Epistemic Modal Logic Meets Algebraic Model Counting

Daxin Liu
National Key Laboratory for Novel Software Technology &
School of Artificial Intelligence,
Nanjing University
Nanjing, China
daxin.liu@nju.edu.cn

Vaishak Belle
School of Informatics
The University of Edinburgh
Edinburgh, United Kingdom
vbelle@ed.ac.uk

ABSTRACT

Algebraic model counting (AMC) generalizes weighted model counting to the semiring setting and supports various types of labels (or weights), including numerical ones as used in weighted model counting, but also sets, boolean formulae, polynomials, and many more. One merit of algebraic model counting is that it can be evaluated efficiently in a succinct form of representation by using knowledge compilation, hence, providing a robust and useful framework that covers many different tasks from a variety of different fields. In this paper, we show that a type of epistemic entailment reasoning in a modal logic of only knowing can be solved by recursively calling a series of algebraic model counting tasks. We show that this approach is not limited to the qualitative setting, but is also applicable to the quantitative modal logic of only-believing. Lastly, we propose an AMC-based regression operator that lifts the approach to even dynamic epistemic reasoning.

CCS CONCEPTS

• **Theory of computation → Modal and temporal logics**; • **Computing methodologies → Reasoning about belief and knowledge**.

KEYWORDS

Reasoning about knowledge and beliefs; Reasoning about probability; Weight model counting; Knowledge compilation

ACM Reference Format:
Daxin Liu and Vaishak Belle. 2026. Epistemic Modal Logic Meets Algebraic Model Counting. In *Proc. of the 25th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2026), Paphos, Cyprus, May 25 – 29, 2026*, IFAAMAS, 10 pages. https://doi.org/10.65109/MSKZ1140

1 INTRODUCTION

Epistemic logic is a growing and important area in philosophy, computer science, game theory, and artificial intelligence. Among other applications, epistemic formalisms play a key role in robot programming, agent-oriented design, and automated planning. In recent years, epistemic notions have become increasingly prevalent in user modeling for explanation and planning, particularly in counterfactual and contrastive plan construction [3], where the popular logic S5 and its variants play a central role [11, 48]. However, as is well known, reasoning (i.e., validity checking) in epistemic logic is computationally hard. For instance, the complexity of reasoning about knowledge in S5, where knowing a proposition also implies its truth, is NP-complete in the single-agent case and PSPACE-complete in the multi-agent case (see Section 3.6 in [22]). If one models the beliefs of a knowledge base (KB) using notions such as minimal knowledge [28] or only-knowing [35], the complexity increases further: even in the single-agent case, it reaches the second level of the polynomial hierarchy [45].

Putting complexity considerations aside, which often reflect worst-case scenarios, there has been relatively little work on how epistemic reasoning can be carried out effectively in practice, particularly in complex agent settings where agents may need to sense, reason about their knowledge, and plan under conditions of their ignorance. Interestingly, the area of knowledge compilation has emerged as a powerful approach for solving various reasoning problems efficiently [10]. The core idea is to transform a propositional language into a data structure that factorizes Boolean connectives. For instance, in the widely used decomposable deterministic negation normal form (d-DNNF), branches of OR connectives are mutually exclusive, and branches of AND connectives share no variables. This structural property allows for local reasoning, enabling us to determine satisfiability or even count the number of satisfying assignments in time linear to the size of the compiled data structure, often referred to as a *circuit*. While constructing such a circuit may incur exponential cost, this is a one-time offline effort, after which evaluating multiple queries against the circuit becomes highly efficient. As demonstrated in systems like ProbLog [43], this mechanism proves to be extremely effective when a knowledge base is available and multiple queries must be evaluated.

Naturally, this raises the question of whether the benefits of knowledge compilation can be extended to epistemic logic. An early influential attempt by Bienvenu et al. [10] applied knowledge compilation directly to the S5 logic. Their findings show that for a very restricted fragment—where disjunctions and negations appear in highly constrained forms (in fact, disjunctions are almost entirely disallowed)—knowledge compilation is feasible and leads to tractable epistemic reasoning, as further demonstrated in [31]. However, this fragment is extremely limited in expressivity. In this paper, we ask whether more expressive fragments of epistemic propositional logic can also benefit from knowledge compilation.

In particular, the framework of algebraic model counting (AMC) [30] offers a promising path forward. AMC generalizes model counting by allowing computations over arbitrary semirings. Once a suitable data structure for Boolean formulas is constructed, AMC enables a range of reasoning tasks—such as SAT solving [1], gradient computation [21, 29], and fuzzy reasoning [26]—to be performed

Proc. of the 25th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2026), C. Amato, L. Dennis, V. Mascardi, J. Thangarajah (eds.), May 25 – 29, 2026, Paphos, Cyprus. © 2026 International Foundation for Autonomous Agents and Multiagent Systems (www.ifaamas.org). https://doi.org/10.65109/MSKZ1140

by merely changing the semiring's elements and operations. Thus, if we were able to establish a strong link between epistemic reasoning and algebraic model counting, it might open up a new solution for epistemic reasoning that might extend itself to not just reasoning about epistemic queries, but potentially probabilistic and/or dynamic reasoning.

In this paper, we show that a successful synthesis between AMC and epistemic reasoning is possible. In particular, we examine epistemic reasoning by AMC in a modal logic of only knowing. Only knowing semantically captures all that the agent knows and allows us to compactly represent the agent's knowledge and ignorance. The way we leverage and relate epistemic logic, specifically propositional *only-knowing logic*, to algebraic model counting is by appealing to a result called the representation theorem [36], a theorem that connects epistemic reasoning and first-order reasoning. We show that, similar to the representation theorem, epistemic reasoning can be solved by recursively calling a series of algebraic model counting tasks, and thereby, we get a strong correspondence between epistemic reasoning and algebraic model counting. Going further, for the very first time, we are able to introduce and relate knowledge compilation for probabilistic epistemic logic. Specifically, we consider the propositional fragment of only knowing labeled with probability distributions as introduced in [8]. We show that under some assumptions about the knowledge base, one can reduce the probabilistic epistemic reasoning task, which might involve computing probabilistic beliefs (even nested beliefs), also to AMC in terms of so-called weighted model counting (WMC), an instance of AMC obtained by applying it to a specific semiring. Lastly, as it is well-known that reasoning about actions in the situation calculus can be reduced to classical first-order reasoning by means of regression [44], we propose an AMC-based regression operator that is capable of handling dynamic probabilistic epistemic reasoning in a probabilistic modal variant of the situation calculus.

The remainder of the paper is organized as follows. In the following section, we review the formal definition of algebraic model counting and its relation to variants of tasks in different fields. In Section 3, we show how algebraic model counting can be applied to perform epistemic reasoning in a propositional modal logic of only knowing. In Section 4, we lift the approach to a probabilistic setting: we show how to use it to connect algebraic model counting with probabilistic epistemic reasoning. Section 5 presents the AMC-based regression operator in dynamic epistemic reasoning. Finally, we discuss related works and conclude in Section 6.

2 PRELIMINARIES

We assume readers know propositional and first-order logic. Here, we review the task of algebraic model counting and briefly review its relation to other tasks in different fields. The underlying mathematical structure is that of a commutative semiring.

Definition 2.1 ((Commutative) Semiring). A *semiring* is a structure $(\mathcal{A}, \oplus, \otimes, e^{\oplus}, e^{\otimes})$, where addition $\oplus$ and multiplication $\otimes$ are associative binary operations over the set $\mathcal{A}$, $\oplus$ is commutative, $\otimes$ distributes over $\oplus$, $e^{\oplus} \in \mathcal{A}$ is the neutral element of $\oplus$, $e^{\otimes} \in \mathcal{A}$ that of $\otimes$, and for all $a \in \mathcal{A}$, $e^{\oplus} \otimes a = a \otimes e^{\oplus} = e^{\oplus}$. For commutative semirings, $\otimes$ is commutative as well.

A *literal* l is a *proposition* (i.e., *binary random variable* in a probabilistic setting) p or the negation of it. A *propositional formula* is a proposition or an expression constructed by propositions and connectives $\wedge, \vee, \neg$. A propositional *theory* T is a set of propositional formulas.

A model of theory T is a *truth assignment to propositions* in T. Syntactically, a model of theory T can be represented by a maximally consistent set of literals from T that satisfies every formula in T. The task of algebraic model counting is now defined as:

Definition 2.2 (AMC Problem). Given

- a propositional logic theory T over a set of atomic propositions Ap;
- a commutative semiring $(\mathcal{A}, \oplus, \otimes, e^{\oplus}, e^{\otimes})$, and
- a labeling function $f : \mathcal{L} \mapsto \mathcal{A}$ mapping literals $\mathcal{L}$ of the propositions in Ap to elements of the semiring set $\mathcal{A}$,

compute

$$\mathrm{A}(T) = \bigoplus_{I \in \mathcal{M}(T)} \bigotimes_{l \in I} f(l), \tag{1}$$

where $\mathcal{M}(T)$ is the set of models of T; if $\mathcal{M}(T) = \emptyset, \mathrm{A}(T) = e^{\oplus}$.

Kimmig et al. [30] showed that many computation tasks are essentially instances of algebraic model counting. The most relevant results of this paper are the instances of SAT and Prob. The former is to decide if a propositional logic theory T over a set of atomic propositions Ap is satisfiable, while the latter is to compute the marginal probability of event T over the joint probability distribution of finite mutually independent binary random variables Ap.

Theorem 2.3 ([30]). *The AMC over semiring $(\mathcal{A}, \oplus, \otimes, e^{\oplus}, e^{\otimes})$ solve the SAT and Prob task respectively, if $(\mathcal{A}, \oplus, \otimes, e^{\oplus}, e^{\otimes})$ are given as*

- $\mathcal{A} = \{\top, \bot\}$, $\oplus = \vee$, $\otimes = \wedge$, $e^{\oplus} = \bot$, $e^{\otimes} = \top$, *and labeling function* $f(l) = \top$ *for all literals; and*
- $\mathcal{A} = \mathbb{Q}^{\geq 0}$, $\oplus = +$, $\otimes = \times$, $e^{\oplus} = 0$, $e^{\otimes} = 1$, *and labeling function* $f(l)$ *for all literals* l *given by*

  $$f(l) = \begin{cases} p_l & l \text{ is positive} \\ 1 - p_{\neg l} & l \text{ is negative} \end{cases},$$

  where p_l *is the probability of literal* l, $\neg l$ *refers to the atom of* l *for negative* l.

