}$. For the guard agent, the set M_g contains all the other agents. Each agent requires to be at distance one from all other agents he cares about. □

3.1 Restricted Topology

The first restriction we study is the restriction of the underlying topology to which we allocate. It is reasonable to assume that if the topology is well-structured, then the problem should become easier. And indeed, in our first result, we support this intuition with

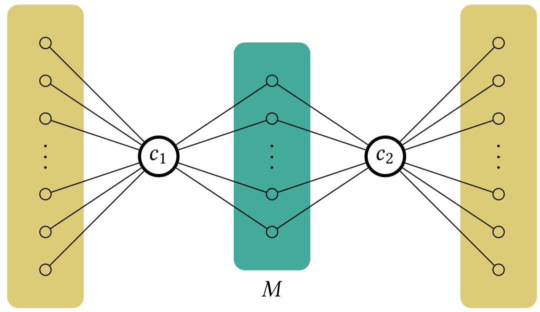

Figure 2: An illustration of the topology used in the proof of Theorem 3.6. Each set of vertices with yellow (light gray) background is of size $B = \frac{3N-k}{2}$ and the set M with green (dark gray) background is of size k. Clearly, the vertices c_1 and c_2 form a vertex cover of this graph of size two.

two polynomial-time solvable cases. Specifically, if the topology is a star graph or a clique, efficient algorithms are possible.

Theorem 3.5. *If the topology G is a star or a clique, the existence of an envy-free allocation can be decided in polynomial time.*

Proof. If the topology is a clique, then all distances are the same. Hence, an arbitrary allocation is envy-free.

When the topology is a star, we have $|A| + 1$ possibilities for $\pi^{-1}(c)$, where c is the center of G: either some agent $a \in A$ is allocated to c or it is empty. As the leaves are symmetric with respect to the distances to other vertices, we can arbitrarily allocate the remaining agents to the leaves for every choice of $\pi^{-1}(c)$ and check whether at least one of these allocations is envy-free. ◻

However, as we demonstrate in the remainder of this subsection, the tractability boundary cannot be pushed much further. Observe that stars are the only family of connected graphs with the vertex cover number one. In the following, we show that on topologies with the vertex cover number two, deciding the existence of an envy-free allocation becomes computationally intractable.

Theorem 3.6. *It is NP-complete to decide whether a distance preservation game Γ admits an envy-free allocation, even if the topology G is of the vertex cover number 2.*

Proof Sketch. We give a polynomial reduction from the Cubic Bisection problem. Here, we are given a cubic graph H with $2N$ vertices and an integer $k \in \mathbb{N}$, and the goal is to decide whether a partition (X, Y) of $V(H)$ exists such that $|X| = |Y| = N$ and there are at most k edges with one endpoint in X and one endpoint in Y. The problem is NP-complete even if there is no partition of $V(H)$ to two equal parts with less than k edges between X and Y, and $N > k > 4$ [23, Theorem 3]. Also note that since H is 3-regular, then in every partition (X, Y) with exactly k edges between X and Y we have exactly $B = \frac{3N-k}{2}$ edges with both endpoints in the same part. Consequently, we can assume that $\frac{3N-k}{2}$ is an integer, as otherwise the Cubic Bisection instance is trivially a No-instance.

Let $\mathcal{I} = (H, k)$ be an instance of the Cubic Bisection problem. We construct an equivalent distance preservation game Γ as follows. The topology G consists of two stars S_1 and S_2 with exactly B leaves each. Let c_1 and c_2 be centers of S_1 and S_2, respectively. We additionally add k parallel edges connecting c_1 and c_2 and subdivide each such edge once. We set M to be the set of vertices resulting from the subdivision. See Figure 2 for an illustration of the topology. The set of agents A contains one *edge agent* $a_{u,v}$ for every pair of $u, v \in V(H)$ such that $\{u, v\} \in E(H)$; i.e., there is one agent for each edge of H. Additionally, we have two *guard agents* g_1 and g_2. Observe that the number of agents is equal to the number of vertices of G. The preferences of our agents are as follows. For each agent $a_{u,v}$, we have $M_{a_{u,v}} = \{a_{w,w'} \mid |\{u, v\} \cap \{w, w'\}| > 0\}$; that is, each agent cares about agents representing adjacent edges. For every guard agent g_i we set $M_{g_i} = A \setminus \{g_1, g_2\}$. The distance edge agents require is always two, while the distance required by the guard agents is always one. ◻

We can also try to generalize stars to general trees, which are not ruled out by the previous hardness result. Nevertheless, this is not possible, as we show in the next construction. Observe that the trees used in this construction are of depth two, and, at the same time, stars (rooted in their centers) are the only family of graphs of depth one, so the result is again tight. Moreover, the preferences are again symmetric.

