Maximizing Index Diversity in Committee Elections

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ABSTRACT

We introduce two models of multiwinner elections with approval preferences and labelled candidates that take the committee's diversity into account. One model aims to find a committee with maximal diversity given a scoring function (e.g. of a scoring-based voting rule) and a lower bound for the score to be respected. The second model seeks to maximize the diversity given a minimal satisfaction for each agent to be respected. To measure the diversity of a committee, we use multiple diversity indices used in ecology and introduce one new index. We define (desirable) properties of diversity indices, test the indices considered against these properties, and characterize the new index. We analyze the computational complexity of computing a committee for both models and scoring functions of well-known voting rules, and investigate the influence of weakening the score or satisfaction constraints on the diversity empirically.

KEYWORDS

labeled multiwinner elections; axiomatic characterization; computational complexity; computational social choice

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1 INTRODUCTION

In the realm of decision making, where alternatives are chosen from a larger pool of options, considerations of both quality and diversity become crucial. This challenge presents itself in varied contexts, such as project funding and academic committee selections. For instance, consider the scenario faced by a city government engaging in participatory budgeting. Here, residents propose community projects characterized by target groups (e.g. children, seniors) and objectives (e.g. environmental protection, education).[1] Similarly, consider the formation of a university hiring committee, where the aim is to select individuals who are not only seen as experts by the colleagues electing them, but also contribute to a diverse range of perspectives (e.g. scientists, non-scientific staff) or disciplines (e.g. math, physics). In both cases, the task becomes to select k alternatives with both a high voter support (which we measure by a total *score* or individual *satisfaction*) and high level of diversity (which we measure by an *index*).

Indeed, both a lower bound on the total score and individual satisfaction are meaningful depending on the application. Consider, for example, an airline selecting k films for its inflight catalogue: films are labelled by genre, language, or region, and passenger groups (families, business travelers) supply an aggregate preference score from historical click or usage data; because these are past, aggregate preferences rather than current voters' utilities, it is natural to enforce a lower bound on the committee score (e.g. total or average past utility) and then maximize catalogue diversity by an index to preserve breadth and guard against sampling bias. Conversely, when planning k outreach programmes at a university, departments are actual voters directly affected by the outcome; activities are labelled by target audience, topic or format, and here one can require per-department satisfaction guarantees (for instance a minimum number of relevant offerings) while maximizing thematic diversity to avoid overrepresentation and broaden impact.

Traditional models, as seen in many previous studies, address this by setting hard quotas or constraints, ensuring a specific representation of specific labels. While such an approach ensures a certain minimum diversity, it often disregards the fluid and multifaceted nature of diversity itself. In particular, when many labels are to be respected, it seems impossible to come up with any meaningful constraints without restricting the possible satisfaction of the voters unpredictably. In contrast, our study introduces two models that incorporate diversity indices rather than strict constraints, while ensuring that either the total score of the committee or the satisfactions of the agents cannot be worsened arbitrarily. This approach, applied to the multiwinner election context, allows for a more nuanced and dynamic assessment of diversity. By employing established ecological indices alongside a newly proposed one, we introduce this elsewhere-established approach into the area of computational social choice.

Our Contributions. We introduce two models for incorporating diversity indices into multiwinner elections with approval preferences and labelled candidates. Our models aim to find a committee with maximal diversity given (1) a scoring function and a lower bound for the score, or (2) a minimal satisfaction for each agent. To measure diversity, we adapt multiple diversity indices used in ecology and propose a new diversity index, the Lexicographic Counting Index, which is designed to measure the diversity of a committee understandably. We provide an analysis of the properties of the diversity indices, including newly introduced properties, capturing, among others, the ability of voters to easily understand why one

[1] Indeed, participatory budgeting instances from Pabulib used in our experiments provide such characteristics. Nevertheless, we focus on the plain multiwinner election scenario without prices and budget in this paper.

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solution is more diverse than another. With these properties, we characterize our new diversity index and are able to differentiate between any two diversity indices considered.

We also analyze the computational complexity of computing a committee for both models: While computing maximally diverse committees is polynomial-time solvable for all indices, it is NP-hard to compute maximally diverse committees that provide some minimum satisfaction for each voter. The computational complexity of computing maximally diverse committees which provide some minimum score depends on the voting rule whose score we consider: E.g. this problem is polynomial-solvable for each of the indices considered when using the score of Approval Voting, but not when using the score of Chamberlin-Courant.

Our numerical experiments provide insights into the influence of weakening the score or satisfaction constraint on the diversity reached. We find that the diversity of the committees determined by scoring-based approval rules can indeed be improved by using a diversity index: For example, allowing the score to be reduced by 10% of the optimal score increases the average percentage of the optimal diversity achieved by 12–19, depending on the index and score considered. However, a reduction of the score by even 50% does not lead to the optimal diversity in some cases.

In the following, proofs marked with ★ are deferred to the long version of the paper [10].

Related Work. Our index-based approach complements established fairness notions in computational social choice. For example, proportional representation axioms [3] focus on representation guarantees, whereas the literature on collective decision making with label-based diversity predominantly enforces diversity via hard constraints (quotas, distributional requirements). By contrast, this paper treats diversity as an optimizable objective via indices and studies maximizing such indices under either an aggregate-score lower bound or per-agent satisfaction guarantees. We nonetheless review and compare constraint-based approaches below.

Celis et al. [11] and Bredereck et al. [8] introduce very similar models wherein candidates possess (possibly structured) labels and diversity is reached through hard distributional constraints. Their approach seeks an optimal committee that maximizes a performance score while meeting specified label occurrence requirements, such as gender quotas. In a similar vein, Ianovski [16] explores a model that accommodates "dominance constraints", requiring certain labels to occur at least as frequently as others, adding a layer of comparative label evaluation. The work by Aziz [2] provides a polynomial-time algorithm computing committees that satisfy two axioms, one ensuring the given diversity constraints are satisfied as much as possible and the other one integrating candidate excellence. Evequoz et al. [13] present an innovative election process where voters first decide on distributional constraints for candidates' attributes before electing candidates under those constraints, demonstrating this method in a Swiss primary election study. In the domain of approval voting, Straszak et al. [31] proposed an integer linear programming (ILP) framework to address diversity constraints across categorized candidate labels, offering computational tools and real-world data applications. Diss et al. [12] approach the integration of diversity constraints within the framework of multiwinner elections by presenting a model where the committee selection prioritizes both high scores and diversity metrics. Their work extends well-known committee scoring rules by tailoring them to meet specified diversity requirements and introduces new axioms for diverse committee selection under constraints.

Exploring applications beyond elections, Gawron and Faliszewski [15] utilize multiwinner voting to refine search systems such as movie recommendations, balancing similarity with diversity without relying on explicit diversity indices. The model from Relia [28] includes attributes for candidates and voters, applying hard distributional constraints and ensuring population-based representation within elected committees, enriching the voting model with demographic considerations. Lastly, Izsak et al. [17] present a framework where alternatives are classified, and inter- and intraclass relations are modeled through synergy functions, aiming to maximize both score-based and relational metrics—a concept parallel to our work, where diversity indices could be interpreted as synergy measures under score and satisfaction constraints. While synergy functions capture relational benefits between alternatives, our approach explicitly adapts ecological diversity indices.

Finding diverse solutions is also important in other contexts of collective decision making. Benabbou et al. [6] analyze diversity constraints in context of (utilitarian) public housing allocation. Aygün and Bó [1] explore diversity constraints for Brazilian college admissions through affirmative action policies, examining the strategic complexities of multidimensional privileges and proposing a fair, strategy-proof mechanism to ensure equitable student selection. Aziz and Sun [5] analyze diversity in the context of school admissions by defining a rank-based diversity concept, where maximal diversity is achieved by prioritizing student matches to seats that fulfill the institution's most crucial diversity criteria, thereby optimizing representation of key groups. Biró et al. [7] analyze the computational complexity of stable-matching-based college admissions and incorporate lower quotas for individual colleges and common quotas for groups of colleges allowing to manage collective diversity targets.

