

Game-Theoretic and Algorithmic Analyses of Multi-Agent Routing under Crossing Costs

Extended Abstract

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ABSTRACT

This paper introduces the Multi-Agent Routing under Crossing Cost model on mixed graphs, a novel framework tailored for asynchronous settings. In our model, instead of treating conflicts as hard constraints, each agent is assigned a path, and the system is evaluated through a cost function that measures potential head-on encounters. This “crossing cost”, which is defined as the product of agents traversing an edge in opposite directions, quantifies the risk of congestion and delay in decentralized execution. Our contributions are both game-theoretic and algorithmic. We model the setting as a congestion game with a non-standard cost function, prove the existence of pure Nash equilibria, and analyze the dynamics leading to them. Equilibria can be found in polynomial time under mild conditions, while the general case is PLS-complete. From an optimization perspective, deciding whether a solution with zero crossing cost exists generalizes the Steiner Orientation problem, making the problem NP-complete. To address this hardness barrier, we design a suite of parameterized algorithms for minimizing crossing cost, with parameters including the number of arcs, edges, agents, and structural graph measures such as vertex cover.

KEYWORDS

Multi-Agent Routing; Computational Complexity; Nash Equilibrium; Parameterized Algorithms

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1 INTRODUCTION

Coordinating the movement of multiple autonomous agents over a shared network is a fundamental challenge in algorithmic robotics, intelligent transportation, and distributed systems. A central abstraction for this task is the Multi-Agent Path Finding (MAPF)

problem, which has numerous real-world applications, e.g., in warehouse management, autonomous vehicle coordination, and multi-robot systems [14, 19, 20]. Several works aim to overcome the computational hardness of MAPF by considering restricted instances and parameterized complexity [5, 6, 8, 10], while others present variations that include constraints such as energy consumption [4], communication [9, 18], rescheduling after delays [13], and many others [2, 15, 17, 21].

In the canonical MAPF formulation, the goal is to compute collision-free paths that prevent agents from simultaneously occupying the same vertex or traversing the same edge in opposite directions, while assuming centralized control and synchronized movement. In contrast, many real-world multi-agent systems, such as fleets of delivery robots, autonomous vehicles on road networks, or agents in communication-limited environments, operate with asynchronous timing and decentralized decision-making. In such settings, it is often impractical to enforce strict synchronization or guarantee globally conflict-free execution. Instead, agents may be provided with routes (that is, sequences of vertices to be followed), while the precise timing of their execution remains unspecified. Consequently, potential conflicts are better viewed not as hard constraints but as quantifiable risks arising from route interactions.

A powerful and well-studied framework for modeling such scenarios is the class of network congestion games, introduced by Rosenthal [16]. In these games, each agent is given a pair of vertices: a source and a destination. Each agent then selfishly selects a path between its source and destination on a network to minimize a cost function based on edge congestion, typically defined as a non-decreasing function of the number of agents traversing an edge. This formulation reflects situations where agents operate independently and may interact only partially or unpredictably during execution. It is well known that network congestion games are guaranteed to possess a pure-strategy Nash equilibrium [16]. While finding such an equilibrium is PLS-complete in general [1, 7], it is tractable in specific cases, such as when all agents share the same source-terminal pair [7].

2 OUR MODEL

In this work, we introduce a novel congestion game model, which we call **Crossing Cost Multi-Agent Routing (CC-MAR)**¹. Unlike traditional network congestion games, the cost is incurred not



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¹See [12] for a full version of the present work.

Table 1: Summary of our game-theoretic contributions

Existence & Stability of Equilibria	Finding a Nash Equilibrium
<ul style="list-style-type: none"> • A Nash Equilibrium always exists • Price of Stability: 1 • Price of Anarchy: unbounded 	<ul style="list-style-type: none"> • Poly-time for polynomially bounded edge weights • PLS-complete for general weights

Table 2: Summary of algorithmic contributions

Parameter	Complexity
# of terminal pairs k	XP (Tight due to [3])
# of edges $ E $	FPT
# of arcs $ A + k$	FPT
# of arcs $ A + \text{diam}(G)$	FPT
vertex cover number $\text{vc} + k$	FPT (Unweighted case)
vertex cover number vc	FPT (Unweighted, no overlapping terminals case)
w.r.t. the size of $(V \times V) \setminus \mathcal{T}$	FPT (Zero cost case)

by the total number of agents on an edge, but only by those traversing it in opposite directions; only potential head-on encounters are penalized. This asymmetric cost structure reflects practical scenarios such as unidirectional flow in narrow corridors or lanes, where same-direction usage is typically non-disruptive.