Theorem 3.7. *It is NP-complete to decide whether a distance preservation game admits an envy-free allocation, even if the preferences are symmetric and the topology is a tree of depth 2.*

Arguably, one of the structurally simplest graphs is a path. Surprisingly, we show that even on such trivial topologies, our problem remains NP-complete.

Theorem 3.8. *It is NP-complete to decide whether a distance preservation game Γ admits an envy-free allocation, even if the topology is a path.*

Proof sketch. We reduce from the 3-Partition problem. In this problem, we are given a multi-set $S = \{s_1, \ldots, s_{3N}\}$ of integers such that $\sum_{i \in [3N]} s_i = N \cdot B$, and the goal is to decide whether a set $\mathcal{X}$ of N pairwise disjoint 3-sized subsets of S exists so that the elements of every $X \in \mathcal{X}$ sum up to exactly B. Let S be a 3-Partition instance. We construct an equivalent distance preservation game Γ as follows. The topology G is a path with $N \cdot B + N + 1$ vertices. The set of agents A contains $N + 1$ *boundary agents* $b_1, \ldots, b_{N+1}$ and s_i-many *element agents* $a_i^1, \ldots, a_i^{s_i}$ for every $s_i \in S$. Every boundary agent b_i, $i \in [2, N+1]$, requires the boundary agent b_{i-1} at distance $B + 1$ and b_1 requires b_2 at distance $B + 1$. Similarly, for every $i \in [3N]$, each agent of the element a_i^j, $j \in [2, s_i]$, must be at distance one from a_i^{j-1}, and a_i^1 must be at distance one from a_i^2. Observe that the number of agents and the number of vertices of the graph are the same, so whenever an agent's preferences are not met by some allocation π, this allocation is not envy-free. ◻

3.2 Bounded Number of Agents

So far, all our hardness proofs have heavily relied on the fact that the number of agents in the instance is equal to the number of vertices of the topology. Due to this, agents were allowed to envy their ideal location. Hence, it is natural to ask whether the hardness of our problem is preserved if some vertices remain empty in every allocation.

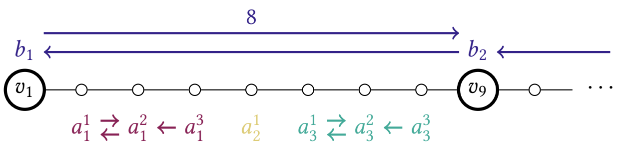

Figure 3: An illustration of the construction used to prove Theorem 3.8. An arrow from an agent a to an agent b represents that $b \in M_a$.

In the first result of this subsection, we show that it is the case. Specifically, we show that the problem remains NP-complete, even if the number of agents is significantly smaller compared to the number of vertices of the topology G.

Theorem 3.9. *It is NP-complete to decide whether a distance preservation game* Γ *admits an envy-free allocation, even if* $|A| \le \frac{3}{5}|V(G)| + 1$.

On a more positive note, if we parameterize by the number of agents, the problem is in complexity class XP. That is, whenever the number of agents is a fixed constant, the problem is polynomial-time solvable.

Theorem 3.10. *There is an algorithm running in* $|V(G)|^{O(|A|)}$ *time that decides whether a distance preservation game* Γ *admits an envy-free allocation.*

Even with considerable effort, we were not able to complement the previous algorithm with a matching complexity lower-bound. Hence, our work leaves an important open question whether, under this parameterization, the previous algorithm is optimal or an FPT is possible.

Open Problem 1. *When parameterized by the number of agents* $|A|$*, is it fixed-parameter tractable or W[1]-hard to decide whether a distance preservation game* Γ *admits an envy-free allocation?*

We conjecture that the correct answer to the previous question is negative, i.e. the problem is W[1]-hard with respect to the number of agents $|A|$.

Since we are not able to affirmatively resolve parameterization by the number of agents, in the remainder of this subsection, we combine parameterization by the number of agents with different restrictions of the topology.