In fact, for the SAT problem, we have T is satisfiable iff $\mathrm{A}(T) = \top$ for the respective semiring and labeling function. Therefore, the propositional entailment problem of deciding if $T \models \phi$ for a propositional ϕ can be solved by checking if $\mathrm{A}(T \wedge \neg\phi) = \bot$. Kimming et al. [30] lists more instances of AMC, including model counting #SAT, weighted model counting, *sensitivity analysis*, *calculating gradient* in machine learning [21, 29], and *finding shortest- and widest-paths* in networks [2], etc, making AMC a versatile abstract computation framework.

3 EPISTEMIC REASONING IN THE LOGIC $\mathcal{OL}$

In this section, we investigate how we can use the algebraic model counting to perform epistemic reasoning. Only-knowing was introduced by Levesque [35] to precisely capture the beliefs of a knowledge base (KB). Levesque proposed the logic $\mathcal{OL}$, where the notion of only-knowing is captured by a modal operator $\boldsymbol{O}$. The logic is a classical (monotonic) logic that captures non-monotonic reasoning mechanisms, such as default extensions via autoepistemic logic [41]. In what follows, we recall the logic $\mathcal{OL}$. The original proposal of $\mathcal{OL}$ is first-order, hence entailment reasoning in general is undecidable; therefore, we focus on its decidable propositional fragment.

The language is identical to propositional logic except that we now have two additional epistemic modal operators $\boldsymbol{K}, \boldsymbol{O}$ to construct formulas. Namely, the vocabulary includes a fixed finite set of propositions Ap, connectives $\wedge, \vee, \neg$, modalities $\boldsymbol{K}, \boldsymbol{O}$, and parentheses. We treat $\supset$ and $\equiv$ as abbreviations. $\boldsymbol{K}\phi$ is intended to be read as "ϕ is known by the agent" while $\boldsymbol{O}\phi$ can be read as "all that is known by the agent is ϕ" and it is intended to capture the maximal knowledge of an agent. A formula is called *objective* if it contains no $\boldsymbol{K}$ and $\boldsymbol{O}$. A formula is called *subjective* if no propositions occur outside $\boldsymbol{K}$ or $\boldsymbol{O}$. A formula is called *basic* if it mentions no $\boldsymbol{O}$. For example, $holdingA \supset \neg holdingB$ is objective; $\boldsymbol{K}(holdingA \supset \neg holdingB)$ and $\boldsymbol{O}(holdingA)$ are subjective with $\boldsymbol{K}(holdingA \supset \neg holdingB)$ being basic; $holdingA \wedge \boldsymbol{K}(holdingA \supset \neg holdingB)$ is neither objective nor subjective.

3.1 Semantics of $\mathcal{OL}$

The semantics is based on possible worlds, and a world w is a mapping from propositions to the truth values, i.e., $w : \text{Ap} \mapsto \{0, 1\}$. Let $\mathcal{W}$ be the set of all such worlds. Essentially, a world w is a subset of Ap and $\mathcal{W} = 2^{\text{Ap}}$. To specify what is known by an agent, we use a set of worlds to model the agent's *epistemic state* $e \subseteq \mathcal{W}$, namely, the agent considers all worlds in the epistemic state e to be possible. Moreover, ϕ is *known* if and only if ϕ holds for *all* worlds in the epistemic state, and ϕ is *only known* if and only if the epistemic state is *just* the set of worlds where ϕ holds. By a model, we mean a pair (e, w). Formally, let ϕ and ψ be any formula, $p \in \text{Ap}$ be a proposition. *Truth* of formulas is given as:

- $e, w \models p$ iff $w[p] = 1$;
- $e, w \models \phi \wedge \psi$ iff $e, w \models \phi$ and $e, w \models \psi$;
- $e, w \models \neg\phi$ iff $e, w \not\models \phi$;
- $e, w \models \boldsymbol{K}\phi$ iff for all $w' \in \mathcal{W}, w' \in e \Rightarrow e, w' \models \phi$;
- $e, w \models \boldsymbol{O}\phi$ iff for all $w' \in \mathcal{W}, w' \in e \Leftrightarrow e, w' \models \phi$.[1]

For a set of formulas Σ, we also understand Σ as the conjunction of all formulas in it. For a set of formulas Σ and formula ϕ, we write $\Sigma \models \phi$, read as "Σ logically *entails* ϕ", to mean for every model (e, w) such that $e, w \models \Sigma$, we have $e, w \models \phi$. We say a formula ϕ is *satisfiable* if there exists a model (e, w) such that $e, w \models \phi$, and ϕ is *valid* if $\{\} \models \phi$.

The logic $\mathcal{OL}$ is *weak S5* when only considering $\boldsymbol{K}$, aka K45. Namely, $\boldsymbol{K}\phi \wedge \boldsymbol{K}(\phi \supset \psi) \supset \boldsymbol{K}\psi$, $\boldsymbol{K}\phi \supset \boldsymbol{K}\boldsymbol{K}\phi$, and $\neg\boldsymbol{K}\phi \supset \boldsymbol{K}(\neg\boldsymbol{K}\phi)$ are valid for all ϕ and ψ. If one only considers models (e, w) such that $w \in e$, then one gets indeed S5, namely, $\boldsymbol{K}\phi \supset \phi$ becomes valid.

The modality $\boldsymbol{O}$ enjoys the expected properties of only-knowing: formula $\boldsymbol{O}\phi \supset \boldsymbol{K}\phi$ and $\boldsymbol{O}\phi \supset \neg\boldsymbol{K}\psi$ are valid if ψ contains propositions that are not in ϕ. Namely, only-knowing entails knowing, and only-knowing something entails not knowing others. For example,

- $\boldsymbol{O}(holdingA \vee holdingB) \models \boldsymbol{K}(holdingA \vee holdingB)$; and
- $\boldsymbol{O}(holdingA \vee holdingB) \models \neg\boldsymbol{K}\, holdingC$.

where *holdingA, holdingB, holdingC* are distinct propositions. That is, only-knowing holding A or B entails that the agent knows it is holding A or B and does not know that it is holding C. The latter is because only-knowing holding A or B means the agent has no knowledge about holding C. Other properties include that: all the known facts that follow from only-knowing an objective theory are just those entailed by the theory. Namely,

Proposition 3.1. *If ϕ, ψ are objective, $\models \boldsymbol{O}\phi \supset \boldsymbol{K}\psi$ iff $\phi \models \psi$.*

3.2 Epistemic Reasoning by AMC in $\mathcal{OL}$

Since the only-knowing modality $\boldsymbol{O}$ captures both knowledge of the agent and its ignorance, specifying the agent's knowledge base by $\boldsymbol{O}$ is much more compact and desirable than specifying the agent's knowledge and ignorance by a long list of formulas of the form $\boldsymbol{K}\phi_1, \boldsymbol{K}\phi_2, \dots \neg\boldsymbol{K}\psi_1, \neg\boldsymbol{K}\psi_2, \dots$. Now that we have a way to specify the knowledge base (KB) of the agent, to perform knowledge-based decision making [38, 40] or epistemic planning [4], one still needs to solve the important task of epistemic reasoning, namely, inferring logical consequences of the KB. In other words, given a knowledge base KB specified by only knowing an objective theory and a subjective query ϕ, the epistemic reasoning task asks if ϕ is entailed by the KB. More precisely,

Definition 3.2 (Epistemic Reasoning). Given a set of objective formulas Σ and a subjective basic query ϕ, the epistemic reasoning task decides if $\boldsymbol{O}\Sigma \models \phi$.

The above reasoning task is fundamentally important if one wishes to develop a cognitive agent using epistemic logic. Answering this problem might not be easy, as the query ϕ might contain nested $\boldsymbol{K}$; this is in contrast to Prop. 3.1, where ψ is objective only. To handle the epistemic reasoning, we propose an operator $\|\cdot\|^{\mathbf{A}}_{\Sigma}$ that recursively calls AMC task over the boolean semiring, i.e., $\mathcal{A} = \{\top, \bot\}$, $\oplus = \vee$, $\otimes = \wedge$, $e^{\oplus} = \bot$, $e^{\otimes} = \top$, and labeling function $f(l) = \top$. We use True to represent tautologies like $p \vee \neg p$, and False for its negation. Formally, we have

Definition 3.3. Given an objective theory Σ and a basic formula ϕ as input, $\|\phi\|^{\mathbf{A}}_{\Sigma}$ is defined by

- $\|\phi\|^{\mathbf{A}}_{\Sigma} := \phi$ if ϕ is an objective formula;
- $\|\neg\phi\|^{\mathbf{A}}_{\Sigma} := \neg\|\phi\|^{\mathbf{A}}_{\Sigma}$;
- $\|\phi \wedge \psi\|^{\mathbf{A}}_{\Sigma} := \|\phi\|^{\mathbf{A}}_{\Sigma} \wedge \|\psi\|^{\mathbf{A}}_{\Sigma}$;
- $\|\boldsymbol{K}(\phi)\|^{\mathbf{A}}_{\Sigma} := \begin{cases} \text{True} & \mathbf{A}(\Sigma \wedge \neg\|\phi\|^{\mathbf{A}}_{\Sigma}) = \bot \\ \text{False} & \mathbf{A}(\Sigma \wedge \neg\|\phi\|^{\mathbf{A}}_{\Sigma}) = \top \end{cases}$.

Theorem 3.4. *Given a finite set of objective formulas Σ and a basic formula ϕ, $\boldsymbol{O}\Sigma \models \phi$ iff $\mathbf{A}(\Sigma \wedge \neg\|\phi\|^{\mathbf{A}}_{\Sigma}) = \bot$.*

The proof is based on Theorem 2.3 and Prop. 3.1 and is by induction on the structure of ϕ. The theorem suggests that there is a symbolic procedure, i.e., by recursively calling a series of algebraic model counting tasks, actually to solve the epistemic reasoning problem in $\mathcal{OL}$.