We use a mixed graph $G = (V, E, A)$ to represent a network and a set \mathcal{T} of k vertex pairs, called terminals, to denote the starting position and destination of each agent. For a set \mathcal{P} of paths that connect the given terminals and an edge $e = \{u, v\} \in E$, $x_{\vec{e}}$ (resp., $x_{\overleftarrow{e}}$) denotes the number of agents that use e in the direction \vec{e} (resp., \overleftarrow{e}). When computing the cost of a path P , it is safe to assume that P uses $\vec{e} = (u, v)$ if $e = \{u, v\} \in E(P)$ where $E(P)$ denotes the set of edges in P .

Crossing Cost Multi-Agent Routing (CC-MAR)

Input: $\Gamma = (G, \mathcal{T})$ where $G = (V, E, A)$ is a (weighted) mixed graph, \mathcal{T} is a multiset that contains k pairs of vertices $(s_i, t_i) \in V \times V, i \in [k]$.

Strategy profile: The set $\mathcal{P} = \{P_1, \dots, P_k\}$ of k paths $P_i = s_i - \dots - t_i, i \in k$.

Cost of an agent: $\text{cost}_{\mathcal{P}}(P_i) = \sum_{e \in E(P_i)} w_e x_{\vec{e}}$

Total cost: $\text{cost}(\mathcal{P}) = \sum_{\{u,v\} \in E} w_{uv} x_{uv} x_{vu}$

Figure 1 presents an example of our model.

3 OUR CONTRIBUTION AND QUESTIONS

We investigate the CC-MAR model from both game-theoretic and algorithmic viewpoints. From the game-theoretic viewpoint, we establish that the game always has a (pure) Nash equilibrium and prove that best-response dynamics are guaranteed to converge within at most $w_{\max} k^2 m$ steps, where w_{\max} is the maximum weight

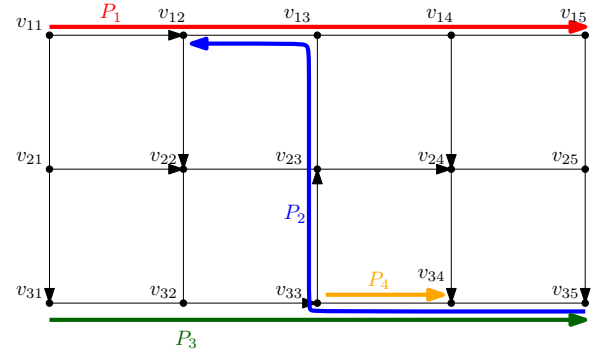


Figure 1: An example of CC-MAR with terminal pairs $(v_{11}, v_{15}), (v_{35}, v_{12}), (v_{31}, v_{35}),$ and (v_{33}, v_{34}) . The strategy of each agent is P_i for $i \in \{1, 2, 3, 4\}$. The cost $\text{cost}_{\mathcal{P}}(P_2)$ is because it crosses P_1 on edge $\{v_{12}, v_{13}\}$, crosses P_3 on edges $\{v_{33}, v_{34}\}, \{v_{34}, v_{35}\}$, and crosses P_4 on edge $\{v_{34}, v_{35}\}$. The total cost of the strategy profile is the sum of all pairwise crossing costs, which is $1(\text{from } P_1, P_2) + 2(\text{from } P_2, P_3) + 1(\text{from } P_2, P_4) = 4$.

among edges and $m = |E|$. This result follows from our proof that any strategy profile with minimum cost is a Nash equilibrium, which implies that the Price of Stability is 1. In contrast, we also present an instance where the Price of Anarchy is unbounded. Next, we prove that a Nash equilibrium can be computed in polynomial time if the weights are bounded by a polynomial in the input size, but the problem becomes PLS-complete otherwise. A summary of these results can be found in Table 1.

From the algorithmic viewpoint, we first note that our model naturally generalizes the Steiner Orientation problem, and therefore several hardness results follow [3, 11]. To complement these, we focus on parameterized algorithms. We first present an XP algorithm parameterized by the number of terminal pairs. We next develop two FPT algorithms: one parameterized by the number of edges, and another by the number of arcs plus the number of terminal pairs. Finally, we consider unweighted graphs and instances without overlapping terminal vertices. A summary of all our algorithmic results can be found in Table 2.

Finally, this framework opens several directions for future research. In particular, while we provide FPT algorithms for parameters involving the number of arcs and edges, the parameterized complexity with respect to $|A|$ alone remains unresolved. Further exploration of efficient heuristics or approximation schemes for practical instances remains an open and practically relevant challenge.

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