We start with the *vertex cover number*, which has been successfully used in the design of efficient algorithms in various areas of artificial intelligence research; see, e.g., [12, 19, 21, 40] for a few examples. The algorithm is based on the so-called *kernelization* technique, which means that we, in polynomial time, preprocess the instance of our problem by dropping non-essential parts of the input and obtain the so-called *kernel*—a smaller instance, which is equivalent to the original instance and whose size is upper-bounded by some function of our parameter. Thanks to its bounded size, we can brute-force all possible solutions in FPT time.

Theorem 3.11. *When parameterized by the number of agents* $|A|$ *and the vertex cover number* $\mathrm{vc}(G)$*, combined, there is an FPT algorithm deciding whether a distance preservation game* Γ *admits an envy-free allocation.*

Proof sketch. Let Γ be a distance preservation game, and C be a vertex cover of the topology G of optimal size $\mathrm{vc}(G) = \vartheta$. Without loss of generality, we can assume that C is a part of the input, as otherwise, we can compute it in $1.25284^{O(\vartheta)} \cdot n^{O(1)}$ time by a known algorithm [33].

As a first step, we partition the vertices of $I = V(G) \setminus C$ into 2^{ϑ} (possibly empty) types according to their neighborhood in C. Let the partitioning be $\mathcal{T} = (T_1, \ldots, T_{2^\vartheta})$. The core of our algorithm is the following reduction rule, which we apply exhaustively.

Reduction Rule 1. *Let there exist* $j \in [2^{\vartheta}]$ *such that* $|T_j| > |A|$ *and let* $T_j = \{v_j^1, \ldots, v_j^{|T_j|}\}$*. Set* $G' = G \setminus \{v_j^i \mid i > |A|\}$ *and continue with* $\Gamma' = (G', A, (M_a)_{a\in A}, (d_a)_{a\in A})$.

Clearly, the previous reduction rule can be applied in polynomial time. Once the rule cannot be applied anymore, we end up with a topology with $|V(G)| \le \vartheta + 2^{O(\vartheta)} \cdot |A|$ vertices. That is, the size of G is bounded by our parameters, and we can brute-force over all possible allocations as we did in Theorem 3.10. The running time of such approach is $(\vartheta + 2^{O(\vartheta)} \cdot |A|)^{O(|A|)}$, which is clearly in FPT. □

The next parameter we study is called *neighborhood diversity* [42]. It is a generalization of the vertex cover number. Intuitively, a graph G is of neighborhood diversity δ, if its vertices can be partitioned into δ types $T_1, \ldots, T_\delta$ such that a) each type induces an independent set or a clique and b) if there is an edge between vertex $v \in T_i$ and $u \in T_j$, then there is a complete bipartite graph between all vertices of T_i and T_j. Neighborhood diversity is an important parameter in various areas, including kidney exchange [34], information diffusion [39], social networks analysis [3, 31, 32], or coalition formation games [25]. This time, the algorithm exploits the partitioning of vertices of the topology G.

Theorem 3.12. *When parameterized by the number of agents* $|A|$ *and the neighborhood diversity* $\mathrm{nd}(G)$*, combined, there is an FPT algorithm deciding whether a distance preservation game* Γ *admits an envy-free allocation.*

Note that neighborhood diversity is also a more general parameter than the distance to clique, so the previous algorithm also yields the following, complementing Theorem 3.5.

Corollary 3.13. *When parameterized by the number of agents* $|A|$ *and the distance to clique* c*, combined, there is an FPT algorithm deciding whether a distance preservation game* Γ *admits an envy-free allocation.*

We can further strengthen the previous results to a more general class of graphs. Specifically, we study topologies of bounded modular-width [28]. This parameter, which is small for dense graphs and is more restrictive than clique-width, comes with a convenient graph decomposition that can be used in the design of dynamic programming algorithms, similarly to standard algorithms over the nice tree decomposition related to the celebrated treewidth. To the best of our knowledge, we are the first to study this parameter in the area of algorithmic game theory and computational social choice.

Theorem 3.14. *When parameterized by the number of agents* $|A|$ *and the modular-width* $\mathrm{mw}(G)$*, combined, there is an FPT algorithm* *deciding whether a distance preservation game* Γ *admits an envy-free allocation.*

It remains unclear how much further we can push the tractability boundary. That is, it would be interesting to provide the complexity classification of the problem with respect to, e.g., tree-width or clique-width of the topology.

