Robust Multiagent Collaboration Through Weighted Max–Min T-Joins

Extended Abstract

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ABSTRACT

Many multiagent tasks—such as fair resource allocation, partnerships between cities, exchanges between schools, and reviewer assignment—require selecting a group of agents such that collaboration remains effective even in the worst case. The *weighted max–min T-join problem* formalizes this challenge by seeking a subset of vertices whose minimum-weight matching is maximized, thereby ensuring robust outcomes against unfavorable pairings.

We study this problem in several directions. First, we design an algorithm that computes an upper bound for the *weighted max–min $2k$-matching problem*, where the chosen set must contain exactly $2k$ vertices. We develop a general algorithm with a $2\ln n$*-approximation guarantee* that runs in $O(n^4)$ time. Second, using ear decompositions, we propose another upper bound for the weighted max–min T-join cost.

KEYWORDS

T-join, Weighted max-min T-join, Weighted max-min $2k$-matching, Approximation algorithms

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1 INTRODUCTION

The T-join problem is an important generalization of matching problems. Let T be an even-sized subset of a finite metric space X. A matching of T pairs up its elements, and the cost of the matching is the sum of the distances within each pair. The classical T-join problem seeks a matching of T with minimum total cost. The max–min T-join problem, in contrast, looks for an even-sized subset T of X whose minimum-cost matching is as large as possible. We denote this maximum of minimum costs by $\mu(X)$.

Let G be a connected weighted graph with positive edge weights. This graph induces a metric on the vertex set V, where the distance between two vertices is the weight of the shortest path connecting them. In such a graph, a subgraph J of G is called *valid* if every cycle C in G satisfies $w(C) \geq 2\,w(C \cap J)$, where $w(C)$ is the total weight of the edges in C, and $w(C \cap J)$ is the total weight of the edges of J that lie in C. The max–min T-join problem for G is equivalent to finding a maximum-weight valid subgraph of G.

Following questions raised by Solé and Zaslawsky [14], Frank [4] studied the case where all edges have weight 1 and designed a polynomial-time algorithm running in $O(|V||E|)$ time. Iwata and Ravi [8] later extended the problem to general edge weights and developed a constant-factor approximation algorithm. Another extension of the weighted max–min T-join problem is the *weighted max–min $2k$-matching problem*, where the goal is to select $2k$ vertices of the graph so that their min-weight matching is maximized.

This problem naturally arises in multiagent systems. For example, in a school exchange program, the coordinator may need to preselect $2k$ schools before pairing them, ensuring that even under worst-case pairings the paired schools are far apart. Our work lies at the intersection of robustness, matching, and subset selection in multiagent systems. Selecting a subset of agents that performs well under worst-case conditions relates to robust optimization, coalition formation, and fairness in multiagent systems. See [1–3, 11–13, 15, 17] for related work.

We present a simple greedy algorithm that provides an upper bound for the cost of the weighted max–min $2k$-matching problem in a metric space. This provides initial progress on an open question posed by [8], who gave a constant-factor approximation algorithm for the weighted max–min T-join problem but left the $2k$-matching case open. Our approach also yields a logarithmic-approximation algorithm for the weighted max–min T-join problem with faster running time. Next, we study the weighted max–min T-join problem using *ear decompositions* of the graph and selects edges according to this decomposition and specific rules. This yields an upper bound, extending the method of [4] from the unweighted to the weighted case. In this paper, we assume that all graphs are weighted. We denote by $\mu(G)$ the cost of the weighted max–min T-join in a graph G. For the weighted max–min $2k$-matching problem, the cost is written as $\mu_{2k}(G)$. Clearly, $\mu(G) = \max_{1\leq i\leq \lfloor n/2\rfloor} \mu_{2i}(G)$. The full version of this paper is available online[1]

2 WEIGHTED MAX–MIN $2k$-MATCHING

We first present an upper bound on the cost of the weighted max–min $2k$-matching problem in a general metric space. Choose an arbitrary vertex $v_1 \in V$ and set $C = \{v_1\}$. At each step, add to C the vertex farthest from C. For a vertex v, its distance from C is defined as $d(v, C) = \min_{v_i \in C} d(v, v_i)$. Repeat this process until all vertices are added. This gives an ordering of the vertices $v_1, v_2, \ldots, v_n$.

