Minimizing Envy and Maximizing Happiness in Graphical House Allocation

Extended Abstract, Main Track

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ABSTRACT

We study graphical house allocation, where agents are connected by a friendship graph and envy can arise only between neighboring agents. Each agent approves a subset of houses and envies a friend if she receives no approved house while the friend is allocated one she approves. We consider two problems: minimizing the number of envious agents, and, among such allocations, maximizing the number of agents receiving an approved house. We provide a detailed complexity analysis, showing that both problems are solvable in polynomial time when each agent approves at most one house, but become NP-hard when agents may approve two houses, establishing a tight tractability boundary. We further present exact algorithms under structural restrictions on the agent graph, including sparsity, small balanced separators, and bounded vertex cover.

KEYWORDS

House Allocation, Fair Division, FPT, Exact Algorithms

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1 INTRODUCTION

Allocating scarce resources fairly and efficiently is a fundamental problem in many domains, such as student–dormitory assignments, public housing, and cloud resource distribution. The classical House Allocation problem offers a simplified abstraction of this task: agents have preferences over houses, and the goal is to assign each agent a house (injectively) so as to minimize dissatisfaction, typically measured via envy, the number of agents who would prefer someone else's allocation over their own. However, this assumption of complete information is often unrealistic: in practice, agents typically judge fairness only within limited social or informational circles, such as friends, colleagues, or teammates. For instance, students compare dorm rooms mainly with close peers, employees benchmark allocations within their teams, and users on digital platforms evaluate outcomes relative to their social networks.

Motivated by this observation, a recent line of work has explored graphical aspects of house allocation, albeit with different objectives [1–3]. Some studies consider allocation under externalities and analyze stability-based notions, while others focus on local envy-freeness i.e., whether an allocation exists with no envy along the edges of the graph. These models typically assume ordinal preferences and primarily study the zero-envy condition.

In this paper, we study the Optimal House Allocation problem, a natural and practically motivated generalization of the classical model. Here, agents are nodes in a graph $\mathcal{G} = (\mathcal{A}, \mathcal{E})$, where edges represent social or informational ties. Let $\mathcal{A}$ be a set of n agents and $\mathcal{H}$ be a set of m houses. Each agent $a \in \mathcal{A}$ is associated with a set of *preferred houses* $\mathcal{P}_a \subseteq \mathcal{H}$; houses outside $\mathcal{P}_a$ are disliked. A *house allocation* is an injective function $\phi : \mathcal{A} \rightarrow \mathcal{H}$, meaning that no two agents are assigned the same house.

Envy is defined *locally* with respect to the graph $\mathcal{G}$. An agent $a \in \mathcal{A}$ envies a neighbor $a' \in N_{\mathcal{G}}(a)$ under an allocation ϕ if and only if (i) $\phi(a) \notin \mathcal{P}_a$, and (ii) $\phi(a') \in \mathcal{P}_a$. Let $\mathcal{E}^{\phi}(a) = \{a' \in N_{\mathcal{G}}(a) \mid \phi(a) \notin \mathcal{P}_a \text{ and } \phi(a') \in \mathcal{P}_a\}$ denote the set of neighbors whom agent a envies under ϕ. We call an agent $a \in \mathcal{A}$ *envious* if $|\mathcal{E}^{\phi}(a)| \geqslant 1$, and *envy-free* otherwise.

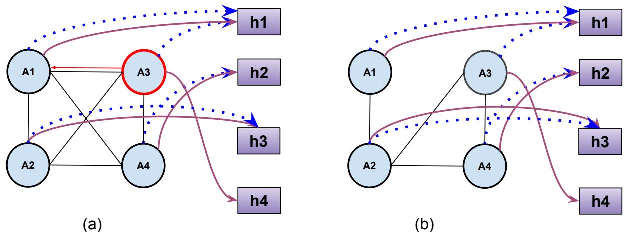

Figure 1: Graphical house allocation for four agents and four houses under different network structures. Blue dotted edges indicate agent preferences, while purple solid arrows represent allocations. (a) shows a fully connected clique graph; agent A3 is envious of A1 (marked by a red edge) because it prefers A1's allocated house. (b) shows a sparse connectivity graph; in this instance, no agent is envious.

This local envy formulation captures a realistic notion of fairness grounded in social perception rather than global comparison. Figure 1 illustrates how the same allocation may induce envy under a dense connectivity graph but remain envy-free when agents have fewer social connections, highlighting a key difference from the classical model and from related graphical formulations such as that of Madathil et al. [4].

While minimizing envy is a fundamental fairness objective, it does not fully capture agent satisfaction. An agent may be non-envious yet still unhappy—for example, if neither she nor any of her neighbors receives a house she prefers. Such outcomes may appear fair locally while providing little real utility. To address this, we distinguish fairness from *happiness*. An agent $a \in \mathcal{A}$ is said to be *happy* under allocation ϕ if $\phi(a) \in \mathcal{P}_a$. The *envy* (resp. *happiness*) of an allocation ϕ is the total number of envious (resp. happy) agents.