2 THE MODEL

Let $\mathbb{N}$ and $\mathbb{N}_0$ be the natural numbers excluding and including zero, respectively, $[t]$ the set $\{1, \dots, t\}$ for any integer t, and $\mathcal{P}(X)$ the power set of any set X. $(x_i)_{i=a}^{b}$ denotes the vector $(x_a, x_{a+1}, \dots, x_b)$ and, for a given vector y, $\dim(y)$ is the number of entries of y, which we refer to as y's dimension.

We consider committee elections with approval preferences and labelled candidates, i.e. elections of the form $\mathcal{E} = (A, C, U, k, L, \lambda)$ consisting of a set A of agents, a candidate set C, an approval profile $U \colon A \to \mathcal{P}(C)$, and a desired committee size $0 < k \leq |C|$. $L = \{l_1, \dots, l_m\}$ is a set of m labels and $\lambda \colon C \to L$ assigns a label to each candidate. In addition, for $i \in [m]$ and a committee $S \subseteq C$, let

$$C_{label}(\mathcal{E}, S, i) \coloneqq \{c \in S : \lambda(c) = l_i\},$$
$$n_i(\mathcal{E}, S) \coloneqq |C_{label}(\mathcal{E}, S, i)|, \text{ and } p_i(\mathcal{E}, S) \coloneqq n_i(\mathcal{E}, S) / |S|$$

be the set, number, and proportion of candidates in S with label l_i, respectively. Furthermore, let

$$\mathrm{distr}(\mathcal{E}, S) \coloneqq (|\{i \in [m] : n_i(\mathcal{E}, S) = j\}|)_{j=0}^{|S|},$$

i.e. $\mathrm{distr}(\mathcal{E}, S)_j$ is the number of labels occurring $j - 1$ times in S.

A rule $\mathcal{R}\colon \mathcal{E} \to \mathcal{P}(\{S \subseteq C : |S| = k\})$ maps an election to at least one k-sized subset of C. We denote by $\mathcal{R}_{\text{vld}}(\mathcal{E}) \coloneqq \{S \subseteq C : |S| = k\}$ the rule that outputs all committees of size k and by $\mathcal{R}^s(\mathcal{E}) = \arg\max_{S \in \mathcal{R}_{\text{vld}}(\mathcal{E})} s(\mathcal{E}, S)$ the rule that outputs all $S \in \mathcal{R}_{\text{vld}}(\mathcal{E})$ with maximal score, where s is a scoring function mapping an election and a committee to $\mathbb{N}$. We only consider scoring functions which take only A, C, U and k into account, i.e. information that is part of a "classical" election. For an $S \in \mathcal{R}_{\text{vld}}(\mathcal{E})$, we measure the satisfaction of an agent a as $\text{sat}(\mathcal{E}, S, a) \coloneqq |S \cap U(a)|$, i.e. as the number of chosen candidates a approves. To measure the diversity, we look at diversity indices which assign a real number to an election $\mathcal{E}$ and committee $S \in \mathcal{R}_{\text{vld}}(\mathcal{E})$.

3 THE DIVERSITY INDICES

In this section, we discuss the diversity indices we consider.

Example 1. As a running example, consider an election $\mathcal{E}$ with projects as candidates, $m = 3$ labels, $L = \{$🤒, 📚, 💪$\}$ (🤒 represents the label "health", 📚 "education", and 💪 "sport")[2], $k = 10$, and the following committees (for each candidate, we indicate its label):

$S' \in \mathcal{R}_{\text{vld}}(\mathcal{E})$	$\text{distr}(\mathcal{E}, S')$
$S'_1 = \{$🤒, 🤒, 🤒, 📚, 📚, 📚, 💪, 💪, 💪, 💪$\}$	$(0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0)$
$S'_2 = \{$🤒, 📚, 💪, 💪, 💪, 💪, 💪, 💪, 💪, 💪$\}$	$(0, 2, 0, 0, 0, 0, 0, 0, 1, 0, 0)$
$S'_3 = \{$📚, 📚, 📚, 📚, 📚, 💪, 💪, 💪, 💪, 💪$\}$	$(1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0)$

Clearly, S'_1 appears as most diverse in the sense that it contains all three different labels and the labels appear as evenly balanced as possible. Whether S'_2 is more diverse than S'_3 depends e.g. on whether someone finds it more important that as many labels as possible are represented or that the labels represented appear as evenly as possible: S'_2 contains three labels, but one occurs much more frequently than the others, while S'_3 contains only two labels, but these occur equally often. ◃

Indices Used in Ecology. In the field of ecology, various indices have been defined to measure the diversity of a community of species—see e.g. [22, 26] for an overview of diversity indices. We directly adapt the following diversity indices often used in ecology so that they receive an election $\mathcal{E} = (A, C, U, k, L, \lambda)$ as well as a committee $S \in \mathcal{R}_{\text{vld}}(\mathcal{E})$ as inputs. For each of the following indices, a higher value indicates a higher diversity.

Richness [32] is a simple diversity index that takes only the number of labels present into account:

$$\begin{aligned} Ri(\mathcal{E}, S) &= |\{i \in [m] : n_i(\mathcal{E}, S) > 0\}| \\ &= \sum_{\ell=1}^{k} \text{distr}(\mathcal{E}, S)_{\ell+1} = m - \text{distr}(\mathcal{E}, S)_1. \end{aligned}$$

The Simpson index [30][3] considers the probability that two candidates chosen independently and at random from the committee have the same label, i.e.

$$Si(\mathcal{E}, S) = -\sum_{i \in [m]} p_i(\mathcal{E}, S)^2 = -\sum_{\ell=1}^{k} \text{distr}(\mathcal{E}, S)_{\ell+1} \cdot \left(\frac{\ell}{k}\right)^2.$$

[2]The emoji graphics are taken from `twemojis` and licensed under CC-BY 4.0. Copyright 2019 Twitter, Inc and other contributors.

[3]In the literature, the Simpson index is usually stated unnegated. We add the negation in order to maximize each index.

Another popular index [23], derived from information theory, is Shannon's entropy [29]. It represents the uncertainty in predicting the label of a randomly chosen candidate from the committee:

$$\begin{aligned} Sh(\mathcal{E}, S) &= -\sum_{i \in [m] : p_i(\mathcal{E}, S) > 0} p_i(\mathcal{E}, S) \cdot \log(p_i(\mathcal{E}, S)) \\ &= -\sum_{\ell=1}^{k} \text{distr}(\mathcal{E}, S)_{\ell+1} \cdot \frac{\ell}{k} \cdot \log\left(\frac{\ell}{k}\right). \end{aligned}$$

Remark 1. The indices rank the committees from Example 1 as follows: $Ri(\mathcal{E}, S'_1) = 3 = Ri(\mathcal{E}, S'_2) > Ri(\mathcal{E}, S'_3) = 2$, $Sh(\mathcal{E}, S'_1) \approx 1.09 > Sh(\mathcal{E}, S'_3) = 0.69 > Sh(\mathcal{E}, S'_2) \approx 0.64$, and $Si(\mathcal{E}, S'_1) = -0.34 > Si(\mathcal{E}, S'_3) = -0.5 > Si(\mathcal{E}, S'_2) = -0.66$. Therefore, Ri classifies S'_3 as least diverse, while Sh and Si both classify S'_2 as the least diverse. ◃

New Index. We introduce a new diversity index, the *Lexicographic Counting Index* (*LC*), which elevates the natural, but simple, diversity index *Ri*. While the idea of lexicographic ordering has been applied in various settings, it has not been used for defining a diversity index before, to the best of our knowledge. The primary goal of *LC* is to maximize the number of labels occurring at least once (like *Ri* does), the secondary goal is to maximize the number of labels occurring at least twice, and so on:

$$LC(\mathcal{E}, S) = \sum_{i=1}^{k} (\min\{m, k\} + 1)^{k+1-i} \cdot |\sigma_i(\mathcal{E}, S)|,$$
$$\text{with } \sigma_i(\mathcal{E}, S) = \{\ell \in [m] : n_\ell(\mathcal{E}, S) \geq i\}.$$

Note that the base is $\min\{m, k\} + 1$, as a committee consists of k candidates and each candidate introduces at most one new label, i.e. $\forall i \in [k] : \sigma_i(\mathcal{E}, S) \leq \min\{m, k\}$. In the paper's long version, we evaluate *LC* with respect to properties adapted from the literature, providing some arguments for calling *LC* a diversity index.