Example 3.5. Suppose we have that:

$$\Sigma := holdingA \vee holdingB$$

$$\phi := \boldsymbol{K}(\neg\boldsymbol{K}(holdingA) \wedge \neg\boldsymbol{K}(holdingB)),$$

---

[1] Note that in case ϕ is objective, the epistemic state e that satisfies $\boldsymbol{O}\phi$ is unique, i.e., e is the set of assignments where ϕ holds. Yet, in general, where ϕ might contain $\boldsymbol{K}$ or $\boldsymbol{O}$, such e might not be unique [36].

it holds that $\boldsymbol{O}\Sigma \models \phi$. This can be proved as follows. It is not hard to see that $\|\boldsymbol{K}(holdingA)\|_{\Sigma}^{\mathbf{A}} = \textsc{False}$ as $\mathbf{A}(\Sigma \wedge \neg holdingA) = \top$ (there exists a model $\mathcal{M} = \{\neg holdingA, holdingB\}$ such that $\mathcal{M} \models \Sigma \wedge \neg holdingA$). Likewise $\|\boldsymbol{K}(holdingB)\|_{\Sigma}^{\mathbf{A}} = \textsc{False}$ and $\|\phi\|_{\Sigma}^{\mathbf{A}} = \|\boldsymbol{K}\textsc{True}\|_{\Sigma}^{\mathbf{A}} = \textsc{True}$. Hence $\mathbf{A}(\Sigma \wedge \neg\|\phi\|_{\Sigma}^{\mathbf{A}}) = \mathbf{A}(\textsc{False}) = \bot$. By Theorem 3.4, $\boldsymbol{O}\Sigma \models \phi$.

One of the implications of Theorem 3.4 is on the complexity of the epistemic reasoning in Definition 3.2.

Corollary 3.6. *For queries with constant size, the time complexity of the epistemic reasoning task in Definition 3.2 is in* $P^{NP[O(1)]}$.

This is because that checking $\boldsymbol{O}\Sigma \models \phi$ reduces to check the entailment of $\Sigma \models \|\phi\|_{\Sigma}^{\mathbf{A}}$ by Theorem 3.4. The operator $\|\cdot\|_{\Sigma}^{\mathbf{A}}$ is a number of constant time reductions and ultimately makes a constant number of calls of propositional entailment checking $\Sigma \models \phi'$, where ϕ' is propositional, hence the time complexity is in $P^{NP[O(1)]}$. This is in contrast to the general propositional entailment reasoning in the logic $\mathcal{OL}$ [45] whose complexity is Σ_2^P-complete, i.e., NP^{NP}-complete.

Besides, the reduction theorem has practical implications as well. The hardness of entailment problems in Def. 3.2 depends on both the size of theory Σ and query ϕ. Typically, the size of the query ϕ is much smaller than the theory Σ, hence the overall complexity is dominated by the size of Σ. By virtue of the knowledge compilation for AMC, it is now possible to compile the theory Σ into circuits once, which might be an exponential cost in time, and then evaluate all future queries ϕ against this circuit in polynomial time [30], hence avoiding repeatedly NP computation for every query.

A subtlety here is that, to solve the epistemic reasoning task in Definition 3.2, we resort to (algebraic) model counting, yet, essentially, what is needed is to perform propositional entailment checking of the form $T \models \phi'$, where ϕ' is propositional. As the task of model counting is usually more difficult than propositional entailment checking, resolving to model counting in Theorem 3.4 seems like overkill. So what are the benefits of resorting to (algebraic) model counting instead of directly applying propositional entailment checking? There are at least two benefits. First, as we will see in Section 4 and Section 5, (algebraic) model counting makes it possible to have a uniform framework to perform both qualitative and quantitative epistemic reasoning. This additionally allows us to reuse the evaluation algorithm as Algorithm 1 in [30] to perform algebraic model counting for various semirings and various types of circuits, including both d-DNNF and *ordered binary decision diagrams* (OBDD), which can be achieved by simply parameterizing the algorithm with the respective semiring. Another benefit is that it enables us to compile the theory T in more succinct types of circuits. A consensus in the knowledge compilation community is that when choosing the type of circuits to compile a theory, one has to trade off the succinctness of the circuit and the tractability in reasoning for variant tasks, see [16] for a summary on this. For example, OBDDs are typically more tractable in propositional entailment checking than d-DNNFs, yet d-DNNFs are typically much more succinct than OBDDs with similar tractability in model counting. Hence, solving the epistemic reasoning task by model counting makes it possible to use compact circuit representations in compiling while maintaining tractability in reasoning.

4 PROBABILISTIC EPISTEMIC REASONING IN THE LOGIC $\mathcal{OBL}$

Having demonstrated the usefulness of AMC in epistemic reasoning for the only-knowing logic $\mathcal{OL}$, in this section, we show that AMC is also valuable in epistemic reasoning for the quantitative only-believing logic $\mathcal{OBL}$ [8]. The logic $\mathcal{OBL}$ is a first-order modal logic of beliefs and only-believing, which can be viewed as a probabilistic extension of $\mathcal{OL}$. In what follows, we first review the logic $\mathcal{OBL}$, thereafter, we show how the previous result of epistemic reasoning can be lifted to a quantitative setting.

While the logic $\mathcal{OBL}$ is first-order, we only consider its propositional fragment. The language includes the usual ingredients of propositional logic, like propositions and connectives. Besides, $\mathcal{OBL}$ has two binary modal operators: $\boldsymbol{B}(\phi\colon t)$ is to be read as "ϕ is believed with a probability t" where t is a rational number. $\boldsymbol{O}(\phi_1\colon t_1, \ldots, \phi_k\colon t_k)$ is to be read as "all that is believed is: ϕ_1 with probability r_1, …, ϕ_k with probability r_k", here ϕ_k mentions no modalities.

4.1 Semantics of $\mathcal{OBL}$

Although focusing on the propositional fragments of $\mathcal{OBL}$ allows us to simplify its semantics, we still need *rational numbers* $\mathbb{Q}$ to talk about probabilities. Now, the vocabulary of $\mathcal{OBL}$ is expanded from $\mathcal{OL}$ to include $\mathbb{Q}$, i.e., the set of rational numbers. [2] Let $\mathbb{Q}_{[0,1]} = \{q \in \mathbb{Q} \mid 0 \leq q \leq 1\}$. *Formulas* are now given by:

$$p \mid \phi \wedge \psi \mid \neg\phi \mid \boldsymbol{B}(\phi\colon t) \mid \boldsymbol{O}(\phi_1\colon t_1, \ldots \phi_k\colon t_k)$$

where $p \in$ Ap are propositions, $t, t_i \in \mathbb{Q}_{[0,1]}$ $(1 \leq i \leq k)$ are number constants; ϕ, ψ, ϕ_i are formulas.

In semantics, the notion of worlds is the same as before, *world* w maps Ap to $\{0, 1\}$. To account for probability, the agent's *epistemic state* e is modeled as a set of *distributions* d over possible worlds, i.e., $d : \mathcal{W} \mapsto \mathbb{Q}$ where $\mathcal{W}$ is the set of all worlds. We will only focus on probability distributions, namely, only consider those d that are *normalized* and satisfy *countable additivity.* Let $\mathcal{D}$ be the set of all such distributions. By a model, we mean a pair (e, w). Now, the truth in $\mathcal{OBL}$ is given by (assuming $p \in$ Ap, $t, t_i \in \mathbb{Q}_{[0,1]}$ for $1 \leq i \leq k$, and ϕ, ψ are formulas):

- $e, w \models p$ iff $w[p] = 1$;
- $e, w \models \phi \wedge \psi$ iff $e, w \models \phi$ and $e, w \models \psi$;
- $e, w \models \neg\phi$ iff $e, w \not\models \phi$;
- $e, w \models \boldsymbol{B}(\phi\colon t)$ iff $\forall d \in \mathcal{D}.d \in e \Rightarrow d(\mathcal{W}_{\phi}^{e}) = t$;
- $e, w \models \boldsymbol{O}(\phi_1\colon t_1, \ldots \phi_k\colon t_k)$ iff $\forall d \in \mathcal{D}.d \in e \Leftrightarrow d(\mathcal{W}_{\phi_1}^{e}) = t_1, \ldots d(\mathcal{W}_{\phi_k}^{e}) = t_k$;

where $\mathcal{W}_{\phi}^{e}$ is defined by $\mathcal{W}_{\phi}^{e} := \{w' \in \mathcal{W} \mid e, w' \models \phi\}$.

That is, the semantics of the non-modal part is exactly the same as in $\mathcal{OL}$; an epistemic state e satisfies $\boldsymbol{B}(\phi\colon t)$ if and only if in all the distributions of e, the probability of ϕ is t (w is irrelevant for $\boldsymbol{B}$ and $\boldsymbol{O}$); e satisfies $\boldsymbol{O}(\phi_1\colon t_1, \ldots \phi_k\colon t_k)$ if and only if e is exactly the set of distribution that ϕ_i has probability t_i for $1 \leq i \leq k$.

The notions of objective, subjective, and basic formulas carry naturally to $\mathcal{OBL}$. Logical entailment, satisfiability, and validity are also defined similarly to $\mathcal{OL}$.

---

[2] We use rationals as constants in the language whose interpretations are fixed and follow the natural sense. Technically, this can be done by featuring the domains with the so-called *rigid designators* or *standard names* [36].

It turns out that the logic $\mathcal{OBL}$ is still *weak S5*, that is, if we define $\boldsymbol{K}\phi$ as $\boldsymbol{B}(\phi\colon 1)$ and read it as "ϕ is known", then we have both positive and negative introspection of knowledge, besides, the following holds as well.

- $\models \boldsymbol{K}\phi \land \boldsymbol{K}(\phi \supset \psi) \supset \boldsymbol{K}\psi$;
- $\models \boldsymbol{B}(\phi\colon t) \supset \boldsymbol{KB}(\phi\colon t)$;
- $\models \neg\boldsymbol{B}(\phi\colon t) \supset \boldsymbol{K}\neg\boldsymbol{B}(\phi\colon t)$ for any t.

$\boldsymbol{O}$ inherits the properties of only-believing. For example,

- $\boldsymbol{O}(\mathit{holdingA}\colon 0.8) \models \boldsymbol{B}(\mathit{holdingA}\colon 0.8)$;
- $\boldsymbol{O}(\mathit{holdingA}\colon 0.8) \models \neg\boldsymbol{B}(\mathit{holdingB}\colon 0.6)$.

for distinct proposition *holdingA* and *holdingB*.
In fact, $\boldsymbol{O}(\mathit{holdingA}\colon 0.8) \models \neg\boldsymbol{B}(\mathit{holdingB}\colon t)$ for any number t.

