

Allocation Proportionality of OWA–Based Committee Scoring Rules

Extended Abstract

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ABSTRACT

The interpretation of proportionality in multiwinner voting differs significantly from that in apportionment. We aim to bridge them by introducing the concept of allocation proportionality: a voting rule is allocation proportional if each party’s share of elected candidates equals that party’s aggregate score. We introduce a new measure of allocation proportionality degree, and use it to experimentally evaluate a number of common OWA–based scoring rules.

KEYWORDS

Multiwinner Voting; Committee Scoring Rules; Proportionality; Party Elections

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1 INTRODUCTION AND BACKGROUND

Proportionality is widely regarded as an important component of fairness in preference aggregation [2]. It posits that each cohesive block of voters should be represented in the election outcome in proportion to its size. This statement admits multiple interpretations, with, in particular, the one in multiwinner voting being significantly different from that in apportionment. To reconcile these two distinct notions of proportionality, we introduce a concept of *allocation proportionality*, based on the party election framework.

The main motivation for this paper arises from one of the fundamental difficulties in defining proportionality for ordinal elections: we have no *a priori* notion of which voter groups should be represented proportionally – and even what it means for a group to be represented if candidates are ranked rather than approved. Proportionality axioms are usually formulated in terms of cohesive voting blocks. For instance, the principal axiom for ordinal voting – *Proportionality for Solid Coalitions (PSC)* [11] – requires that any sufficiently large group of voters who rank the same set of candidates above all others receive representation commensurate with its size. A similar approach is taken by approval proportionality axioms such as the *Justified Representation (JR)* family [1, 17, 22].



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However, empirical work (e.g., [9, 18]) shows solid coalitions rarely occur in real-world data. Thus, axioms based on such definitions tell us little about the behavior of voting systems in typical conditions, motivating the shift toward quantitative or robust proportionality assessments. More robust variants of PSC and EJRC were proposed in [10]. Quantitative measures of ordinal proportionality were developed by Skowron [19], Janson [15], and Bardal et al. [3]. They are, however, incompatible with proportionality measures for apportionment rules that are widely used in social sciences [23]. To fill that gap, we propose a measure of *allocation proportionality degree* that transposes apportionment-based notions of proportionality into ordinal voting. We then use it to experimentally compare six common OWA–based scoring rules.

An extended version of this paper is available on arXiv [5].

2 FRAMEWORK

Our focus is on proportionality as a property of voting rules, rather than as a quantity to be measured for particular elections. Thus, we can avoid the problem of cohesive voting groups not being known by redefining them and enforcing cohesive behavior through the statistical culture, using the framework of *party elections* [8].

A party election is an election endowed with an additional structure – every candidate is mapped to some party:

DEFINITION 1 (PARTY ELECTION). A *party election* (C, P, a, w) is a tuple consisting of a set of candidates, C ; a set of parties, P ; a surjective *affiliation function*, $a : C \rightarrow P$, that maps each candidate to a party; and a *profile* (i.e., a collection of linear orders on C), w .

Throughout the paper, we use $n := |w|$ for the number of voters, m for the number of candidates, and k for the committee size.

We consider the class of *OWA–Based Committee Scoring Rules* [20], which are a subclass of *committee scoring rules* [12–14]. The intuition behind OWA-based rules is to define committee scoring functions in terms of weighted sums of individual candidate scores while introducing flexibility in how voter satisfaction is defined. Each vote is translated into a vector of candidate scores according to a fixed scoring vector, and these scores are aggregated using an *ordered weighted averaging* (OWA) operator [26] with weights depending on the candidates’ relative positions in the voter’s ranking.

For a linear order \prec , let $\text{pos}_{\prec}(i)$ denote the rank position of a candidate i in \prec , i.e., the number of candidates $j \in C$ such that $i \preceq j$. For a k -committee $S \subseteq C$, the *position vector*, $\mathbf{p}_{\prec}(S)$, is a vector of positions of candidates of S in \prec permuted so that its coordinates are sorted in the increasing order.

DEFINITION 2 (OWA–BASED SCORING RULE). An *OWA–based scoring rule induced by an OWA weight vector* $\mathbf{z} \in [0, 1]^k$ and a

scoring vector $\mathbf{s} \in [0, 1]^m$ is a social choice function that maps a party election to the set of all k -subsets of C maximizing the expression $\sum_{\langle c \in \mathbf{w} \rangle} \sum_{i=1}^k z_i s_{p_{\langle c \rangle, i}(S)}$, where $p_{\langle c \rangle, i}(S)$ is the i -th coordinate of position vector $\mathbf{p}_{\langle c \rangle}(S)$, over $S \in \{Z \in \mathcal{P}(C) : |Z| = k\}$ [14, 20].

rule	scoring vector	OWA vector
SNTV	1-approval	1-approval
k -Borda	Borda	k -approval
Bloc Voting	k -approval	k -approval
Chamberlin–Courant (CC)	Borda	1-approval
Harmonic Borda (HB)	Borda	harmonic
Proportional k -Approval (PAV)	k -approval	harmonic

Table 1: Examples of OWA-based scoring rules.