We conclude this subsection with one more positive result. This time, we combine the number of agents with the diameter of the topology. Note that small-world property (and, therefore, bounded diameter) has been observed, both theoretically and empirically, in many real-world networks [50, 51].

The algorithm is again based on kernelization. However, this time, we use a surprising connection between our problem and a famous theorem of Ramsey, which is very well-known from combinatorics.

Lemma 3.15 (Ramsey's Theorem (see [6, 24]). *Let* G *be a complete graph, whose edges are colored with* q *different colors, and* $k \in \mathbb{N}$. *If* $|V(G)| \geq q^{qk}$, *then there exists* $S \subseteq V(G)$ *such that* $|S| \geq k$ *and all edges between the vertices in* S *have the same color.*

Informally speaking, Ramsey's theorem states that for every integer k, each large enough edge-colored graph G contains a monochromatic clique of size k. We use this as follows. If our graph is large enough with respect to our parameters, using the previous lemma on an auxiliary graph, we find a set of vertices of size $|A|$ such that the vertices are equidistant. Then, any allocation of the agents on such locations is an envy-free solution. Otherwise, the size of our graph is bounded with respect to the parameters, and we can again brute-force over all possible allocations.

Theorem 3.16. *When parameterized by the number of agents* $|A|$ *and the diameter of the topology* $\mathrm{diam}(G)$, *combined, there is an FPT algorithm deciding whether a distance preservation game* Γ *admits an envy-free allocation.*

3.3 Restricted Preferences

The last dimension of the problem we can exploit in order to obtain tractable algorithms are agent's preferences. One natural restriction could be a bound on the size of relationship sets. And indeed, if each relationship set is empty, an arbitrary allocation is a solution. However, even though it was not stated formally, the construction used in the proof of Theorem 3.8 is so that the size of each relationship set is at most one. Therefore, this direction is not promising.

Consequently, we focus on even more structured preference graphs. In particular, in our first result, we assume that D_{pref} is an in-star. This means that there is one "super-star" agent a^* and for each agent $b \in A$ we have $M_b = \{a^*\}$ or $M_b = \emptyset$. The algorithm is, relatively to the restriction, surprisingly non-trivial dynamic programming.

Theorem 3.17. *If* D_{pref} *is an in-star, there is a polynomial time algorithm deciding whether a distance preservation game* Γ *admits an envy-free allocation*

In the previous result, we assumed that there is exactly one agent the other agents care about. Next, we turn our attention to a similarly restricted case. In particular, we show that if there is exactly one agent which is not indifferent, then a polynomial time algorithm is possible.

Algorithm 1 An algorithm finding an EF allocation if the preference graph D_{pref} is an out-star.

1: Fix an arbitrary allocation π
2: **while** $\exists b \in A \setminus \{a\}$ s. t. $\mathrm{cost}_a(\pi) > \mathrm{cost}_a(\pi^{a\leftrightarrow b})$ **do**
3: $\quad c = \arg\max_{b \in A}(\mathrm{cost}_a(\pi) - \mathrm{cost}_a(\pi^{a\leftrightarrow b}))$
4: $\quad \pi = \pi^{a\leftrightarrow c}$
5: **end while**
6: **return** Envy-free allocation π

Theorem 3.18. *If* D_{pref} *is an out-star, there is a polynomial time algorithm deciding whether a distance preservation game* Γ *admits an envy-free allocation.*

Proof. The algorithm is a simple swap dynamics that starts with an arbitrary allocation π and, through a sequence of swaps improving agent's a cost, it reaches an envy-free allocation; see Algorithm 1 for a formal description.

For correctness, it is easy to see that the algorithm always returns an envy-free allocation, since it terminates only if the condition on line 2, which checks for envy-freeness for agent a, is not satisfied. For running time, after each execution of the loop in line 2, the cost of agent a is reduced by at least one (and all other agents are indifferent, so their cost is constant zero). Moreover, the initial cost is at most $\mathrm{diam}(G) \cdot |A| \in \mathcal{O}\left(n^2\right)$, and the cost cannot be less than zero. That is, after at most a quadratic number of improving steps, the algorithm returns an envy-free allocation. ◻

We conclude with one more intractability result. Specifically, we show that even if the number of agents with a non-empty relationship set is our parameter, the problem is W[1]-hard. This result is complemented by an XP algorithm similar to that of Theorem 3.10, but requires additional steps to ensure that the important agents do not envy any of the indifferent agents.