Remark 2. *LC* classifies S'_1 as the most and S'_3 as the least diverse committee: $LC(\mathcal{E}, S'_1) = 4^{10} \cdot 3 + 4^9 \cdot 3 + 4^8 \cdot 3 + 4^7 = 4145152 > LC(\mathcal{E}, S'_2) = 4^{10} \cdot 3 + 4^9 + 4^8 + 4^7 + 4^6 + 4^5 + 4^4 + 4^3 = 3495232 > LC(\mathcal{E}, S'_3) = 4^{10} \cdot 2 + 4^9 \cdot 2 + 4^8 \cdot 2 + 4^7 \cdot 2 + 4^6 \cdot 2 = 2793472.$ ◃

4 WHICH DIVERSITY INDICES TO USE?

Next, we want to distinguish formally between the aforementioned indices by defining properties and testing the indices against them. Note that not all properties that we define should necessarily be satisfied by a diversity index: The properties rather draw attention to differences between the indices, which should be taken into account when picking a diversity index to be used. This is of interest when electing committees consisting of at least six candidates, because *Sh*, *Si*, and *LC* behave the same for small committees when deciding which committee is more diverse:

Observation 1 (★). *For all elections $\mathcal{E}$ with $k \leq 5$, it holds that $\forall r, r' \in \{Sh, Si, LC\}, \diamond \in \{<, >, =\}, S_1, S_2 \in \mathcal{R}_{\text{vld}}(\mathcal{E}) : r(\mathcal{E}, S_1) \diamond r(\mathcal{E}, S_2) \Leftrightarrow r'(\mathcal{E}, S_1) \diamond r'(\mathcal{E}, S_2)$. For all elections $\mathcal{E}$ with $k \leq 7$, it holds that $\forall S_1, S_2 \in \mathcal{R}_{\text{vld}}(\mathcal{E}), \diamond \in \{<, >, =\} : Sh(\mathcal{E}, S_1) \diamond Sh(\mathcal{E}, S_2) \Leftrightarrow LC(\mathcal{E}, S_1) \diamond LC(\mathcal{E}, S_2)$.*

However, the indices can behave differently for larger k. One reason for this is that only *Ri* and *LC* consider the number of labels present to be more important than the evenness of the distribution of the labels present, while *Si* and *Sh* do not—this can be seen e.g. in Example 1. Thus, the first question that one has to answer is whether having as many labels in the committees as possible has the highest priority—this could be the case when electing a team working on an interdisciplinary project, where having an expert from a discipline that is not covered otherwise is very valuable. We express this through the following property:

Property 1 (Present Label Maximization)**.** A diversity index D satisfies *Present Label Maximization* if, for all elections $\mathcal{E}$ and $S_1, S_2 \in \mathcal{R}_{\text{vld}}(\mathcal{E})$ for which $\text{distr}(\mathcal{E}, S_1)_1 < \text{distr}(\mathcal{E}, S_2)_1$ (i.e. S_1 contains more different labels than S_2), it holds that $D(S_1) > D(S_2)$.

Observation 2 (★)**.** *Ri and LC satisfy Present Label Maximization, Si and Sh do not.*

In contrast, it should hold for each diversity index that increasing the frequency of a label by decreasing those of a more frequent label by at least two leads to a higher (*Occurrence Balancing*) or at least the same (*Weak Occurrence Balancing*) diversity:

Property 2 (Occurrence Balancing)**.** A diversity index D satisfies (*Weak*) *Occurrence Balancing* if for all elections $\mathcal{E}$ and $S' \in \mathcal{R}_{\text{vld}}(\mathcal{E})$ for which there are $i, j \in [m]$ with $n_i(\mathcal{E}, S') + 2 \leq n_j(\mathcal{E}, S')$, it holds that $\forall c_i \in C_{label}(\mathcal{E}, C, i) \setminus S', c_j \in C_{label}(\mathcal{E}, S', j) : D(S') < D(S'')$ $(D(S') \leq D(S''))$ with $S'' = S' \setminus \{c_j\} \cup \{c_i\}$.

The following result not only separates *Ri* from the other three indices, but is also the basis for showing in Section 5.1 that finding a committee with the highest diversity is in **P**.

Observation 3 (★)**.** *All diversity indices considered satisfy Weak Occurrence Balancing. Si, Sh, and LC satisfy Occurrence Balancing.*

The properties introduced so far allow us to differentiate between any pair of diversity indices considered apart from *Si* and *Sh*, which both take the evenness of the distribution into account. They disagree, however, on whether balancing two labels is preferable for rare or for dominant labels. We first define which pairs of labels whose occurrences differ by d are *balancable*: $\text{balancable}(\mathcal{E}, S, d)$ returns, for an election $\mathcal{E}$ and a committee $S \in \mathcal{R}_{\text{vld}}(\mathcal{E})$, all pairs (i, j) so that $n_i(\mathcal{E}, S) + d = n_j(\mathcal{E}, S)$ and $n_i(\mathcal{E}, S) + \lfloor \frac{d}{2} \rfloor \leq |C_{label}(\mathcal{E}, C, i)|$ (i.e. the first label of the pair occurs d fewer times than the second one, but its frequency could be increased by $\lfloor \frac{d}{2} \rfloor$). The function $\text{balance}(\mathcal{E}, S, d, (i, j))$ actually balances such a label pair $(i, j) \in \text{balancable}(\mathcal{E}, S, d)$ by returning a committee $S' \in \mathcal{R}_{\text{vld}}(\mathcal{E})$ so that $n_i(\mathcal{E}, S') = n_i(\mathcal{E}, S) + \lfloor \frac{d}{2} \rfloor, n_j(\mathcal{E}, S') = n_j(\mathcal{E}, S) - \lfloor \frac{d}{2} \rfloor$ and $\forall e \in [m] \setminus \{i, j\} : n_e(\mathcal{E}, S') = n_e(\mathcal{E}, S)$, i.e. only the number of l_i and l_j are balanced. Based on this, we can define the following property:

Property 3 (Prioritization of Rare Label Balancing)**.** A diversity index D satisfies *Prioritization of Rare Label Balancing* if, for all elections $\mathcal{E}, S \in \mathcal{R}_{\text{vld}}(\mathcal{E})$ and $d \geq 2$ for which there are $(i, j), (k, l) \in \text{balancable}(\mathcal{E}, S, d)$ with $n_i(\mathcal{E}, S) < n_k(\mathcal{E}, S)$, it holds for $S_{(i,j)} \coloneqq \text{balance}(\mathcal{E}, S, d, (i, j))$ and $S_{(k,l)} \coloneqq \text{balance}(\mathcal{E}, S, d, (k, l))$ regarding the diversity that $D(\mathcal{E}, S_{(i,j)}) > D(\mathcal{E}, S_{(k,l)})$.

Observation 4 (★)**.** *Sh and LC satisfy Prioritization of Rare Label Balancing, Si and Ri do not.*

In some sense, this makes *Sh* more similar to *LC* than *Si*, as *Sh* and *LC* both prefer to increase the frequency of the rarest among the four labels, while *Si* rates both options as equally good.

Next, we want to look at another property that distinguishes between our indices and takes into account that the index is to be used in an election: One important goal is to ensure that voters (or other stakeholders) are able to understand why a committee has been chosen over a different one, which is likely to promote acceptance of the result or at least a more informed debate about it (e.g. when maximizing diversity is incorporated into elections). Hence, we want to focus on the *explainability* of the diversity indices next, or, in other words, the effort required to decide which of two given committees is more diverse.

For this, let $\mathcal{E}$ be an election with $S_1, S_2 \in \mathcal{R}_{\text{vld}}(\mathcal{E})$. To decide which of S_1 and S_2 is more diverse (or whether they are equally diverse), the only information required by the diversity indices considered is how many labels occur how often, which is provided by the distr vectors. When comparing $\text{distr}(\mathcal{E}, S_1)$ and $\text{distr}(\mathcal{E}, S_2)$, e.g. for S_2' and S_3' from Example 1, it seems natural to look only at the entries at which the distr vectors differ. Indeed, it is not difficult to see from the definitions of the considered diversity indices that any $i \in [k+1]$ with $\text{distr}(\mathcal{E}, S_1)_i = \text{distr}(\mathcal{E}, S_2)_i$ is irrelevant.