3 ALLOCATION PROPORTIONALITY

Ordinal voting lacks a natural equivalent to the number of votes from single-choice voting systems. Nevertheless, rule-specific measures can be defined. In particular, for any OWA-based rule \mathcal{R} induced by a scoring vector \mathbf{s} , we can define *aggregate party score*:

DEFINITION 3 (AGGREGATE PARTY SCORE). Let \mathcal{R} be any OWA-based scoring rule induced by a scoring vector \mathbf{s} , and let $(C, P, \mathbf{a}, \mathbf{w})$ be a party election. For every party $i \in P$, its *aggregate party score* under \mathcal{R} is the normalized sum of its candidate scores:

$$\psi_i^{\mathcal{R}} := \frac{\sum_{\langle c \in \mathbf{w} \rangle} \sum_{j \in a^{-1}(i)} s_{\text{pos}_{\langle c \rangle}(j)}}{\sum_{\langle c \in \mathbf{w} \rangle} \sum_{j \in C} s_{\text{pos}_{\langle c \rangle}(j)}} = \frac{\sum_{\langle c \in \mathbf{w} \rangle} \sum_{j \in a^{-1}(i)} s_{\text{pos}_{\langle c \rangle}(j)}}{\|\mathbf{w}\| \|\mathbf{s}\|_1}. \tag{3.1}$$

With aggregate party scores as our counterpart to vote shares, we can finally introduce our definition of *allocation proportionality*:

DEFINITION 4 (ALLOCATION PROPORTIONALITY). An OWA-based scoring rule is *allocation proportional* for a given party election if for every $i \in P$ the share of committee seats obtained by candidates of the i -th party equals the aggregate party score of that party.

We establish that our aggregate party score ψ is a good *vote surrogate* for ordinal profiles as it satisfies the following desiderata:

- (1) **Normalization:** $\sum_{i \in P} \psi_i^{\mathcal{R}} = 1$ for every rule \mathcal{R} .
- (2) **Anonymity and neutrality:** ψ is invariant under permuting voters and under any relabeling of parties and candidates that preserves \mathbf{a} .
- (3) **Additivity over unions of electorates.**
- (4) **Weak monotonicity:** If each candidate of party i is (weakly) moved up in every ballot, then $\psi_i^{\mathcal{R}}$ does not decrease.
- (5) **Independence from OWA vector:** ψ depends only on \mathbf{s} .

In addition, allocation proportionality is a natural extension of both apportionment-based proportionality notions and PSC: under SNTV aggregate party score is equivalent to single-choice vote share, while for sufficiently top-heavy scoring vectors allocation proportionality is consistent with PSC seat guarantees.

However, we do not expect any voting rule to satisfy allocation proportionality, as we have the classic problem of seats being discrete and (usually) much less numerous than voters. Instead, we are thinking primarily in terms of an *allocation proportionality degree*.

A number of *proportionality degree indices* have been developed in the field of apportionment proportionality [23] to measure deviation from perfect proportionality. We use the *Kullback–Leibler divergence* [16] of \mathbf{v} from \mathbf{s} :

$$D_{KL}(\mathbf{v} \parallel \mathbf{s}) = \sum_{i=1}^d v_i (\log v_i - \log s_i), \tag{3.2}$$

as it has been proven a theoretically superior alternative to norm-based proportionality indices [24, 25].

4 EXPERIMENTS

Even for realistic apportionment elections, proportionality is difficult to model analytically [6]. Thus, we study the problem experimentally. We need to ensure that the probabilistic model for generating voter preferences does not treat each party’s candidates as independent, but instead groups them, reflecting the assumptions about cohesive voter blocks. While this is not a feature of most statistical cultures used in computational social choice [4, 21], we use *1- and 2-dimensional Euclidean party election models* by [7], which do have this feature. 2-D Euclidean is more realistic, while 1-D Euclidean is single-peaked and thus of particular interest.

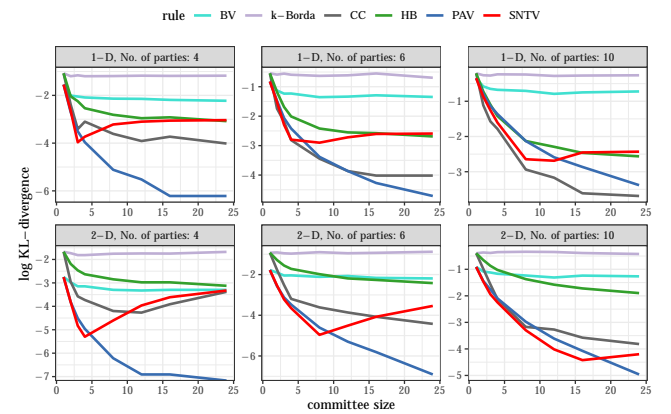


Figure 1: Allocation proportionality degree.

Among the evaluated rules, k -PAV emerges as the most robustly allocation-proportional across diverse settings. Chamberlin–Courant achieves high proportionality in multiparty elections – particularly under 1-D models – though its performance diminishes if parties are few. Harmonic Borda occupies an intermediate position, closer to highly proportional rules under 1-D models and to disproportional ones under 2-D models. While the proportionality of PAV, CC, and HB generally improves as a function of committee size k , SNTV exhibits a non-linear trajectory: it shows initial improvement but becomes increasingly disproportional (egalitarian) when k exceeds the number of parties. Conversely, Block Voting and especially k -Borda remain consistently disproportional, characterized by a systemic bias toward majority parties.

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