Theorem 3.19. *When parameterized by the number of agents with non-zero degree in* D_{pref}, *it is W[1]-hard and in XP to decide whether a distance preservation game* Γ *admits an envy-free allocation.*

Note that parameterization by the number of indifferent agents cannot lead to a tractability result, as we have NP-completeness already for instances with no such agent (cf. Theorem 3.4).

4 JUMP AND SWAP STABILITY

In the remainder of the paper, we briefly explore the stability notions of jump and swap stability. First, we show that these notions are also not always satisfiable. We start with jump stability and show that, already with two agents, stable allocations may fail to exist.

Proposition 4.1. *A jump stable allocation is not guaranteed to exist, even if* $|A| = 2$ *and the topology is a path.*

Proof. Assume that the topology is a path on at least four vertices, we have two agents a_1 and a_2 interested in each other, and $\mathrm{d}_{a_1}(a_2) = 1$ and $\mathrm{d}_{a_2}(a_1) = 2$. For the sake of contradiction, assume that there is an envy-free allocation. If a_2 is at distance one from a_1, then a_2 prefers to jump to a location at distance two from $\pi(a_1)$, and such a vertex always exists, as the path is of length at least four. Similarly, if a_1 is not at the distance exactly one from a_2, a_1 prefers to jump to a location at distance one from a_2, which again always exists and is empty. These are the only possible allocations, so the proposition holds. □

For swap stability, instances with two agents always admit a swap-stable solution, as such instances always admit an envy-free solution. Hence, we need at least three agents to show non-existence.

Proposition 4.2. *A swap stable allocation is not guaranteed to exist, even if* $|A| = 3$ *and the topology is a path.*

Proof sketch. We provide just the construction. Let $A = \{a_1, a_2, a_3\}$ and the topology be a path with three vertices v_1, v_2, v_3. For every agent $a \in A$, we set $M_a = A \setminus \{a\}$. The distance functions are then as follows: $\mathrm{d}_{a_1}(a_2) = 1$, $\mathrm{d}_{a_1}(a_3) = 2$, $\mathrm{d}_{a_2}(a_1) = 2$, $\mathrm{d}_{a_2}(a_3) = 1$, $\mathrm{d}_{a_3}(a_1) = 1$, and $\mathrm{d}_{a_3}(a_2) = 2$. That is, each agent wants one agent at the distance one and the other at distance two, but these relations are not symmetric. □

If the assumed stability notion is envy-freeness, it is NP-complete to decide the existence even if the preferences are symmetric. As we show in our first algorithmic result, the situation with jump and swap stability is much more positive. Specifically, by a potential argument, a simple best-response dynamic converges in polynomial time.

Theorem 4.3. *If the preferences are symmetric, a jump stable and a swap stable allocation always exist and can be found in polynomial time.*