Thus, we denote the set of all indices that matter as $I_R(\mathcal{E}, S_1, S_2) = \{i \in [k+1] : \text{distr}(\mathcal{E}, S_1)_i \neq \text{distr}(\mathcal{E}, S_2)_i\}$. Based on this, we define the *reduced distribution vector* $\text{rdistr}(\mathcal{E}, S_1, S_2)$ as the distr vector of S_1 at the indices in $I_R(\mathcal{E}, S_1, S_2)$, with ρ as the vector of the elements of $I_R(\mathcal{E}, S_1, S_2)$ in ascending order:

$$\text{rdistr}(\mathcal{E}, S_1, S_2) \coloneqq \left(\text{distr}_{\rho_i}(\mathcal{E}, S_1)\right)_{i=1}^{|I_R(\mathcal{E}, S_1, S_2)|}.$$

Consequently, $\text{rdistr}(\mathcal{E}, S_1, S_2)$ and $\text{rdistr}(\mathcal{E}, S_2, S_1)$ correspond to $\text{distr}(\mathcal{E}, S_1)$ and $\text{distr}(\mathcal{E}, S_2)$, respectively, where the entries at indices at which the distr vectors are equal are removed. Note that this implies $\dim(\text{rdistr}(\mathcal{E}, S_1, S_2)) = \dim(\text{rdistr}(\mathcal{E}, S_2, S_1)) = \dim(\rho)$.

Remark 3. For the committees S_2' and S_3' in Example 1, we have

$$I_R(\mathcal{E}, S_2', S_3') = I_R(\mathcal{E}, S_3', S_2') = \{1, 2, 6, 9\}, \rho = (1, 2, 6, 9)$$
$$\text{rdistr}(\mathcal{E}, S_2', S_3') = (0, 2, 0, 1), \text{rdistr}(\mathcal{E}, S_3', S_2') = (1, 0, 2, 0) \quad \triangleleft$$

Based on this, we call a diversity index *n-explainable* if, roughly speaking, at most n indices from $\text{rdistr}(\mathcal{E}, S_1, S_2)$, $\text{rdistr}(\mathcal{E}, S_2, S_1)$, or ρ need to be used to decide which of S_1, S_2 is more diverse or whether they are equally diverse. Clearly, if a diversity index uses only a few indices, the decision is easier to follow than when many are used, especially when the dimension of the vectors becomes larger. Thus, n represents a degree of explainability, with a small value indicating a high explainability. We want to formalize this with the help of a function that makes this decision: Let T be all possible triples (a, b, c) for which there are an election $\mathcal{E}$ and $S_1, S_2 \in \mathcal{R}_{\text{vld}}(\mathcal{E})$ so that $a = \text{rdistr}(\mathcal{E}, S_1, S_2), b = \text{rdistr}(\mathcal{E}, S_2, S_1)$, and c is the ρ vector, i.e. all the inputs such a function needs to be able to process. For a given diversity index D, a function $e : T \to \{\textit{less}, \textit{more}, \textit{equal}\}$ is called *D-explainable using at most n indices* if

(1) e decides whether the first argument describes a more or less diverse committee than the second one, i.e.

$$e(\text{rdistr}(\mathcal{E}, S_1, S_2), \text{rdistr}(\mathcal{E}, S_2, S_1), \rho) =$$
$$\begin{cases} \textit{less} & \text{if } D(\mathcal{E}, S_1) < D(\mathcal{E}, S_2) \\ \textit{equal} & \text{if } D(\mathcal{E}, S_1) = D(\mathcal{E}, S_2) \\ \textit{more} & \text{if } D(\mathcal{E}, S_1) > D(\mathcal{E}, S_2) \end{cases}$$

(2) $e((),(),()) = \textit{equal}$, which happens if two distr vectors are the same—a scenario in which each diversity index considered classifies the committees as equally diverse.
(3) For each $r \in \mathbb{N}$, there are two index sets $I_o, I_d \subseteq [r]$ so that
   - for all $(\text{rdistr}^{(1)}, \text{rdistr}^{(2)}, \rho) \in T$ with $\dim(\rho) = r$, it holds that $\{i \in [r] : e \text{ uses } \text{rdistr}_i^{(1)} \text{ or } \text{rdistr}_i^{(2)} \text{ or both}\} \subseteq I_d$ and $\{i \in [r] : e \text{ uses } \rho_i\} \subseteq I_o$, i.e. at most the indices from I_o are accessed in ρ and at most those from I_d are accessed in the rdistr vectors for any vectors of dimension r,
   - $|I_o| + |I_d| \leq n$, i.e. at most n indices are used.

   Thus, we count the use of both $\text{rdistr}_i^{(1)}$ and $\text{rdistr}_i^{(2)}$ only once, as it is natural to assume that we need to look at the same indices in both vectors to compare them. In addition, we allow the index sets to differ with varying r. This is necessary e.g. when the diversity index needs to access nearly all indices, so that the index sets become larger with growing r.

Based on this, we define the following properties:

Property 4 (n-Explainability)**.** A diversity index D is *n-explainable* if there is a D-explainable function e that uses at most n indices for any argument $(a, b, c) \in T$ with $\dim(a) \geq 1$[4].

As, for each of e's arguments $t = (a, b, c) \in T$, the number of indices used is bounded upwards by $2\dim(a) = 2\dim(b) = 2\dim(c)$, we denote the dimension of a for $t = (a, b, c) \in T$ as r. By using all the available information of each argument $t \in T$, each of the considered diversity indices is clearly $2r$*-explainable*. Next, we give the smallest possible n so that the index is n*-explainable*, for each of the diversity indices considered.

Theorem 1 (★)**.** *(1) LC is* 1*-explainable,*
(2) Ri is 2*-explainable, but not* 1*-explainable,*
(3) Si and Sh are $2r-2$*-explainable, but not* $2r-3$*-explainable.*

While the proof is deferred to the long version of the paper due to the space constraints, we want to give some insights here: For LC it holds—due to its lexicographic nature—that $\text{rdistr}(\mathcal{E}, S_1, S_2)_1 < \text{rdistr}(\mathcal{E}, S_2, S_1)_1 \Leftrightarrow LC(\mathcal{E}, S_1) > LC(\mathcal{E}, S_2)$ so that

$$e_{LC}(\text{rdistr}^{(1)}, \text{rdistr}^{(2)}, \rho) = \begin{cases} \textit{less} & \text{if } \dim(\rho) \geq 1 \text{ and } \text{rdistr}_1^{(1)} > \text{rdistr}_1^{(2)} \\ \textit{equal} & \text{if } \dim(\rho) = 0 \\ \textit{more} & \text{if } \dim(\rho) \geq 1 \text{ and } \text{rdistr}_1^{(1)} < \text{rdistr}_1^{(2)} \end{cases}$$

can be chosen as an LC-explainable function, which uses at most index 1 in the rdistr vectors.

For Ri it follows directly from its definition that

$$e_{Ri}(\text{rdistr}^{(1)}, \text{rdistr}^{(2)}, \rho) = \begin{cases} \textit{less} & \text{if } \dim(\rho) \geq 1 \text{ and } \rho_1 = 1 \text{ and } \text{rdistr}_1^{(1)} > \text{rdistr}_1^{(2)} \\ \textit{equal} & \text{if } \dim(\rho) = 0 \text{ or } \rho_1 > 1 \\ \textit{more} & \text{if } \dim(\rho) \geq 1 \text{ and } \rho_1 = 1 \text{ and } \text{rdistr}_1^{(1)} < \text{rdistr}_1^{(2)} \end{cases}$$

can be chosen as a Ri-explainable function, using at most 2 indices. However, Ri is not 1*-explainable*. One way to explain this is that a diversity index being 1*-explainable* and fulfilling *Present Label Maximization* (which Ri does) categorizes two committees as equally diverse if and only if their distr vectors are equal and hence the rdistr vectors have the dimension zero, which is clearly not the case for Ri (consider e.g. S_1' and S_2' from Example 1).