4.2 Probabilistic Epistemic Reasoning by AMC in $\mathcal{OBL}$

Much like the situation in $\mathcal{OL}$, a fundamental task in $\mathcal{OBL}$ is to do epistemic reasoning. Namely, given a probabilistic knowledge base (PKB) specified by only believing and a belief query, we need to decide if the query is entailed by the knowledge base. More formally, we are interested in the following form of probabilistic epistemic reasoning:

Definition 4.1 (Probabilistic Epistemic Reasoning). Given a PKB := $\boldsymbol{O}(p_1\colon t_1, \dots, p_k\colon t_k, -)$, and subjective basic formula ϕ, the probabilistic epistemic reasoning task is to decide if PKB $\models \phi$ where "$-$" denotes the concatenation of expressions such as "$p_i \land p_j\colon t_i \times t_j$", indicating that propositions $p_1, \dots, p_k$ are *mutually independent*.

Essentially, the PKB asserts that the agent believes only a joint distribution over independent atomic variables $p_1, \dots, p_k$ with probabilities $t_1, \dots, t_k$. The reason we consider this special type of probabilistic knowledge bases is that reasoning in such PKBs has a direct correspondence with AMC (as soon below), as in AMC, weights are usually assigned to literals only.

Now, we show how probabilistic epistemic entailment reasoning against such PKBs can be solved by AMC.

Definition 4.2 (AMC Prob.). Given a probabilistic knowledge base PKB := $\boldsymbol{O}(p_1\colon t_1, \dots, p_k\colon t_k, -)$ and an objective formula ϕ, we define $\mathrm{A}^{\mathrm{PKB}}(\phi)$ as the algebraic model counting task over the semiring $\mathcal{A} = (\mathbb{Q}^{\geq 0}, +, \times, 0, 1)$ and the labeling function $f(l)$ with

$$f(l) = \begin{cases} t_i & l \text{ is equivalent to } p_i \text{ for some } i \\ 1 - t_i & \neg l \text{ is equivalent to } p_i \text{ for some } i \end{cases}.$$

Proposition 4.3. *Given a probabilistic knowledge base* PKB := $\boldsymbol{O}(p_1\colon t_1, \dots, p_k\colon t_k, -)$ *and an objective formula* ϕ, *let* $\mathrm{A}^{\mathrm{PKB}}(\phi)$ *as above, then for any* $t \in \mathbb{Q}_{[0,1]}$,

$$\text{PKB} \models \boldsymbol{B}(\phi\colon t) \text{ iff } \mathrm{A}^{\mathrm{PKB}}(\phi) = t$$

The result is a direct consequence of Theorem 2.3 and the fact that the $\boldsymbol{B}$ modality is indeed a probability over possible worlds.

Now, we can define the operator $\|\cdot\|^{\mathrm{A}}_{\mathrm{PKB}}$ to solve the probabilistic epistemic reason problem just as what we did in the logic $\mathcal{OL}$.

Definition 4.4. Given a PKB := $\boldsymbol{O}(p_1\colon t_1, \dots, p_k\colon t_k, -)$ and a basic formula ϕ, $\|\phi\|^{\mathrm{A}}_{\mathrm{PKB}}$ is defined by

- $\|\phi\|^{\mathrm{A}}_{\mathrm{PKB}} := \phi$ if ϕ is an objective formula;
- $\|\neg\phi\|^{\mathrm{A}}_{\mathrm{PKB}} := \neg\|\phi\|^{\mathrm{A}}_{\mathrm{PKB}}$;
- $\|\phi \land \psi\|^{\mathrm{A}}_{\mathrm{PKB}} := \|\phi\|^{\mathrm{A}}_{\mathrm{PKB}} \land \|\psi\|^{\mathrm{A}}_{\mathrm{PKB}}$;
- $\|\boldsymbol{B}(\phi\colon t)\|^{\mathrm{A}}_{\mathrm{PKB}} := \begin{cases} \textsc{True} & \mathrm{A}^{\mathrm{PKB}}(\|\phi\|^{\mathrm{A}}_{\mathrm{PKB}}) = t \\ \textsc{False} & \text{otherwise} \end{cases}.$

Theorem 4.5. *For any* PKB := $\boldsymbol{O}(p_1\colon t_1, \dots, p_k\colon t_k, -)$, *basic formula* ϕ, $\boldsymbol{O}(p_1\colon t_1, \dots, p_k\colon t_k, -) \models \phi$ *iff* $\models \|\phi\|^{\mathrm{A}}_{\mathrm{PKB}}$.

The proof is based on Prop. 4.3 and is by induction on the structure of ϕ. The theorem implies that the probabilistic epistemic reasoning problem can be solved by recursively calling a series of algebraic model counting tasks as well.

Example 4.6. Suppose we have that

$$\text{PKB} := \boldsymbol{O}(\mathit{holdingA}\colon 0.8; \mathit{holdingB}\colon 0.6, -)$$
$$\phi := \boldsymbol{B}(\mathit{holdingA} \land \boldsymbol{B}(\mathit{holdingA} \land \mathit{holdingB}\colon 0.48)\colon 0.8),$$

it holds that PKB $\models \phi$. Since $\|\boldsymbol{B}(\mathit{holdingA} \land \mathit{holdingB}\colon 0.48)\|^{\mathrm{A}}_{\mathrm{PKB}}$ =True (the equality is because $0.48 = t_{\mathit{holdingA},\mathit{holdingB}} = 0.8 \times 0.6$), we have $\|\phi\|^{\mathrm{A}}_{\mathrm{PKB}} = \|\boldsymbol{B}(\mathit{holdingA}\colon 0.8)\|^{\mathrm{A}}_{\mathrm{PKB}} = \textsc{True}$. By Theorem 4.5, $\boldsymbol{O}(\mathit{holdingA}\colon 0.8; \mathit{holdingB}\colon 0.6, -) \models \phi$.

As an implication of Theorem 4.5, we have that:

Corollary 4.7. *For queries with constant size, the time complexity of the epistemic reasoning task in Definition 4.1 is in* $P^{\#P[O(1)]}$.

Again, the operator $\|\cdot\|^{\mathrm{A}}_{\mathrm{PKB}}$ is a number of constant time reductions and ultimately makes a number of calls to the sorts $\mathrm{A}^{\mathrm{PKB}}(\phi') = t$, where ϕ is propositional. This is essentially probabilistic reasoning and whose complexity is $\#P$-complete [13], as one needs to both check if there is a model for the propositional formula ϕ' and compute the weights or probabilities of such a model.

Besides, the reduction theorem above also has practical implications. One could indeed compile the probabilistic knowledge base into circuits once, and it is possible to carry out efficient (still polynomial time, but subject to different assumptions on the circuits) probabilistic reasoning for all future queries [30].

Another remark is that while there is a representation theorem by [36] that deals with the propositional epistemic reasoning in Definition 3.2 by appealing to entailment reasoning, there is no such result that discusses how one can lift it to a probabilistic setting. Our result above is the first generalisation of this sort.

Lastly, we remark that the requirement on PKBs to be a joint distribution of independent variables is not a must. We have it simply for the direct correspondence as in Def. 4.2 between the probabilistic epistemic reasoning task and the *literal-weighted model counting*. Literal-weighted model counting requires weights of a model can be factorized according to the literals of the model (essentially, independent variables) so that evaluation is efficient based on this property. We could lift the approach to general probability distributions Pr in two ways. First, one can always introduce a new set of literals Lit, and distribution Pr′, and formula ψ such that Pr is factorizable according to Ap ∪ Lit, and for any query ϕ, we have $\mathsf{Pr}(\phi) = \mathsf{Pr}'(\phi \land \psi)$, then we do literal-weighted model counting on distribution Pr′ [13]. Besides converting the task, we could also modify the meaning of weights assigned to literals, hence the weights are no longer probabilities but conditional probabilities or others. [20] uses this idea and proposes an encoding of a Bayesian network to an *algebraic decision diagram* (an algebraic extension of OBDD), then performs literal-weighted model counting therein to solve probabilistic reasoning. We could use this approach to encode a general probability distribution as well.

5 REASONING IN THE DYNAMIC PROBABILISTIC LOGIC $\mathcal{DS}$

The versatility of the AMC framework in epistemic reasoning is not limited to the static setting. It is well-known that reasoning about actions in the situation calculus [44] can be reduced to classical first-order reasoning by means of regression [44]. Here, we propose an AMC-based regression operator to handle probabilistic epistemic reasoning in a dynamic setting. We use the dynamic extension of $\mathcal{OBL}$ logic, i.e., logic $\mathcal{DS}$ [7, 39], where every action is stochastic. Dynamic epistemic reasoning extends epistemic reasoning to dynamic domains where the agent's epistemic knowledge changes with actions and sensing [49]. In what follows, we first review the logic $\mathcal{DS}$; thereafter, we show how to use AMC to do dynamic epistemic reasoning.

5.1 The logic $\mathcal{DS}$

Here, we review the logic $\mathcal{DS}$, a probabilistic modal variant of the situation calculus and a dynamic variant of the logic $\mathcal{OBL}$.

Syntax. We will focus on the propositional fragment of $\mathcal{DS}$, nevertheless, we still need to retain two of its special predicates (introduced below). To account for actions, $\mathcal{DS}$ expands the vocabulary of $\mathcal{OBL}$ with a fixed finite set Act of *actions* which syntactically look like constants. Besides, to model a stochastic domain where actions might have non-deterministic effects, $\mathcal{DS}$ is equipped with two special binary *predicates* $alt(a, a')$, $l(a, t)$. The idea is that instead of saying a stochastic action has non-deterministic effects, $\mathcal{DS}$ views a *stochastic action* as a set of actions (mutual *alternatives*) that are *observationally indistinguishable* to the agent and each has a *deterministic effect*. Moreover, the predicate $l(a, t)$ just specifies that the likelihood of action a is t. For example, $alt(TossCoinHead, TossCoinTail)$ indicates that the actions $TossCoinHead$ and $TossCoinTail$ are mutual alternatives, and $l(TossCoinHead, 0.5)$ says the likelihood of $TossCoinHead$ is 0.5.