Proof. We prove the theorem by showing the existence of a polynomially bounded potential, and so a simple best-response dynamic converges in polynomial time. For an allocation π, we let $P(\pi) = \sum_{a\in A}(\mathrm{cost}(a,\pi) - \sum_{b\in M_a} \max\{0, \mathrm{d}_a(b) - |V(G)|\})$ be the sum of costs of all agents, where we normalize all distance functions to be at most $|V(G)|$ by letting $\mathrm{d}_a(b) = |V(G)|$ whenever $\mathrm{d}_a(b) > |V(G)|$.It follows that $0 \le P(\pi) \le |A|^2 \cdot |V(G)|$. Now, let us compute how the value of $P(\pi)$ changes in the case of a single jump or a single swap. First, note that $\sum_{a\in A}\sum_{b\in M_a} \max\{0, \mathrm{d}_a(b) - |V(G)|\}$ does not depend on π, so we only need to consider how the sum of costs changes. Let us first consider the jump of an agent a to a vertex v. Since only a and the agents b with $a \in M_b$ are affected and since $a \in M_b$ if and only if $b \in M_a$ in a symmetric instance, we get $P(\pi) - P(\pi^{a\mapsto v}) = \mathrm{cost}(a,\pi) - \mathrm{cost}(a,\pi^{a\mapsto v}) + \sum_{b\in M_a}(|\,\mathrm{d}_b(a) - \mathrm{dist}(\pi(a),\pi(b))| - |\,\mathrm{d}_b(a) - \mathrm{dist}(\pi^{a\mapsto v}(a),\pi^{a\mapsto v}(b))|)$. Note that $\mathrm{d}_b(a) = \mathrm{d}_a(b)$, because the preferences are symmetric. So $\sum_{b\in M_a}|\,\mathrm{d}_b(a) - \mathrm{dist}(\pi(a),\pi(b))| = |\,\mathrm{d}_a(b) - \mathrm{dist}(\pi(a),\pi(b))| = \mathrm{cost}(a,\pi)$ and similarly, $\sum_{b\in M_a}|\,\mathrm{d}_b(a) - \mathrm{dist}(\pi^{a\mapsto v}(a),\pi^{a\mapsto v}(b))| = \mathrm{cost}(a,\pi^{a\mapsto v})$. Therefore, $P(\pi) - P(\pi^{a\mapsto v}) = 2(\mathrm{cost}(a,\pi) - \mathrm{cost}(a,\pi^{a\mapsto v}))$. It follows that indeed P is a polynomially bounded potential. Hence, we start from an arbitrary allocation π and follow a simple best response dynamic, where 1) we check for every agent whether they can improve their cost by jumping to an empty vertex, 2) if so, let the agent jump there and repeat. By the above argument, in every jump, the value of $P(\pi)$ decreases by two times the improvement of the cost of the agent that jumped. Hence, after at most $|A|^2|V(G)|$ many jumps, we reach a jump stable allocation. By an analogous argument, we get that if a and b swap, then $P(\pi) - P(\pi^{a\leftrightarrow b}) = 2(\mathrm{cost}(a,\pi) - \mathrm{cost}(a,\pi^{a\leftrightarrow b}) + \mathrm{cost}(b,\pi) - \mathrm{cost}(b,\pi^{a\leftrightarrow b}))$ and starting from arbitrary allocation and letting agents swap if they both prefer the swap converges to a swap stable allocation in polynomially-many swaps. □

Our last result settles the polynomial-time solvability of the problem for jump and swap stability in the case where the preferences are acyclic. The core idea of the algorithm is to find a topological ordering according to agents' preferences and let them pick their best location in the opposite of this ordering. The proof is similar to that of Aziz et al. [2] for an analogous setting in the original DPGs.

Theorem 4.4. *If the preference graph* D_{pref} *is acyclic, a jump stable and a swap stable allocation always exist and can be found in polynomial time.*

5 DISCUSSION

Our paper provides an almost complete landscape of the complexity of envy-free allocations in graphical distance preservation games. Our results illustrate that restrictions in the input instances are required to reach tractability. We believe that Open Problem 1 is the most important question that remains open on this front, which deserves further study. Additionally, we highlight a few more future directions more fundamentally varying the model.

"At most" and "at least" distances. Two very natural classes of ideal distances are what we term *"at least"* and *"at most"* distances. These are two-step functions where one of the steps is zero, and the other one is linear. There, if agent i wants their distance from j to be at least (resp. at most) d, then the cost is zero if under the allocation the condition is satisfied, and defined as normal otherwise. Observe that when an agent has ideal distance at most 1, then it is equivalent to having ideal distance exactly 1, hence the NP-completeness for envy-free allocations from Theorem 3.4 holds for this case. However, the problem remains open when agents have "at least" ideal distances, which is arguably a very natural class. Does our problem become easier under these distance functions?

Jump and Swap Stability. Our initial results for jump and stable allocations leave ample space for further study. Of course, there are the directions of studying the complexity of the problem, for which we conjecture it is NP-complete in general, under the same lens as envy-free allocations. Study how it behaves when we constrain the topology, or the graph, or agent's preferences. One less obvious direction that we would like to highlight is the existence of jump and swap stable allocations on paths under "at least" and "at most" distance functions. Despite our efforts to resolve the question, in both directions, the problem remained hard to crack.

Maximize Social Welfare. An orthogonal research direction is to study the maximization of social-welfare in graphical distance games; this objective was extensively studied in the model of Aziz et al. [4]. Observe that this is a very interesting problem, since it is at least as hard as Subgraph Isomorphism, an important problem for various AI subfields [35, 44]; this is when D_{pref} is symmetric and the ideal distance for every agent is 1. What are the graph classes that allow for efficient algorithms for this problem?

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