While LC and Ri hence only need to use a small and constant number of indices and are therefore easy to comprehend, this is not the case for Si and Sh. They are $2r-2$*-explainable*, because the last element in ρ and in the rdistr vectors simply do not need to be used because they can be reconstructed based on the other entries. Thus, the result that there is no Si- and Sh-explainable function using at most $2r-3$ indices is more interesting, which clearly distinguishes Sh and Si from Ri and LC.

To conclude this section, we answer whether the behavior of LC—which is the only index fulfilling all the properties introduced—can be characterized by (some of) the properties introduced so far:

Theorem 2 (★)**.** *LC is characterized by* 1*-Explainability and Present Label Maximization.*

5 INCORPORATING DIVERSITY INDICES INTO ELECTIONS

In this section, we propose two ways to incorporate diversity (indices) into elections. One way is to maximize the diversity given a minimal satisfaction for each agent:

Definition 1 (MAX-D-DSAT)**.** Given an $\mathcal{E} = (A, C, U, k, L, \lambda)$ and a function $h : A \to \mathbb{N}_0$, find a committee with maximal diversity with respect to the diversity index D among all committees $S \in \mathcal{R}_{\text{vld}}(\mathcal{E})$ for which $\forall a \in A : \text{sat}(\mathcal{E}, S, a) \geq h(a)$.

On the one hand, this provides a certain amount of freedom in the search for a diverse committee, but on the other hand, it gives the voters the certainty that their satisfaction cannot be worsened arbitrarily. One possibility for defining $h(a)$ is to compute a committee S with a well-known voting rule and to define $h(a)$ as the satisfaction of agent a with S minus one (which we will consider in our numerical experiments) or to ensure the same minimum satisfaction for each agent.

A different way to incorporate diversity is to maximize the diversity given a scoring function (e.g. of a scoring-based voting rule) and a minimum committee score:

Definition 2 (MAX-(D, s)-DSCR)**.** Given an election $\mathcal{E}$ and a bound $\beta \in \mathbb{N}_0$, find a committee with maximal diversity with respect to diversity index D among all committees $S \in \mathcal{R}_{\text{vld}}(\mathcal{E})$ for which $s(\mathcal{E}, S) \geq \beta$, where s is a scoring function.

We will show results for the following, well-known scoring-based approval voting rules in the following sections: Multi-Winner Approval Voting (AV), Satisfaction Approval Voting (SAV), Proportional Approval Voting (PAV), and Approval Chamberlin-Courant (CC) (see e.g. [19] for definitions of these rules).

We will refer to the score of the scoring-based voting rule $\mathcal{R}$ as $\text{score}_{\mathcal{R}}$, and to the decision variant of MAX-D-DSAT and MAX-(D, s)-DSCR, in which the goal is to find a committee with a diversity which is at least a given value δ, as D-DSAT and (D, s)-DSCR, respectively.

[4]For $\dim(a) = 0$, each D-explainable function returns *equal* and hence does not need to use any index.

5.1 Complexity Results

First, we consider the computational complexity of finding a "diversity optimal" committee without additional constraints. For this, we can exploit the fact that each index satisfies *Weak Occurrence Balancing*. Therefore, the highest diversity is reached when starting with an empty committee and iteratively adding a candidate with a label that, among the labels with unselected candidates, occurs least often in the current committee:

Observation 5 (★). *Given an election $\mathcal{E}$ and one of the diversity indices considered, choosing a committee $S \in \mathcal{R}_{\text{vld}}(\mathcal{E})$ with the highest diversity is polynomial-time solvable.*

Yet, D-DSAT is **NP**-hard for these indices. The same holds for (D, s)-DSCR if finding winning committees is **NP**-hard for $\mathcal{R}^s$. We show this result for a broader class of diversity indices satisfying the following, clearly desirable property:

Property 5 (Uniqueness Optimality)**.** A diversity index D satisfies *Uniqueness Optimality* if for each election $\mathcal{E}$ with $m \geq k$, it holds that $S \in \mathcal{R}_{\text{vld}}(\mathcal{E})$ has optimal diversity if and only if no two candidates in S have the same label.

Clearly, each of *LC*, *Si*, and *Sh* satisfy this property because they satisfy *Occurrence Balancing*, and *Ri* satisfies it because it is defined as $m - \text{distr}(\mathcal{E}, S)_1$ and $\text{distr}(\mathcal{E}, S)_1$ gets smaller if fewer labels do not occur at all. For such indices, we have the following:

Observation 6. *If D is a diversity index that satisfies Uniqueness Optimality, (1) (D, s)-DSCR is* **NP***-hard if the decision problem of $\mathcal{R}^s$ is* **NP***-hard and (2) D-DSAT is* **NP***-hard.*

Proof. (1) The reduction from the **NP**-hard decision problem of $\mathcal{R}^s$, i.e. where given an election $\mathcal{E} = (A, C, U, k)$, the task is to decide whether there is $S \in \mathcal{R}_{\text{vld}}(\mathcal{E})$ with $s(\mathcal{E}, S) \geq s^*$, works as follows: Choose the election $\mathcal{E}' = (A, C, U, k, L', \lambda')$ with $L' = \{l_c : c \in C\}$ as the possible labels and $\lambda'(c) = l_c$ (i.e. each candidate has a different label). Set $\delta = D(\mathcal{E}, S^*)$, where S^* is a committee for $\mathcal{E}$ in which k labels occur exactly once, and $\beta = s^*$. Thus, each committee of size k for $\mathcal{E}'$ has the diversity $D(\mathcal{E}, S^*)$. Hence, (D, s)-DSCR has a solution if and only if there is a committee of score at least s^*.
(2) The reduction from the problem of finding a committee in which each agent has a satisfaction of at least one, which is **NP**-hard [27], works analogously by giving each candidate a different label, setting δ to $D(\mathcal{E}, S^*)$ with S^* being a committee with k labels, and $h(a) = 1$ for all agents $a \in A$. □

As the decision problems of CC and PAV are **NP**-hard [4, 27], we have that $(D, \text{score}_{\text{CC}})$-DSCR and $(D, \text{score}_{\text{PAV}})$-DSCR are **NP**-hard, where D is one of the indices considered.

In the remainder of this section, we want to focus on separable scoring functions:

Definition 3. A scoring function s is called *separable* if there is a polynomial-time computable function w mapping an election and a candidate to $\mathbb{N}$ such that for every election $\mathcal{E}$ and for every $S \subseteq C$, we have that $s(\mathcal{E}, S) = \sum_{c \in S} w(\mathcal{E}, c)$.

Clearly, a committee of $\mathcal{R}^s$ can be computed in polynomial time if s is separable. The scoring function of AV is such a separable function with $w(\mathcal{E}, c) = |\{a \in A : c \in U(a)\}| \leq |A|$, and the scoring function of SAV can be transformed into a separable function with $w(\mathcal{E}, c) = \sum_{a \in A : c \in U(a)} (\ell / |U(a)|)$, where ℓ is the least common multiple of $\bigcup_{a \in A} \{|U(a)|\}$, which can be computed in polynomial time using binary representation. For separable functions, we have:

Theorem 3 (★). MAX-(D, s)-DSCR *is in* **P** *if s is a separable function and $D \in \{Ri, LC\}$.*

Therefore, for $D \in \{Ri, LC\}$, it holds that MAX-$(D, \text{score}_{\text{AV}})$-DSCR and MAX-$(D, \text{score}_{\text{SAV}})$-DSCR are in **P**. While the algorithm for Theorem 3 utilizes the lexicographic nature of *LC*, we show a result for a class of diversity indices which includes *Si* and *Sh* next:

Theorem 4. MAX-(D, s)-DSCR *is in* **P** *if*

- *s is a separable function and there is an $\alpha \in \mathbb{N}_0$ polynomial in the input size so that, for each $c \in C$, $w(\mathcal{E}, c) \leq \alpha$,*
- *the index D can be expressed as $\sum_{l \in [m]} \sum_{i=1}^{n_l} t(i)$ and, for all $i \in [k]$, it holds that $0 < t(i)$ and $t(i)$ can be computed in polynomial time, and t is strictly monotonically decreasing.*