In dynamic settings, the truth of formulas will change as the effects of actions, hence we also call atomic propositions *fluents*. To capture the dynamic feature, the language is expanded with two action modalities $[a]\phi$ and $\Box\phi$ that are read as: "ϕ holds after action a" and "ϕ holds after any action sequence" respectively.

Formulas ϕ of $\mathcal{DS}$ are given by:

$$\begin{aligned} AF &:= p \mid alt(a, a') \mid l(a, t) \\ \phi &:= AF \mid \phi_1 \wedge \phi_2 \mid \neg\phi \mid [a]\phi \mid \Box\phi \\ &\quad \mid \boldsymbol{B}(\phi\colon t) \mid \boldsymbol{O}(\phi_1\colon t_1, \ldots \phi_k\colon t_k) \end{aligned}$$

where $p \in$ Ap are atomic propositions, $t, t_i \in \mathbb{Q}_{[0,1]}$ $(1 \leq i \leq k)$ are rational numbers, $a, a' \in$ Act are actions, and ϕ, ϕ_i are formulas.

Atomic formulas AF can be: (1) a proposition p; or (2) constructed by predicate $alt(a, a')$ and $l(a, t)$ where a, a' are actions $\in$ Act and $t \in \mathbb{Q}_{[0,1]}$ is a rational number. Formulas can also be constructed by connectives $\wedge, \vee, \neg$, and modality $\boldsymbol{B}, \boldsymbol{O}$ and modality $[a]$ and $\Box$. For action sequence $z = a_1 \cdots a_k$, we write $[z]\phi$ to mean $[a_1] \cdots [a_k]\phi$.

A formula is a *fluent formula* if it contains only propositions $p \in$ Ap, modal $[\cdot]$, $\boldsymbol{B}$, and boolean connectives $\wedge, \vee, \neg$. E.g. $holdingA \vee \neg holdingB$ is objective; $[pickupA]\boldsymbol{B}(holdingA \wedge holdingB\colon 0.125)$, $\boldsymbol{B}([pickupA]\, holdingA\colon 1)$ are all fluent formula, but $l(pickupA, 0.5)$, $\Box[pickupA]\, holdingA$, and $\boldsymbol{O}(holdingA\colon 1)$ are not.

Semantics. In a dynamic setting, a world has to account for both what holds initially and after any action sequence. Let Act* be the set of all action sequences, including the empty sequence $\langle\rangle$.

By a *world* w, we mean a mapping $w : AF \times \text{Act}^* \mapsto \{0, 1\}$ from atomic formulas AF and action sequence Act* to $\{0, 1\}$. Intuitively, the action sequence here serves as the *history* of execution. Again, we use $\mathcal{W}$ to denote the set of all such worlds. We assume that the interpretation of $l(a, t)$ is functional in t, i.e., $\forall a \in \text{Act}, z \in \text{Act}^*$, there is exactly one number $t^a_{w,z}$ representing the likelihood of a under w, z, such that $w[l(a, t^a_{w,z}), z] = 1$.

Besides, the alternative relationship among stochastic actions is captured by an equivalence relation $alt : \text{Act} \times \text{Act} \mapsto \{0, 1\}$. A world $w \in \mathcal{W}$ is *compatible* with an alternative relationship alt, i.e., $w \bowtie alt$ if

- $w[alt(a, a'), z] = 1$ iff $\langle a, a'\rangle \in alt, \forall z \in \text{Act}^*$; and
- $\forall z \in \text{Act}^*$, $a \in$ Act, $\sum_{\{a' : \langle a, a'\rangle \in alt\}} t_{a'} = 1$ where $t_{a'}$ satisfies $w[l(a', t_{a'}), z] = 1$;

That is, $w \bowtie alt$ if the interpretation of predicate alt in w faithfully captures relation alt and the likelihood of actions in the same equivalence class sums up to 1.

Clearly, we can extend the relation alt and likelihood function l (in a world w) from single actions to action sequences, denoted by alt^* and l^*_w respectively. Namely, for finite action sequences z, z', $\langle z, z'\rangle \in alt^*$ iff the actions at each index of z, z' are mutual alternatives. Moreover, $l^*_w(z) = l^*_w(z') \times t$ supposing $z = z' \cdot a$ and $w[l(a, t), z'] = 1$, with $l^*_w(\langle\rangle) = 1$. Namely, the likelihood of action sequence z in a world w is just the product of the likelihood of each action in it.

Now, the agent's *epistemic state* e is modeled as a set of probability distributions d over possible worlds as before. Let $\mathcal{D}$ be the set of all such distributions. By a *model*, we mean a 4-tuple (e, w, z, alt). We only consider models where $w \bowtie alt$ and e satisfies $\forall w' \in e, w' \bowtie alt$. [3] Hence, for a model (e, w, z, alt), alt can be implicitly assumed, and it can be written as (e, w, z) instead.

Now, given a world $w \in \mathcal{W}$, an epistemic state e, and a formula ϕ, we define $e, w \models \phi$ as $e, w, \langle\rangle \models \phi$, where for any $z \in \text{Act}^*$ (assuming ap is an atom in AF, ϕ, ψ are formulas, $t, t_i \in \mathbb{Q}_{[0,1]}$ for $1 \leq i \leq k$, and a is an action in Act):

- $e, w, z \models ap$ iff $w[ap, z] = 1$;
- $e, w, z \models \phi \wedge \psi$ iff $e, w, z \models \phi$ and $e, w, z \models \psi$;
- $e, w, z \models \neg\phi$ iff $e, w, z \not\models \phi$;
- $e, w, z \models [a]\phi$ iff $e, w, z \cdot a \models \phi$;
- $e, w, z \models \Box\phi$ iff $e, w, z \cdot z' \models \phi$ for all $z' \in \text{Act}^*$;
- $e, w, z \models \boldsymbol{B}(\phi\colon t)$ iff $\forall d \in \mathcal{D}$, $d \in e \Rightarrow \sum_{(w',z') \in \|\phi\|^{d,alt}_{w,z}} d(w') \cdot l^*_{w'}(z') = t$.
- $e, w, z \models \boldsymbol{O}(\phi_1\colon t_1, \ldots, \phi_k\colon t_k)$ iff $\forall d \in \mathcal{D}$, $d \in e \Leftrightarrow \sum_{(w',z') \in \|\phi_i\|^{d,alt}_{w,z}} d(w') \cdot l^*_{w'}(z') = t_i$ for $1 \leq i \leq k$.

---

[3] It is observed in [39] that a fixing alt relation across all worlds is the key to ensuring that the modality of $\boldsymbol{B}$ enjoys positive introspection dynamically, namely, if we take $\boldsymbol{K}(\phi)$ to be $\boldsymbol{B}(\phi\colon 1)$, then $\Box\boldsymbol{B}(\phi\colon r) \supset \boldsymbol{K}(\boldsymbol{B}(\phi\colon r))$ is valid for any ϕ. Allowing alt to be incomparable with w means worlds might interpret predicate $alt(\cdot, \cdot)$ in a different way, hence, losing this property.

where $\|\phi\|^{d,alt}_{w,z} = \{(w', z') \mid \{d\}, w', z' \models \phi, w' \bowtie alt, \langle z, z' \rangle \in alt^*\}$ is the alternative world and action pairs that might result in ϕ under d and alt^*.

Truth of formulas is similar to truth in $\mathcal{OBL}$, except for the modalities. Truth of $[a]$ and $\square$ simply push the action and action sequences into the model, while truth of $\boldsymbol{B}$ needs to consider the action sequence in the model now. Intuitively, $e, w, z \models \boldsymbol{B}(\phi\colon t)$ if and only if after the action sequence z, the probability of ϕ is t in all distributions $d \in e$, for which one needs to consider all the alternative pairs (w', z') of worlds and action sequence of that might result in ϕ, i.e., $(w', z') \in \|\phi_i\|^{d,alt}_{w,z}$. This extends to $\boldsymbol{O}$. Logical entailment, satisfiability, and validity are given similarly as in $\mathcal{OBL}$.

5.2 Projection by AMC in $\mathcal{DS}$

In what follows, we propose an AMC-based regression operator to solve the projection problem in $\mathcal{DS}$.

Projection. An important task in reasoning about actions is *projection*, which is to decide if a query is entailed by a knowledge base after a given action sequence. This task is fundamentally important for AI applications such as planning and decision-making. In order to do projection, $\mathcal{DS}$ uses a variant of the *basic action theories* [44] to specify the dynamics of a domain. Intuitively, they specify the universal rules obeyed by the world, which include:

- Successor state axioms Σ_{ssa}, one for each fluent proposition $p \in \mathrm{Ap}$ and action $a \in \mathrm{Act}$ of the form [4] $\square[a]p \equiv \gamma_{a,p}$, where $\gamma_{a,p}$ is a fluent formula without modalities. That is, it is always the case that after an action a, p holds, if and only if before the action, $\gamma_{a,p}$ holds;
- action alternatives axioms Σ_{alt}, one for each $a, a' \in \mathrm{Act}$, of the form $\square alt(a, a') \equiv \eta_{a,a'}$, where $\eta_{a,a'}$ is either True or False, s.t. that alt forms an equivalence relation in Act;
- likelihood axioms Σ_l, one for each action $a \in \mathrm{Act}$, of the form $\square l(a, t)$, where t is a rational number, subject to that $\sum_{a_j \in \mathrm{Act}_i} t_j = 1$ for all i and equivalent class Act_j induced by Σ_{alt} where $\Sigma_l \models l(a_j, t_j)$.

We lump these axioms as Σ, i.e., $\Sigma = \Sigma_{ssa} \cup \Sigma_{alt} \cup \Sigma_l$. Clearly, one can easily syntactically identify the set of alternatives of a given sequence $z = a_1 \cdots a_k$, i.e., $alt^*(z)$ and the likelihood of z, i.e., $l^*(z)$: $alt^*(z) = \{a'_1 \cdots a'_k \in \mathrm{Act}^* \mid \square alt(a_1, a'_1) \equiv \textsc{True}$ appears in Σ_{alt}, …, and $\square alt(a_k, a'_k) \equiv \textsc{True}$ appears in $\Sigma_{alt}\}$, $l^*(z) = \prod_i t_i$ where $\square l(a_1, t_1)$ appears in $\Sigma_l, \ldots,$ and $\square l(a_k, t_k)$ appears in Σ_l.