[1] https://arxiv.org/abs/2602.07720

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For a set $A \subseteq V(G)$ with an even number of vertices, let $mwm(A)$ denote the cost of a minimum-weight perfect matching on A. Define $opt_{2k} = \max_{1\leq i\leq k} mwm(v_1, \ldots, v_{2i})$. Clearly, $opt_2 \leq opt_4 \leq \cdots \leq opt_{2k}$. We obtain the following upper bound for the cost of $\mu_{2k}(G)$.

Theorem 2.1. *Suppose $\{v'_1, \ldots, v'_{2k}\} \subseteq V(G)$ is an optimal set for the weighted max–min 2k-matching problem, i.e., $\mu_{2k}(G) = mwm(v'_1, \ldots, v'_{2k})$ Then, $\mu_{2k}(G) = mwm(v'_1, \ldots, v'_{2k}) \leq 2(1 + H_{k-1})opt_{2k}$.*

Proof sketch: The proof is by induction on k and uses the fact that, by the greedy algorithm, the distance from each point $u' \in \{v_{2k+1}, \ldots, v_n\}$ to its closest point in $\{v_1, \ldots, v_{2k}\}$ is smaller than the distance between any pair of points in $\{v_1, \ldots, v_{2k}\}$. Otherwise, u' would have been included among the points in $\{v_1, \ldots, v_{2k}\}$.

Theorem 2.1 provides an upper bound for the cost of weighted max–min $2k$-matching problem. Starting from v_1, in each step we select the farthest point. Repeating this for $2k$ steps takes $O(nk)$ time. Computing the minimum weight perfect matching for a set $\{v_1, \ldots, v_{2i}\}$ takes $O(i^3)$ time. So, overall, the running time for computing this upper bound is $O(nk + k^4)$. Note that the algorithm produces a set of size at most $2k$ whose minimum-weight matching gives an upper bound to $\mu_{2k}(G)$. Using Theorem 2.1, we present a logarithmic-factor approximation algorithm for the weighted max–min T-join problem.

Theorem 2.2. *Let*

$$mwm(v_1, \ldots, v_{2t}) = \max_{1\leq i\leq \lfloor n/2\rfloor} mwm(v_1, v_2, \ldots, v_{2i}),$$

and suppose $\{v^*_1, \ldots, v^*_{2\ell}\}$ is the optimal solution for the weighted max–min T-join problem. Then

$$\begin{aligned} mwm(v_1, \ldots, v_{2t}) &\leq mwm(v^*_1, \ldots, v^*_{2\ell}) \\ &\leq 2(1 + H_{\lfloor n/2\rfloor -1})mwm(v_1, \ldots, v_{2t}). \end{aligned}$$

By Theorem 2.2, we compute $opt_{2\lfloor n/2\rfloor}$, so the algorithm runs in $O(n^4)$ time. The weighted max–min T-join problem also admits a constant-factor approximation algorithm [8], but it relies on the ellipsoid method and has no explicit running-time bound. Our algorithm is much simpler, and experiments show that the values opt_{2k} give tighter bounds in practice.

3 EAR DECOMPOSITION AND WEIGHTED MAX–MIN T-JOIN PROBLEM

In this section, we present an algorithm that computes an upper bound for the weighted max–min T-join problem. The bound applies even when the points do not form a metric space.

Given a connected weighted graph G, the goal is to select a set of edges with maximum total weight such that, in every cycle, the total weight of the selected edges is at most half of the total weight of the cycle. As shown in [8], this is equivalent to choosing a set of vertices whose minimum-weight matching is maximized. We call any set of edges satisfying this condition a *valid set of edges*. Thus, the task is to find a valid set of edges with maximum total weight. By definition, every bridge edge (that is, an edge whose removal disconnects the graph) must be included in any optimal valid set. Our approach extends the work of Frank [4], who studied the unweighted case using ear decompositions.