Proof. Given an instance I_D of MAX-(D, s)-DSCR with an election $\mathcal{E}$ with n candidates, we construct an instance I_K of the 0-1 Knapsack problem in polynomial time. We assume that $\beta \leq k \cdot \alpha$ (β being the lower bound for the score, see Definition 2), since there is no solution for I_D otherwise. For each candidate c_i, we add an item x_i with the weight $w_K(x_i) \coloneqq n\alpha + 1 - w(\mathcal{E}, c_i)$ and the value $v(x_i) = t(\pi(c_i)) + \eta$, where $\eta \coloneqq k \cdot t(1) + 1$ and π outputs c_i's position in a descending ordering of the candidates with the same label as c_i based on w. The knapsack's bound is $B \coloneqq k(n\alpha + 1) - \beta$. Let, for a solution X of I_K, $S(X) = \{c_i \mid x_i \in X\}$, $v(X)$ and $w_K(X)$ the value and weight of X, and, for a solution S of I_D, $X(S) = \{x_i : c_i \in S\}$. We claim that I_K has a solution X with value at least $k \cdot \eta$ if and only if $S(X)$ is a solution to I_D and that I_D is infeasible otherwise:

Let X be a solution to I_K with value at least $k \cdot \eta$. It follows that $|X| \geq k$. Assume that $|X| > k$, then $w_K(X) \geq (k+1)(n\alpha + 1) - w(\mathcal{E}, S(X)) \geq (k+1)(n\alpha+1) - n\alpha = k(n\alpha+1) + 1 > B$, a contradiction. Thus, $|X| = k$ and $w_K(X) \leq B \Leftrightarrow k(n\alpha + 1) - w(\mathcal{E}, S(X)) \leq k(n\alpha + 1) - \beta \Leftrightarrow \beta \leq w(\mathcal{E}, S(X))$. Thus, $S(X)$ is of size exactly k and respects the score constraint. If X is a solution to I_K with value less than $k \cdot \eta$, then $|X| < k$. Since X is maximal, there is no size-k solution respecting the capacity constraints and hence no solution to I_D that respects the score constraint. Analogously, I_K has no solution with value at least $k \cdot \eta$ if I_D has none.

Finally, we show that, for an optimal solution X_K^* of I_K with value at least $k \cdot \eta$, $S(X_K^*)$ is an optimal solution of I_D. If, for a label l_j, n_j many items x_i with $\lambda(c_i) = l_j$ are chosen in X_K^*, the n_j items which come first in the order π are always chosen and thus $v(X_K^*) = D(\mathcal{E}, S(X_K^*)) + k \cdot \eta$. Next, assume that I_D has an optimal solution $S_D^* \neq S(X_K^*)$ with $D(\mathcal{E}, S_D^*) > D(\mathcal{E}, S(X_K^*))$. Consider the committee S^* with $n_l(\mathcal{E}, S^*) = n_l(\mathcal{E}, S_D^*)$ for $l \in [m]$ and $c \in S^* \Leftrightarrow \pi(c) \leq n_j(\mathcal{E}, S_D^*)$ with $l_j = \lambda(c)$. Thus, $D(\mathcal{E}, S^*) = D(\mathcal{E}, S_D^*) = v(X(S^*)) - k \cdot \eta > D(\mathcal{E}, S(X_K^*)) = v(X_K^*) - k \cdot \eta$, a contradiction.

Therefore, the problem can be solved, e.g. by the solving the 0-1 Knapsack instance with dynamic programming in $O(nB) = O(n\,(k(n\alpha + 1) - \beta))$ time. □

Si fulfills the condition in Theorem 4 by using $t(i) = 2k + 1 - 2i$, *Sh* through $t(i) = -i \log(i) + (i-1)\log(i-1) + \log(k) + 2$ with $t(1) = \log(k) + 2$. Thus, we have:

Corollary 1 (★). MAX-$(Si, \text{score}_{\text{AV}})$-DSCR *is in* **P** *and, if considering a computational model in which the logarithm of natural numbers and addition and multiplication including a logarithm can be computed in polynomial time,* MAX-$(Sh, \text{score}_{\text{AV}})$-DSCR *is in* **P**.

However, we cannot apply Theorem 4 for SAV with the previously mentioned approach to transform SAV's score into a separable function, as it leads to weights that could not be bounded by a value polynomial in the input size. Therefore, we show the following, which is also applicable to SAV:

Theorem 5 (★). (D, s)-DSCR *is in* **P** *if s is a separable function and D has the same properties as those in Theorem 4 and there is a $\zeta \in \mathbb{N}$ polynomial in the input size such that $0 < t(i) \leq \zeta$ for all $i \in [k]$.*

The proof of Theorem 5 is similar in nature to the proof of Theorem 4 by constructing an instance of the 0-1 knapsack problem, but the diversity constraint is expressed through the weights and the score constraint through the values of the items. While *Sh* does not fulfill the imposed conditions of Theorem 5—leaving the question open whether $(Sh, \text{score}_{\text{SAV}})$-DSCR is in **P**—the previously mentioned choice of $t(i)$ for *Si* fulfills them. Therefore:

Corollary 2. $(Si, \text{score}_{\text{SAV}})$-DSCR *is in* **P**.

5.2 Experiments

To evaluate the problems, we use datasets with approval preferences from Pabulib [14]—a collection of participatory budgeting data, a scenario in which incorporating diversity may be desirable—, in which categories (e.g. urban greenery) and/or targets (e.g. adults) are assigned to the candidates: For each such instance, we create up to three new instances of our model by assigning to a candidate as the label (1) the categories, (2) the targets, or (3) the union of the categories and targets (e.g. {urban greenery, adults} as a label and different sets forming different labels). We also transformed two datasets [20, 21] with approval preferences from PrefLib [24, 25] about the French presidential election in 2002, consisting of seven instances overall, by assigning the combination of gender and political leaning as the label to each candidate. The dimensions of the experimental data can be seen in Fig. 1. We removed instances with $|C| = m$ because each committee leads to the optimal diversity for them, as each diversity index considered satisfies *Uniqueness Optimality*. Here, we show results for $k = 10$ for which we discarded instances with $|C| \leq k$, leading to 687 instances. We conducted the same experiments for $k \in \{6, 8\}$ as well, shown in the paper's long version, which support the main takeaways presented here.

The code for creating the instances and running the experiments is published on GitHub [9].

Experimental Setup. To investigate MAX-(D, s)-DSCR, i.e. the influence of weakening the score constraint on the diversity reached, we consider score_{AV} and $\text{score}_{\text{SAV}}$. For a given diversity index, let $\mathcal{R}^{p}_{\text{scr}}$ be the rule returning the committees with the highest diversity among the committees reaching at least p% of the highest value of $\text{score}_{\mathcal{R}}$.

To examine MAX-D-DSAT, i.e. the influence of the satisfaction constraints, we additionally consider CC, PAV, the Method of Equal Shares (Rule X), and Phragmén's sequential rule (seq-Phragmén) (see [19] for definitions). For each rule $\mathcal{R}$ considered, we first compute one committee S using the Python library *abcvoting* [18] with

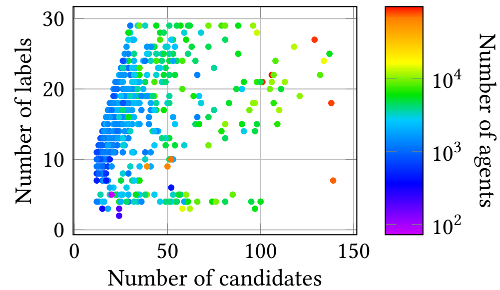

Figure 1: The dimensions of the experimental data, where the color of each point represents the average number of agents of all instances with the given number of labels and candidates.

default parameters and refer to the rule returning this committee as $\mathcal{R}$. Based on the satisfactions of the agents with S, we look at the rule that returns the committees with the highest diversity reachable when the satisfaction of each agent can be decreased by at most one, which we denote as $\mathcal{R}^{-1}_{\text{sat}}$.