Example 5.1. The following is a basic action theories for a dynamic pick-holding domain. The two fluents $\{holdingA, holdingB\}$ describe that objects A, B are holding respectively. They can be affected by actions $\{pickupA, pickupB\}$.

$$\square[pickupA]\, holdingA \equiv \neg holdingB \vee holdingA \tag{2}$$

$$\square[pickupB]\, holdingA \equiv \textsc{False} \tag{3}$$

$$\square[pickupA]\, holdingB \equiv holdingB \tag{4}$$

$$\square[pickupB]\, holdingB \equiv \textsc{True} \tag{5}$$

$$\square alt(pickupA, pickupA),\ \square alt(pickupB, pickupB) \tag{6}$$

$$\square alt(pickupA, pickupB),\ \square alt(pickupB, pickupA) \tag{7}$$

$$\square l(pickupA, 0.5),\ \square l(pickupB, 0.5) \tag{8}$$

In English, after picking up A, A is held if and only if B was not held or A was already held before the action (Eq. (2)); picking up B will cause A no longer being held (Eq. (3)); picking up A has no effects on *holdingB* (Eq. (4)); picking up B will cause B being held (Eq. (5)); The actions $\mathrm{Act}_{pickup} = \{pickupA, pickupB\}$ are mutual alternatives and observationally indistinguishable to the agent (Eq. (6) (7)); both actions might happen with likelihood 0.5 (Eq. (8)).

We are interested in the epistemic version of projection reasoning, i.e., dynamic epistemic reasoning, which is formulated as:

Definition 5.2 (Projection Reasoning). Given a probabilistic knowledge base $\mathrm{PKB} := \boldsymbol{O}(p_1\colon t_1, \ldots, p_k\colon t_k, -, \Sigma\colon 1)$ and a fluent formula ϕ, the projection reasoning task is to decide if $\mathrm{PKB} \models \phi$ where "$-$" denotes the concatenation of expressions such as "$p_i \wedge p_j\colon t_i \times t_j$", indicating that propositions $p_1, \ldots, p_k$ are mutually independent.

Note that query ϕ might contain nested modal $\boldsymbol{B}$ and modal $[\cdot]$.

Projection by AMC-based Regression. A classical mechanism to solve the projection task is regression [34, 44], which recursively replaces sub-formulas of the form $[a]p$ in the query by the right-hand side of the appropriate successor state axioms of fluent p and action a to eliminate action modalities $[a]$. Yet, this does not apply to queries with $\boldsymbol{B}$. Here, we propose a variant of regression by using the algebraic model counting techniques to handle queries with $\boldsymbol{B}$ even nested $\boldsymbol{B}$.

Definition 5.3 (Regression). Given a probabilistic knowledge base $\mathrm{PKB} := \boldsymbol{O}(p_1\colon t_1, \ldots, p_k\colon t_k, -, \Sigma\colon 1)$, a fluent formula ϕ, we define $\mathfrak{R}_{\mathrm{PKB}}[\phi]$ as $\mathfrak{R}_{\mathrm{PKB}}[\langle\rangle, \phi]$ with $\mathfrak{R}_{\mathrm{PKB}}[z, \phi]$ for any action sequence $z \in \mathrm{Act}^*$ defined as:

- $\mathfrak{R}_{\mathrm{PKB}}[z, \phi] := \phi$ if ϕ is True or False;
- $\mathfrak{R}_{\mathrm{PKB}}[z, \neg\phi] := \neg\mathfrak{R}_{\mathrm{PKB}}[z, \phi]$;
- $\mathfrak{R}_{\mathrm{PKB}}[z, \phi \wedge \psi] := \mathfrak{R}_{\mathrm{PKB}}[z, \phi] \wedge \mathfrak{R}_{\mathrm{PKB}}[z, \psi]$;
- $\mathfrak{R}_{\mathrm{PKB}}[z, [a]\phi] := \mathfrak{R}_{\mathrm{PKB}}[z \cdot a, \phi]$ for $a \in \mathrm{Act}$;
- $\mathfrak{R}_{\mathrm{PKB}}[z, p]$ is defined inductively for fluent proposition p:
  (1) $\mathfrak{R}_{\mathrm{PKB}}[\langle\rangle, p] := p$;
  (2) $\mathfrak{R}_{\mathrm{PKB}}[z \cdot a, p] := \mathfrak{R}_{\mathrm{PKB}}[z, \gamma_{a,p}]$, where $\gamma_{a,p}$ the RHS of Σ_{ssa} of fluent proposition p and action a.
- for $\mathfrak{R}_{\mathrm{PKB}}[z, \boldsymbol{B}(\phi\colon t)]$, we have: $\mathfrak{R}_{\mathrm{PKB}}[\cdot] :=$
  $$\begin{cases} \textsc{True} & t = \sum_{z' \in alt^*(z)} \mathrm{A}^{\mathrm{PKB}}(\mathfrak{R}_{\mathrm{PKB}}[z', \phi]) \times l^*(z') \\ \textsc{False} & otherwise \end{cases}.$$
  where $\mathrm{A}^{\mathrm{PKB}}(\phi)$ is defined as in Def. 4.2.

With these definitions, we have a main result:

Theorem 5.4. *Given a* $\mathrm{PKB} := \boldsymbol{O}(p_1\colon t_1, \ldots, p_k\colon t_k, -, \Sigma\colon 1)$, *a fluent query* ϕ, $\mathrm{PKB} \models \phi$ iff $\models \mathfrak{R}_{\mathrm{PKB}}[\phi]$.

The proof is based on Theorem 2.3 and is by induction on the structure of ϕ. The theorem suggests that the probabilistic projection of a certain knowledge base can be solved by combining regression and AMC.

Example 5.5. Let Σ be as in Example 5.1, PKB be a probabilistic knowledge base as $\mathrm{PKB} := \boldsymbol{O}(holdingA\colon \frac{1}{2}; holdingB\colon \frac{1}{2}, -, \Sigma\colon 1)$. Then we have PKB entails

$$[pickupA]\boldsymbol{B}(holdingA \wedge [pickupA]\boldsymbol{B}(\neg holdingB\colon 0.125)\colon 0.375).$$

---

[4] The $\square$ modality has lower syntactic precedence than the connectives, and $[\cdot]$ has the highest priority. Hence, the formula should be read as $\square\left(([a]p) \equiv \gamma_{a,p}\right)$.

	holdingA	*holdingB*	t_0	$t_{pickupA}$	$t_{pickupA \cdot pickupA}$
θ_1	0	0	0.25	0	0
θ_2	0	1	0.25	0.625	0.8125
θ_3	1	0	0.25	0.25	0.125
θ_4	1	1	0.25	0.125	0.0625

Table 1: Joint probability distributions in Example 5.5 over actions

The joint distribution of propositions *holdingA* and *holdingB* is listed in Table 1, where $\theta_1, \ldots \theta$ are the truth assignments of the propositions. Column t_0 lists the initial joint distribution of PKB, Column $t_{pickupA}$ and Column $t_{pickupA \cdot pickupA}$ list the distributions obtained after executing stochastic action *pickupA* and the action sequence *pickupA* · *pickupA* respectively. The result holds because (in the following, we use *hA* for *holdingA* and *hB* for *holdingB*)

$$\begin{aligned}&\mathfrak{R}_{\text{PKB}}[[pickupA]\boldsymbol{B}(hA \wedge [pickupA]\boldsymbol{B}(\neg hB\colon 0.125)\colon 0.375)]\\ =&\mathfrak{R}_{\text{PKB}}[pickupA, \boldsymbol{B}(hA \wedge [pickupA]\boldsymbol{B}(\neg hB\colon 0.125)\colon 0.375)]\end{aligned}$$

which equals (by definition of $\mathfrak{R}_{\text{PKB}}$) True if $0.375 =$

$$\sum_{a \in \text{Act}_{pickup}} \mathbf{A}^{\text{PKB}}(\mathfrak{R}_{\text{PKB}}[a, hA \wedge [pickupA]\boldsymbol{B}(\neg hB\colon 0.125)]) \cdot 0.5 \quad (9)$$

and False otherwise. Now, we show Eq. (9) indeed holds.

$$\begin{aligned}&\mathfrak{R}_{\text{PKB}}[pickupA, hA \wedge [pickupA]\boldsymbol{B}(\neg hB\colon 0.125)]\\ =&\mathfrak{R}_{\text{PKB}}[pickupA, hA]\\ =&\neg hB \vee hA\end{aligned}$$

The first "=" is due to $\mathfrak{R}_{\text{PKB}}[pickupA, [pickupA]\boldsymbol{B}(\neg hB\colon 0.125)] =$ True. Likewise,

$$\begin{aligned}&\mathfrak{R}_{\text{PKB}}[pickupB, hA \wedge [pickupA]\boldsymbol{B}(\neg hB\colon 0.125)]\\ =&\mathfrak{R}_{\text{PKB}}[pickupB, hA] = \text{False}\end{aligned}$$

Hence,

$$\begin{aligned}&\sum_{a \in \text{Act}_{pickup}} \mathbf{A}^{\text{PKB}}(\mathfrak{R}_{\text{PKB}}[a, hA \wedge [pickupA]\boldsymbol{B}(\neg hB\colon 0.125)]) \cdot 0.5\\ =&\ \mathbf{A}^{\text{PKB}}(\neg hB \vee hA) \cdot 0.5 = (0.25 + 0.25 + 0.25) \cdot 0.5 = 0.375\end{aligned}$$

Thus, $\text{PKB} \models [pickupA]\boldsymbol{B}(hA \wedge [pickupA]\boldsymbol{B}(\neg hB\colon 0.125)\colon 0.375)$.