We recall the necessary definitions. An *ear decomposition* of G is a sequence of subgraphs $G_0, G_1, \ldots, G_t = G$, where G_0 consists of a single vertex and no edges, and each G_i is obtained from G_{i-1} by adding a path or a cycle P_i. The endpoints of P_i (which coincide when P_i is a cycle) lie in G_{i-1}, while its remaining vertices are new. Each P_i is called an *ear*; if P_i is a single edge, it is called *trivial*. The collection $P = \{P_1, \ldots, P_t\}$ is also referred to as an ear decomposition, and the length of an ear is the number of its edges. It is well known that a graph admits an ear decomposition if and only if it is 2-edge-connected. A graph is 2-edge-connected if it is connected and remains connected after the removal of any single edge—equivalently, it is a connected graph with no bridge edges.

Frank [4] proved that for an unweighted graph G (i.e. the weight of each edge is 1), one has $\mu(G) = \frac{\varphi(G)+|V|-1}{2}$, where $\varphi(G)$ denotes the minimum number of edges whose contraction yields a factor-critical graph. A graph G is called factor-critical if, for every vertex $v \in V(G)$, the graph $G - v$ has a perfect matching Frank's proof begins by showing that $\varphi(G) \geq 2\mu(G) - |V| + 1$. We extend this result to the weighted graphs. Given a graph G, we first contract all bridges to get a 2-edge-connected graph G''. If we compute the optimal solution for the weighted max–min T problem on G'', then adding back the bridge edges gives the optimal solution for G. Thus, without loss of generality, we may assume that G is 2-connected, and therefore admits an ear decomposition.

In an ear decomposition of G, for an ear P, let $w(P)$ be the sum of its edge weights. We start with the following lemma.

Lemma 3.1. *Let $E' = \{e_1, \ldots, e_t\}$ be a valid set of edges of G, and let u and v be two vertices of G. Choose P to be a path from u to v (or a cycle if $u = v$) such that $(2\sum_{e\in P\cap E'} w(e)) - w(P)$ is minimized among all possible paths from u to v (or cycles if $u = v$). Let $E_1 = P\cap E'$ and $E_2 = P \setminus E_1$. Then $(E' \setminus E_1) \cup E_2$ is also a valid set.*

Proof. Assume that $(E' \setminus E_1) \cup E_2$ is not valid. This mean that there is a cycle that crosses the edges of P such that is violates the condition. So, we can find another path P' connecting u and v such that $2\sum_{e\in P'\cap E'(G)} - w(P')$ is less than $2\sum_{e\in P\cap E'(G)} - w(P)$ which is a contradiction, □

Now, for a given path P, define $\max(P)$ as the largest sum of edge weights from P that does not exceed $w(P)/2$. If $\{P^*_1, \ldots, P^*_t\}$ is an ear decomposition of G minimizing $\sum_{i=1}^{t} \max(P^*_i)$, then we have $\mu(G) \leq \sum_{i=1}^{t} \max(P^*_i)$. We show this inequality by proving the following theorem.

Theorem 3.2. *Let $\{G_0, \ldots, G_t = G\}$ be an ear decomposition of G, where each G_i is obtained from G_{i-1} by adding a path P_i. Then, $\mu(G) \leq \sum_{i=1}^{t} \max(P_i)$.*

Proof sketch: The proof proceeds by induction on the number of ears.

According to Theorem 3.2, for a given graph G, we can compute an upper bound by first contracting the bridge edges, then constructing an ear decomposition, and finally computing $\max(P)$ for each ear P. Note that in general, given a path P, computing $\max(P)$ is NP-hard, as it is equivalent to the knapsack problem [5, 9]. However, there exists an FPTAS [7, 16] with running time $O(n^2/\epsilon)$ that can compute a $(1 - \epsilon)$-approximation for $\max(P)$ for any $\epsilon > 0$.

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