We also compute all winning committees for these rules with *abcvoting* to investigate whether the diversity index could serve as a tiebreaker between them.

As the results for SAV, PAV, Rule X, and seq-Phragmén are very similar to the results for AV and due to lack of space, we focus on AV and CC here and show the results for the other rules in the paper's long version.

Experimental Results. We want to highlight the following observations, based on Fig. 2: On average, both AV and CC (without score or satisfaction constraints) achieve roughly 70% of the optimal diversity when measured with *Si* and 80% when using the other indices. Thus, the diversity can indeed be improved. Choosing the winning committee with the highest diversity rarely makes a difference for AV, which has multiple winning committees for only around 5% of the instances. In contrast, the diversity differs between the winning committees of CC—which has multiple winning committees for around 30% of the instances—for around 20% of the instances. Furthermore, CC achieves, even when choosing a winning committee with the smallest diversity, a slightly higher proportion of the optimal diversity than AV on average. Similarly, $\text{CC}^{-1}_{\text{sat}}$ rarely performs worse than $\text{AV}^{-1}_{\text{sat}}$ (which already reaches 78%–88% of the optimal diversity on average, depending of the index): $\text{CC}^{-1}_{\text{sat}}$ reaches a higher diversity than $\text{AV}^{-1}_{\text{sat}}$ for around 38–44% and at least the same diversity for around 83–87% of the instances (depending on the index).

We also compare $\text{AV}^{90}_{\text{scr}}$ with $\text{AV}^{-1}_{\text{sat}}$ and AV: On average, $\text{AV}^{90}_{\text{scr}}$ reaches 4–7% more of the optimal diversity than $\text{AV}^{-1}_{\text{sat}}$. $\text{AV}^{90}_{\text{scr}}$ can also lead to a noticeable change compared to AV: The percentage of the optimal diversity achieved on average increases by 12–19. It is also interesting that the gain in diversity is larger overall for $\text{AV}^{90}_{\text{scr}}$ compared to AV than for $\text{AV}^{80}_{\text{scr}}$ compared to $\text{AV}^{90}_{\text{scr}}$. The same applies for the gain from $\text{AV}^{90}_{\text{scr}}$ to $\text{AV}^{80}_{\text{scr}}$ compared to that from $\text{AV}^{80}_{\text{scr}}$ to $\text{AV}^{70}_{\text{scr}}$ (see the paper's long version for the plots), which suggests that there are diminishing returns when weakening the score constraint. In addition, for each diversity index, there are

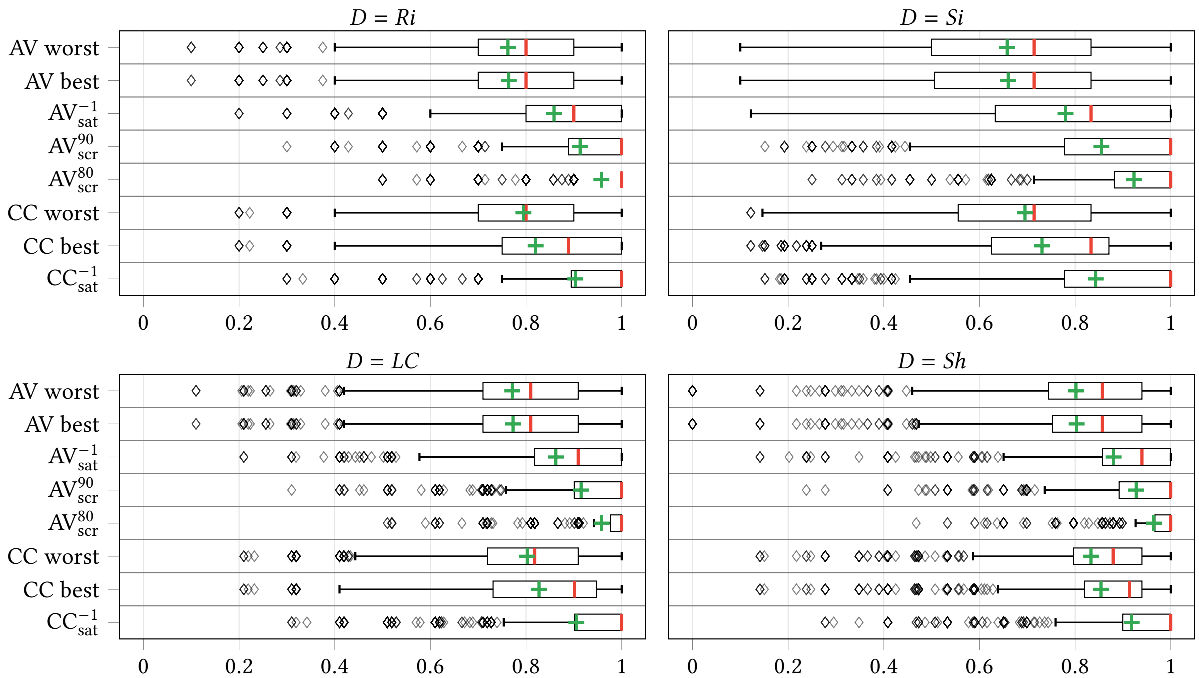

Figure 2: The proportion of the optimal diversity reached on the experimental data when using the specified diversity index D. "$\mathcal{R}$ best" ("$\mathcal{R}$ worst") refers to the rule choosing the committees with the highest (lowest) diversity among the winning committees of $\mathcal{R}$. The red line indicates the median, the green cross the mean.

instances for which even allowing a score reduction of 50% does not lead to the optimal diversity.

When comparing the four diversity indices, it seems most challenging to achieve the optimal diversity when using *Si* for each rule, as visualized in Fig. 2.

6 EPILOGUE

We adapted several diversity indices used in ecology to the context of committee elections and introduced a new diversity index. We also introduced properties of diversity indices which allow us to differentiate between any pair of indices and to characterize the new index via two of our properties. The underlying model assumes that each candidate has one label: While this allows to define a label as a set of "sub-labels" (e.g. {urban greenery, adults} as one label), all indices we consider do not take the (dis)similarity of labels into account. Further research could investigate diversity indices that incorporate such distances, which also requires thinking about how such distances are determined.

Furthermore, we treat all labels as equally important, which is in line with how the diversity indices considered treat different species in the literature. However, there are scenarios in which labels have different importance/weights. Our preliminary results for weighted labels, presented in the paper's long version, show that, after adapting the indices straight-forwardly to weighted labels, only the adapted *Si* index satisfies a desirable property that we define for the weighted scenario. In addition, we show an analogous result to Corollary 1, using the same technique as in Theorem 4 with only small adjustments. This shows that some of our results for the non-weighted scenario could be useful for a weighted scenario.

From an algorithmic point of view, we proved that (D, s)-DSCR is in **P** in some cases if s is a separable scoring function. This includes the score of AV and SAV for each diversity index considered apart from *Sh* in case of SAV—we left open whether (Sh, score_{SAV})-DSCR is polynomial-time solvable. However, there are other s for which we prove that (D, s)-DSCR is **NP**-hard, which is also the case for D-DSAT. Further work may study parameterized complexity or approximation algorithms for these problems.

Our experiments revealed interesting trade-offs between satisfaction/score guarantees and diversity, showing e.g. that the diversity of committees can indeed be improved. It would also be interesting to investigate how much the diversity indices differ on real world data or to evaluate past elections regarding their scoring-diversity performance. This could be particularly interesting in the context of participatory budgeting, which calls for an extension of our model in which the costs of the projects and the respective budget becomes the third objective (next to voter satisfaction and diversity).

Finally note that, while ecological indices provide rigorous measures of diversity, their practical interpretability varies, particularly in settings with high label cardinality. Our axiomatic characterization aims to enhance transparency, though systematic guidance for index selection in specific application domains remains an important direction for future work.