Theorem 5.4 has implications on the complexity of the projection task in Def. 5.2. There are many factors involved, including: (1) the size of query ϕ; (2) the size of Σ_{ssa}, $\mathfrak{R}_{\text{PKB}}$ is no longer a constant number of reductions, $\mathfrak{R}_{\text{PKB}}[z, \cdot[a]p]$ in fact will expand and result in a lager formula $\mathfrak{R}_{\text{PKB}}[z, \gamma_{a,p}]$; (3) the size of Act which bounds the size of equivalent class of actions induced by Σ_{alt} and it determines the number of AMC calls when handling $\boldsymbol{B}$. Hence, we assume these factors are relatively small constants, then we have:

Corollary 5.6. *For queries ϕ, successor state axioms Σ_{ssa}, and set of action* Act *with constant size, the time complexity of the epistemic reasoning task in Definition 5.2 is in $P^{\#P[O(1)]}$.*

We remark that our approach can be easily lifted to PKB with arbitrary distributions as before. Besides, the proposed regression operator is essentially an adaptation of the regression operator in [39], where the regression of $\boldsymbol{B}$ reduces to logical formulas with infinite "$\sum$" as logical terms. Although the regression operator in [39] is more general in the sense that it can handle domains with sensing or stochastic actions whose likelihood depends on fluents, yet, evaluation of these formulas is, in general, undecidable, and in practice, one might need to resort to Monte Carlo Sampling [9]. In contrast, our proposed regression operator is a terminating symbolic procedure that relies on AMC and knowledge compilation.

6 DISCUSSION AND CONCLUSION

To the best of our knowledge, this paper is the first to connect algebraic model counting (AMC) with epistemic modal logic. We briefly review relevant work in epistemic reasoning and AMC.

In epistemic logic, the foundational work by Fagin et al. [22] systematically studies knowledge and its computational properties. The notion of only-knowing was introduced by Levesque [35], and further explored in relation to total knowledge [42] and minimal knowledge [28]. This concept has since been extended to handle actions [33], multi-agent systems [6, 23, 27], and probabilistic settings [7, 25, 37]. The complexity of epistemic reasoning and only-knowing has been studied in [22, 45], while Levesque [36] proved a representation theorem that inspires our Theorem 3.4. Our symbolic AMC-based operator "$| \cdot |_{\Sigma}^{\mathbf{A}}$" extends the semantic operator "$| \cdot |_{\Sigma}$" in that work: "$\| \cdot \|_{\Sigma}$" resolves to semantic entailment while our "$\| \cdot \|_{\Sigma}^{\mathbf{A}}$" is a symbolic procedure that relies on AMCs. Additionally, Lakemeyer [32] showed that epistemic reasoning after actions can be reduced to first-order entailment, a result extended to probabilistic and multi-agent cases [39]. Our work differs by offering a syntactic, decidable alternative based on AMC.

On the AMC side, weighted model counting (WMC) has proven effective across formalisms such as Bayesian networks [14], factor graphs [15], probabilistic programming [24], and probabilistic databases [47]. WMC extends model counting (#SAT) [1] by associating weights to models, and is supported by SAT solvers and knowledge compilation techniques [17, 19, 46]. Approximate inference methods use local search [50] or sampling [12]. Extensions to relational domains, such as weighted first-order model counting [18], allow more compact representations.

Algebraic model counting generalizes WMC by using arbitrary semirings (e.g., max, min, ∧, ∨) instead of numeric summation, enabling a wider range of reasoning tasks [30]. Belle and De Raedt [5] further extend this to semiring programming over continuous domains. Our work builds on these ideas to explore how AMC can support epistemic reasoning in a sound, symbolic manner.

To sum up, we show that algebraic model counting (AMC) can be applied to certain forms of epistemic reasoning within the modal logic of only-knowing. We further extended this idea to probabilistic epistemic reasoning in both static and dynamic contexts.

For future work, we plan to implement the proposed procedure for epistemic reasoning and systematically compare it to variants of existing modal logic theorem provers. Besides, extending the approach to a multi-agent setting is also an interesting direction.

ACKNOWLEDGMENTS

Daxin is funded by an NSFC Grant No. 6250071156; Vaishak is funded by a Royal Society University Research Fellowship.

REFERENCES

[1] Fahiem Bacchus, Shannon Dalmao, and Toniann Pitassi. 2009. Solving# SAT and Bayesian inference with backtracking search. *Journal of Artificial Intelligence Research* 34 (2009), 391–442.

[2] John Baras and George Theodorakopoulos. 2022. *Path problems in networks.* Springer Nature, Switzerland AG.

[3] Vaishak Belle. 2024. Counterfactual Explanations as Plans. In *Proceedings 40th International Conference on Logic Programming, ICLP 2024, University of Texas at Dallas, Dallas Texas, USA, October 14-17 2024 (EPTCS, Vol. 416)*, Pedro Cabalar, Francesco Fabiano, Martin Gebser, Gopal Gupta, and Theresa Swift (Eds.). Cambridge University Press, Dallas, Texas, United States, 153–167. https://doi.org/10.4204/EPTCS.416.14

[4] Vaishak Belle, Thomas Bolander, Andreas Herzig, and Bernhard Nebel. 2023. Epistemic planning: Perspectives on the special issue. *Artif. Intell.* 316 (2023), 103842. https://doi.org/10.1016/J.ARTINT.2022.103842

[5] Vaishak Belle and Luc De Raedt. 2020. Semiring programming: A semantic framework for generalized sum product problems. *International Journal of Approximate Reasoning* 126 (2020), 181–201.

[6] Vaishak Belle and Gerhard Lakemeyer. 2010. Multi-Agent Only-Knowing Revisited. In *Principles of Knowledge Representation and Reasoning: Proceedings of the Twelfth International Conference, KR 2010, May 9-13, 2010*, Fangzhen Lin, Ulrike Sattler, and Miroslaw Truszczynski (Eds.). AAAI Press, Toronto, Ontario, Canada, 49–59. https://aaai.org/papers/12-1361-multi-agent-only-knowing-revisited/

[7] Vaishak Belle and Gerhard Lakemeyer. 2017. Reasoning about Probabilities in Unbounded First-Order Dynamical Domains. In *Proceedings of the Twenty-Sixth International Joint Conference on Artificial Intelligence, IJCAI 2017, Melbourne, Australia, August 19-25, 2017*, Carles Sierra (Ed.). ijcai.org, Melbourne, Australia, 828–836. https://doi.org/10.24963/IJCAI.2017/115

[8] Vaishak Belle, Gerhard Lakemeyer, and Hector J. Levesque. 2016. A First-Order Logic of Probability and Only Knowing in Unbounded Domains. In *Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence, February 12-17, 2016*, Dale Schuurmans and Michael P. Wellman (Eds.). AAAI Press, Phoenix, Arizona, USA, 893–899. https://doi.org/10.1609/AAAI.V30I1.10108

[9] Vaishak Belle and Hector J. Levesque. 2015. ALLEGRO: Belief-Based Programming in Stochastic Dynamical Domains. In *Proceedings of the Twenty-Fourth International Joint Conference on Artificial Intelligence, IJCAI 2015, July 25-31, 2015*, Qiang Yang and Michael J. Wooldridge (Eds.). AAAI Press, Buenos Aires, Argentina, 2762–2769. http://ijcai.org/Abstract/15/391

[10] Meghyn Bienvenu, Hélène Fargier, and Pierre Marquis. 2010. Knowledge Compilation in the Modal Logic S5. In *Proceedings of the Twenty-Fourth AAAI Conference on Artificial Intelligence, AAAI 2010, July 11-15, 2010*, Maria Fox and David Poole (Eds.). AAAI Press, Atlanta, Georgia, USA, 261–266. https://doi.org/10.1609/AAAI.V24I1.7587

[11] Thomas Caridroit, Jean-Marie Lagniez, Daniel Le Berre, Tiago de Lima, and Valentin Montmirail. 2017. A SAT-Based Approach for Solving the Modal Logic S5-Satisfiability Problem. In *Proceedings of the Thirty-First AAAI Conference on Artificial Intelligence, February 4-9, 2017*, Satinder Singh and Shaul Markovitch (Eds.). AAAI Press, San Francisco, California, USA, 3864–3870. https://doi.org/10.1609/AAAI.V31I1.11128

[12] Supratik Chakraborty, Daniel J. Fremont, Kuldeep S. Meel, Sanjit A. Seshia, and Moshe Y. Vardi. 2014. Distribution-Aware Sampling and Weighted Model Counting for SAT. In *Proceedings of the Twenty-Eighth AAAI Conference on Artificial Intelligence, July 27 -31, 2014*, Carla E. Brodley and Peter Stone (Eds.). AAAI Press, Québec City, Québec, Canada, 1722–1730. https://doi.org/10.1609/AAAI.V28I1.8990

[13] Mark Chavira and Adnan Darwiche. 2008. On probabilistic inference by weighted model counting. *Artif. Intell.* 172, 6-7 (2008), 772–799. https://doi.org/10.1016/J.ARTINT.2007.11.002

[14] Mark Chavira and Adnan Darwiche. 2008. On probabilistic inference by weighted model counting. *Artificial Intelligence* 172, 6-7 (2008), 772–799.

[15] Arthur Choi, Doga Kisa, and Adnan Darwiche. 2013. Compiling Probabilistic Graphical Models Using Sentential Decision Diagrams. In *Symbolic and Quantitative Approaches to Reasoning with Uncertainty - 12th European Conference, ECSQARU 2013, July 8-10, 2013. Proceedings (Lecture Notes in Computer Science, Vol. 7958)*, Linda C. van der Gaag (Ed.). Springer, Utrecht, The Netherlands, 121–132. https://doi.org/10.1007/978-3-642-39091-3_11

[16] Adnan Darwiche and Pierre Marquis. 2002. A Knowledge Compilation Map. *J. Artif. Intell. Res.* 17 (2002), 229–264. https://doi.org/10.1613/JAIR.989

[17] Adnan Darwiche and Pierre Marquis. 2002. A knowledge compilation map. *Journal of Artificial Intelligence Research* 17 (2002), 229–264.