This motivates the study of two optimization problems. The first seeks to minimize envy, while the second refines this objective by maximizing happiness among all minimum-envy allocations.

Definition 1 (Optimal House Allocation)**.** *Given a set $\mathcal{A}$ of n agents, a graph $\mathcal{G} = (\mathcal{A}, \mathcal{E})$ on $\mathcal{A}$, a set $\mathcal{H}$ of m houses, and preference sets $(\mathcal{P}_a)_{a\in\mathcal{A}}$, compute an allocation ϕ that minimizes* $\left|\{a \in \mathcal{A} \mid \mathcal{E}^{\phi}(a) \neq \emptyset\}\right|$.

Definition 2 (Optimally Happy House Allocation of Agent Network)**.** *Given a set $\mathcal{A}$ of n agents, a graph $\mathcal{G}$ on $\mathcal{A}$, a set $\mathcal{H}$ of m houses, and preference sets $(\mathcal{P}_a)_{a\in\mathcal{A}}$, compute an allocation ϕ that minimizes the number of envious agents, and among all such allocations, maximizes* $|\{a \in \mathcal{A} \mid \phi(a) \in \mathcal{P}_a\}|$.

2 OUR CONTRIBUTIONS

We investigate the problems of Optimal House Allocation and Optimally Happy House Allocation of Agent Network. We design various novel algorithms which solve these problems efficiently under conditions such as restricted preference structures and simpler graph classes, and complement these with matching NP-hardness results under other natural restrictions. Together, these results place the problems in appropriate positions within several complexity landscapes. We summarize our contributions below.

We first consider instances where every agent prefers exactly one house, i.e., $|\mathcal{P}_a| = 1$ for all $a \in \mathcal{A}$. In this setting, each agent can be envious of at most one neighbor. We reduce the problem to a bipartite matching instance that captures feasible assignments and counts envy violations. A minimum cost maximum matching yields an optimal solution in polynomial time for both Optimal House Allocation and Optimally Happy House Allocation of Agent Network. However, when $\mathcal{P}_a \leqslant 2$, we prove that Optimal House Allocation becomes NP-hard even on complete bipartite graphs. We also show NP-hardness of Optimal House Allocation on 3-regular graphs even when $|\mathcal{A}| = |\mathcal{H}|$ and agents have identical preferences, i.e., for all $a, a' \in \mathcal{A}, \mathcal{P}_a = \mathcal{P}_{a'}$. This is in stark contrast with the existing results on complete graphs, where either $|\mathcal{A}| = |\mathcal{H}|$ or identical preferences lead to polynomial-time algorithms [4]. Combined with the previous result, this establishes a tight boundary between tractable and intractable cases based solely on preference size.

We design exact exponential algorithms that improve over the naive brute-force bound $2^{O(|\mathcal{A}| \log |\mathcal{H}|)}$. Our first algorithm runs in time $O(2^{|\mathcal{A}|+2|\mathcal{E}|} \cdot (|\mathcal{A}| + |\mathcal{H}|)^{O(1)})$, and is particularly efficient for sparse graphs. The key observation is that envy is purely local: an agent can only be envious of its neighbors in the agent graph. We therefore guess, for each agent a, the subset of neighbors it is envious of, yielding $2^{\deg(a)}$ possibilities per agent, and additionally guess which non-envious agents receive preferred houses. Once these choices are fixed, all remaining constraints are captured by agent-wise feasibility sets, and the existence of a consistent allocation reduces to a polynomial-time bipartite matching problem.

We further design an exact algorithm for graphs admitting $f(n)$-balanced separators, such as planar and bounded-genus graphs. By guessing the house assignments and envy status of agents in a separator of size $f(n)$, the instance decomposes into two independent subinstances of size at most $2n/3$. This yields a recursive algorithm with running time $O(2^{m+O(f(n)\log n+n)})$, leading to improved bounds such as $2^{O(n)}$ for planar graphs and $2^{O(\mathrm{tw}\log n+n)}$ for graphs of treewidth tw.

We also design an exact algorithm parameterized by the vertex cover number k of the agent graph. The algorithm runs in time $(2^m)^k \cdot (|\mathcal{A}| + |\mathcal{H}|)^{O(1)}$ by explicitly guessing the house assignments of agents in the vertex cover and their resulting envy status. Once these guesses are fixed, all remaining agents form an independent set, and their envy interactions are entirely determined by the assignments to the vertex cover. This allows the problem on the remaining agents to be solved via a single minimum-cost bipartite matching instance. As a consequence, the algorithm yields a quasi-polynomial-time solution for graphs with vertex cover size $\log^{O(1)} |\mathcal{A}|$. To complement this positive result, we prove that bounding the vertex cover size by $|\mathcal{A}|^{\varepsilon}$ does not yield tractability. Specifically, Optimal House Allocation remains NP-hard even on bipartite and split graphs whose vertex cover size is at most $|\mathcal{A}|^{\varepsilon}$ for any constant $\varepsilon \in (0, 1)$. This establishes a sharp separation between quasi-polynomial-time solvability for very small vertex covers and intractability once the vertex cover grows polynomially.

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