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REFERENCES

[1] Orhan Aygün and Inácio Bó. 2021. College Admission with Multidimensional Privileges: The Brazilian Affirmative Action Case. *American Economic Journal: Microeconomics* 13, 3 (2021), 1–28. https://doi.org/10.1257/mic.20170364

[2] Haris Aziz. 2019. A Rule for Committee Selection with Soft Diversity Constraints. *Group Decision and Negotiation* 28, 6 (2019), 1193–1200. https://doi.org/10.1007/s10726-019-09634-5

[3] Haris Aziz, Markus Brill, Vincent Conitzer, Edith Elkind, Rupert Freeman, and Toby Walsh. 2017. Justified representation in approval-based committee voting. *Social Choice and Welfare* 48, 2 (2017), 461–485. https://doi.org/10.1007/s00355-016-1019-3

[4] Haris Aziz, Serge Gaspers, Joachim Gudmundsson, Simon Mackenzie, Nicholas Mattei, and Toby Walsh. 2015. Computational Aspects of Multi-Winner Approval Voting. In *Proceedings of the 14th International Conference on Autonomous Agents and Multiagent Systems (AAMAS '15)*. IFAAMAS, 107–115. https://doi.org/10.65109/nhty2001

[5] Haris Aziz and Zhaohong Sun. 2025. Multi-rank smart reserves: A general framework for selection and matching diversity goals. *Artificial Intelligence* 339 (2025), 104274. https://doi.org/10.1016/j.artint.2024.104274

[6] Nawal Benabbou, Mithun Chakraborty, Xuan-Vinh Ho, Jakub Sliwinski, and Yair Zick. 2018. Diversity Constraints in Public Housing Allocation. In *Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS '18)*. IFAAMAS, 973–981. https://doi.org/10.65109/xvtn6533

[7] Péter Biró, Tamás Fleiner, Robert W. Irving, and David F. Manlove. 2010. The College Admissions problem with lower and common quotas. *Theoretical Computer Science* 411, 34 (2010), 3136–3153. https://doi.org/10.1016/j.tcs.2010.05.005

[8] Robert Bredereck, Piotr Faliszewski, Ayumi Igarashi, Martin Lackner, and Piotr Skowron. 2018. Multiwinner Elections with Diversity Constraints. In *Proceedings of the 32nd AAAI Conference on Artificial Intelligence (AAAI '18)*. AAAI Press, 933–940. https://doi.org/10.1609/aaai.v32i1.11457

[9] Paula Böhm, Robert Bredereck, and Till Fluschnik. 2026. diversity-indices-in-elections. Published on GitHub: https://github.com/paubo14/diversity-indices-in-elections.

[10] Paula Böhm, Robert Bredereck, and Till Fluschnik. 2026. Maximizing Index Diversity in Committee Elections. arXiv:2602.11400 [cs.GT] https://arxiv.org/abs/2602.11400

[11] L. Elisa Celis, Lingxiao Huang, and Nisheeth K. Vishnoi. 2018. Multiwinner Voting with Fairness Constraints. In *Proceedings of the 27th International Joint Conference on Artificial Intelligence (IJCAI '18)*. International Joint Conferences on Artificial Intelligence Organization, 144–151. https://doi.org/10.24963/ijcai.2018/20

[12] Mostapha Diss, Clinton Gubong Gassi, and Issofa Moyouwou. 2025. Combining Diversity and Excellence in Multiwinner Elections. *Group Decision and Negotiation* 34, 4 (2025), 683–713. https://doi.org/10.1007/s10726-025-09928-x

[13] Florian Evequoz, Johan Rochel, Vijay Keswani, and L. Elisa Celis. 2022. Diverse Representation via Computational Participatory Elections – Lessons from a Case Study. In *Proceedings of the 2nd ACM Conference on Equity and Access in Algorithms, Mechanisms, and Optimization (EAAMO '22)*. ACM, 1–11. https://doi.org/10.1145/3551624.3555297

[14] Piotr Faliszewski, Jarosław Flis, Dominik Peters, Grzegorz Pierczyński, Piotr Skowron, Dariusz Stolicki, Stanisław Szufa, and Nimrod Talmon. 2023. Participatory Budgeting: Data, Tools and Analysis. In *Proceedings of the 32nd International Joint Conference on Artificial Intelligence (IJCAI '23)*. International Joint Conferences on Artificial Intelligence Organization, 2667–2674. https://doi.org/10.24963/ijcai.2023/297

[15] Grzegorz Gawron and Piotr Faliszewski. 2024. Using multiwinner voting to search for movies. *Theory and Decision* (2024). https://doi.org/10.1007/s11238-024-10012-0

[16] Egor Ianovski. 2022. Electing a committee with dominance constraints. *Annals of Operations Research* 318, 2 (2022), 985–1000. https://doi.org/10.1007/s10479-021-04128-7

[17] Rani Izsak, Nimrod Talmon, and Gerhard Woeginger. 2018. Committee Selection with Intraclass and Interclass Synergies. In *Proceedings of the 32nd AAAI Conference on Artificial Intelligence (AAAI '18)*. AAAI Press, 1071–1078. https://doi.org/10.1609/aaai.v32i1.11479

[18] Martin Lackner, Peter Regner, and Benjamin Krenn. 2023. abcvoting: A Python package for approval-based multi-winner voting rules. *Journal of Open Source Software* 8, 81 (2023), 4880. https://doi.org/10.21105/joss.04880

[19] Martin Lackner and Piotr Skowron. 2023. *Multi-Winner Voting with Approval Preferences*. Springer. https://doi.org/10.1007/978-3-031-09016-5

[20] Jean-François Laslier and Karine Van der Straeten. 2004. Une expérience de vote par assentiment lors de l'élection présidentielle française de 2002. *Revue française de science politique* 54, 1 (2004), 99–130. https://doi.org/10.3917/rfsp.541.0099

[21] Jean-François Laslier and Karine Van der Straeten. 2008. A live experiment on approval voting. *Experimental Economics* 11, 1 (2008), 97–105. https://doi.org/10.1007/s10683-006-9149-6

[22] Tom Leinster. 2021. *Entropy and Diversity: The Axiomatic Approach*. Cambridge University Press. https://doi.org/10.1017/9781108963558

[23] Tom Leinster and Christina A. Cobbold. 2012. Measuring diversity: the importance of species similarity. *Ecology* 93, 3 (2012), 477–489. https://doi.org/10.1890/10-2402.1

[24] Nicholas Mattei and Toby Walsh. 2013. *PrefLib: A Library for Preferences http://www.preflib.org*. Springer, 259–270. https://doi.org/10.1007/978-3-642-41575-3_20

[25] Nicholas Mattei and Toby Walsh. 2017. A PrefLib.org Retrospective: Lessons Learned and New Directions. *Trends in Computational Social Choice. AI Access Foundation* (2017), 289–309.

[26] E.C. Pielou. 1975. *Ecological Diversity*. John Wiley & Sons.

[27] Ariel D. Procaccia, Jeffrey S. Rosenschein, and Aviv Zohar. 2008. On the complexity of achieving proportional representation. *Social Choice and Welfare* 30, 3 (2008), 353–362. https://doi.org/10.1007/s00355-007-0235-2

[28] Kunal Relia. 2022. DiRe Committee : Diversity and Representation Constraints in Multiwinner Elections. In *Proceedings of the 31st International Joint Conference on Artificial Intelligence (IJCAI '22)*. International Joint Conferences on Artificial Intelligence Organization, 5143–5149. https://doi.org/10.24963/ijcai.2022/714

[29] Claude E. Shannon. 1948. A mathematical theory of communication. *The Bell System Technical Journal* 27, 3 (1948), 379–423. https://doi.org/10.1002/j.1538-7305.1948.tb01338.x

[30] Edward H. Simpson. 1949. Measurement of Diversity. *Nature* 163, 4148 (1949), 688–688. https://doi.org/10.1038/163688a0

[31] Andrzej Straszak, Marek Libura, Jarostaw Sikorski, and Dariusz Wagner. 1993. Computer-assisted constrained approval voting. *Group Decision and Negotiation* 2, 4 (1993), 375–385. https://doi.org/10.1007/bf01384490

[32] Robert H. Whittaker. 1972. Evolution and Measurement of Species Diversity. *TAXON* 21, 2-3 (1972), 213–251. https://doi.org/10.2307/1218190