[18] Guy Van den Broeck, Wannes Meert, and Adnan Darwiche. 2014. Skolemization for Weighted First-Order Model Counting. In *Principles of Knowledge Representation and Reasoning: Proceedings of the Fourteenth International Conference, KR 2014, July 20-24, 2014*, Chitta Baral, Giuseppe De Giacomo, and Thomas Eiter (Eds.). AAAI Press, Vienna, Austria, 111–120. https://aaai.org/papers/0111-8012-skolemization-for-weighted-first-order-model-counting/

[19] Guy Van den Broeck, Ingo Thon, Martijn van Otterlo, and Luc De Raedt. 2010. DTProbLog: A Decision-Theoretic Probabilistic Prolog. In *Proceedings of the Twenty-Fourth AAAI Conference on Artificial Intelligence, AAAI 2010, July 11-15, 2010*, Maria Fox and David Poole (Eds.). AAAI Press, Atlanta, Georgia, USA, 1217–1222. https://doi.org/10.1609/AAAI.V24I1.7755

[20] Paulius Dilkas and Vaishak Belle. 2021. Weighted model counting with conditional weights for Bayesian networks. In *Proceedings of the Thirty-Seventh Conference on Uncertainty in Artificial Intelligence, UAI 2021, 27-30 July 2021 (Proceedings of Machine Learning Research, Vol. 161)*, Cassio P. de Campos, Marloes H. Maathuis, and Erik Quaeghebeur (Eds.). AUAI Press, Virtual Event, 386–396. https://proceedings.mlr.press/v161/dilkas21a.html

[21] Jason Eisner. 2002. Parameter Estimation for Probabilistic Finite-State Transducers. In *Proceedings of the 40th Annual Meeting of the Association for Computational Linguistics, July 6-12, 2002.* ACL, Philadelphia, PA, USA, 1–8. https://doi.org/10.3115/1073083.1073085

[22] Ronald Fagin, Joseph Y Halpern, Yoram Moses, and Moshe Vardi. 2004. *Reasoning about knowledge.* MIT press, Cambridge, Massachusetts.

[23] Qihui Feng and Gerhard Lakemeyer. 2024. Probabilistic Multi-agent Only-Believing. In *Proceedings of the 23rd International Conference on Autonomous Agents and Multiagent Systems, AAMAS 2024, May 6-10, 2024*, Mehdi Dastani, Jaime Simão Sichman, Natasha Alechina, and Virginia Dignum (Eds.). International Foundation for Autonomous Agents and Multiagent Systems / ACM, Auckland, New Zealand, 571–579. https://doi.org/10.5555/3635637.3662908

[24] Daan Fierens, Guy Van den Broeck, Joris Renkens, Dimitar Shterionov, Bernd Gutmann, Ingo Thon, Gerda Janssens, and Luc De Raedt. 2015. Inference and learning in probabilistic logic programs using weighted boolean formulas. *Theory and Practice of Logic Programming* 15, 3 (2015), 358–401.

[25] Alfredo Gabaldon and Gerhard Lakemeyer. 2007. ESP: A Logic of Only-Knowing, Noisy Sensing and Acting. In *Proceedings of the Twenty-Second AAAI Conference on Artificial Intelligence, July 22-26, 2007.* AAAI Press, Vancouver, British Columbia, Canada, 974–979. http://www.aaai.org/Library/AAAI/2007/aaai07-155.php

[26] Todd J. Green, Gregory Karvounarakis, and Val Tannen. 2007. Provenance semirings. In *Proceedings of the Twenty-Sixth ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems, June 11-13, 2007*, Leonid Libkin (Ed.). ACM, Beijing, China, 31–40. https://doi.org/10.1145/1265530.1265535

[27] Joseph Y Halpern and Gerhard Lakemeyer. 2001. Multi-agent Only Knowing. *Journal of Logic and Computation* 11, 1 (2001), 41–70.

[28] Joseph Y Halpern and Yoram Moses. 1985. *Towards a theory of knowledge and ignorance: Preliminary report.* Springer, Berlin, Heidelberg.

[29] Angelika Kimmig, Guy Van den Broeck, and Luc De Raedt. 2011. An Algebraic Prolog for Reasoning about Possible Worlds. In *Proceedings of the Twenty-Fifth AAAI Conference on Artificial Intelligence, AAAI 2011, August 7-11, 2011*, Wolfram Burgard and Dan Roth (Eds.). AAAI Press, San Francisco, California, USA, 209–214. https://doi.org/10.1609/AAAI.V25I1.7852

[30] Angelika Kimmig, Guy Van den Broeck, and Luc De Raedt. 2017. Algebraic model counting. *Journal of Applied Logic* 22 (2017), 46–62.

[31] Gerhard Lakemeyer and Yves Lespérance. 2012. Efficient Reasoning in Multiagent Epistemic Logics. In *ECAI 2012 - 20th European Conference on Artificial Intelligence. Including Prestigious Applications of Artificial Intelligence (PAIS-2012) System Demonstrations Track, August 27-31 , 2012 (Frontiers in Artificial Intelligence and Applications, Vol. 242)*, Luc De Raedt, Christian Bessiere, Didier Dubois, Patrick Doherty, Paolo Frasconi, Fredrik Heintz, and Peter J. F. Lucas (Eds.). IOS Press, Montpellier, France, 498–503. https://doi.org/10.3233/978-1-61499-098-7-498

[32] Gerhard Lakemeyer and Hector J. Levesque. 2004. Situations, Si! Situation Terms, No!. In *Principles of Knowledge Representation and Reasoning: Proceedings of the Ninth International Conference (KR2004), June 2-5, 2004*, Didier Dubois, Christopher A. Welty, and Mary-Anne Williams (Eds.). AAAI Press, Whistler, Canada, 516–526. https://aaai.org/papers/KR04-054-situations-si-situation-terms-no/

[33] Gerhard Lakemeyer and Hector J. Levesque. 2009. A Semantical Account of Progression in the Presence of Defaults. In *IJCAI 2009, Proceedings of the 21st International Joint Conference on Artificial Intelligence, July 11-17, 2009*, Craig Boutilier (Ed.). ijcai.org, Pasadena, California, USA, 842–847. http://ijcai.org/Proceedings/09/Papers/144.pdf

[34] Gerhard Lakemeyer and Hector J. Levesque. 2011. A semantic characterization of a useful fragment of the situation calculus with knowledge. *Artif. Intell.* 175, 1 (2011), 142–164. https://doi.org/10.1016/J.ARTINT.2010.04.005

[35] Hector J Levesque. 1990. All I know: a study in autoepistemic logic. *Artificial intelligence* 42, 2-3 (1990), 263–309.

[36] Hector J Levesque and Gerhard Lakemeyer. 2001. *The logic of knowledge bases.* Mit Press, Cambridge, Massachusetts.

[37] Daxin Liu and Qihui Feng. 2023. On the progression of belief. *Artificial Intelligence* 322 (2023), 103947.

[38] Daxin Liu, Qinfei Huang, Vaishak Belle, and Gerhard Lakemeyer. 2023. Verifying Belief-Based Programs via Symbolic Dynamic Programming. In *ECAI 2023 - 26th European Conference on Artificial Intelligence, September 30 - October 4, 2023 (Frontiers in Artificial Intelligence and Applications, Vol. 372)*, Kobi Gal, Ann Nowé, Grzegorz J. Nalepa, Roy Fairstein, and Roxana Radulescu (Eds.). IOS Press, Kraków, Poland, 1497–1504. https://doi.org/10.3233/FAIA230429

[39] Daxin Liu and Gerhard Lakemeyer. 2021. Reasoning about Beliefs and Meta-Beliefs by Regression in an Expressive Probabilistic Action Logic. In *Proceedings of the Thirtieth International Joint Conference on Artificial Intelligence, IJCAI 2021, 19-27 August 2021*, Zhi-Hua Zhou (Ed.). ijcai.org, Virtual Event / Montreal, Canada, 1951–1958. https://doi.org/10.24963/IJCAI.2021/269

[40] Daxin Liu and Gerhard Lakemeyer. 2025. A Framework for Belief-based Programs and Their Verification. *J. Artif. Intell. Res.* 82 (2025), 1205–1242. https://doi.org/10.1613/JAIR.1.15796

[41] Robert C Moore. 1985. Semantical considerations on nonmonotonic logic. *Artificial intelligence* 25, 1 (1985), 75–94.

[42] Ian Pratt-Hartmann. 2000. Total Knowledge. In *Proceedings of the Seventeenth National Conference on Artificial Intelligence and Twelfth Conference on on Innovative Applications of Artificial Intelligence, July 30 - August 3, 2000*, Henry A. Kautz and Bruce W. Porter (Eds.). AAAI Press / The MIT Press, Austin, Texas, USA, 423–428. http://www.aaai.org/Library/AAAI/2000/aaai00-065.php

[43] Luc De Raedt, Angelika Kimmig, and Hannu Toivonen. 2007. ProbLog: A Probabilistic Prolog and Its Application in Link Discovery. In *IJCAI 2007, Proceedings of the 20th International Joint Conference on Artificial Intelligence, January 6-12, 2007*, Manuela M. Veloso (Ed.). ijcai.org, Hyderabad, India, 2462–2467. http://ijcai.org/Proceedings/07/Papers/396.pdf

[44] Raymond Reiter. 2001. *Knowledge in action: logical foundations for specifying and implementing dynamical systems.* MIT press, Cambridge, Massachusetts.

[45] Riccardo Rosati. 2000. On the decidability and complexity of reasoning about only knowing. *Artificial Intelligence* 116, 1-2 (2000), 193–215.

[46] Tian Sang, Paul Beame, and Henry A. Kautz. 2005. Performing Bayesian Inference by Weighted Model Counting. In *Proceedings, The Twentieth National Conference on Artificial Intelligence and the Seventeenth Innovative Applications of Artificial Intelligence Conference, July 9-13, 2005*, Manuela M. Veloso and Subbarao Kambhampati (Eds.). AAAI Press / The MIT Press, Pittsburgh, Pennsylvania, USA, 475–482. http://www.aaai.org/Library/AAAI/2005/aaai05-075.php

[47] Dan Suciu, Dan Olteanu, Christopher Ré, and Christoph Koch. 2022. *Probabilistic databases.* Springer Nature, Heidelberg, Germany.

[48] Johan van Benthem, Jan van Eijck, Malvin Gattinger, and Kaile Su. 2018. Symbolic model checking for Dynamic Epistemic Logic - S5 and beyond. *J. Log. Comput.* 28, 2 (2018), 367–402. https://doi.org/10.1093/LOGCOM/EXX038

[49] Hans P. van Ditmarsch, Wiebe van der Hoek, and Barteld P. Kooi. 2008. *Dynamic Epistemic Logic.* Springer, Berlin, Heidelberg, Germany.

[50] Wei Wei and Bart Selman. 2005. A New Approach to Model Counting. In *Theory and Applications of Satisfiability Testing, 8th International Conference, SAT 2005, June 19-23, 2005, Proceedings (Lecture Notes in Computer Science, Vol. 3569)*, Fahiem Bacchus and Toby Walsh (Eds.). Springer, St. Andrews, UK, 324–339. https://doi.org/10.1007/11499107_24