The Landscape of Almost Equitable Allocations

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ABSTRACT

Equitability is a fundamental notion in fair division which requires that all agents derive equal value from their allocated bundles. We study, for general (possibly non-monotone) valuations, a popular relaxation of equitability known as *equitability up to one item* (EQ1). An EQ1 allocation may fail to exist even with additive non-monotone valuations; for instance, when there are two agents, one valuing every item positively and the other negatively. This motivates a mild and natural assumption: all agents agree on the sign of their value for the *grand bundle*. Under this assumption, we prove the existence and provide an efficient algorithm for computing EQ1 allocations for two agents with general valuations. When there are more than two agents, we show the existence and polynomial-time computability of EQ1 allocations for valuation classes beyond additivity and monotonicity, in particular for (1) *doubly monotone* valuations and (2) *submodular* (resp. *supermodular*) valuations where the value for the grand bundle is *non-negative* (resp. *non-positive*) for all agents. Furthermore, we settle an open question of Bilò et al. by showing that an EQ1 allocation always exists for non-negative (resp. non-positive) valuations, i.e., when every agent values each subset of items non-negatively (resp. non-positively). Finally, we complete the picture by showing that for general valuations with more than two agents, EQ1 allocations may not exist even when agents agree on the sign of the grand bundle, and that deciding the existence of an EQ1 allocation is computationally intractable.

KEYWORDS

Fair Division; Equitability; Non-monotone Valuations; Submodular Valuations; Subadditive Valuations

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1 INTRODUCTION

A central problem in multiagent systems concerns the fair allocation of resources, tasks, or items among agents with heterogeneous preferences. At its core lies a simple yet fundamental question: *how should such resources be divided fairly?* The field of fair division provides a formal framework for studying this question and has introduced variety of fairness axioms, particularly in settings involving indivisible items. Among these, *equitability* [15] is a compelling and well-studied notion, requiring that all agents derive equal subjective value from their allocated bundles. Motivated by empirical evidence [19, 20], equitability captures an *interpersonal* notion of fairness by emphasizing equality in experienced happiness among agents.

The vast majority of existing work assumes that agents' valuations are *monotonic*, i.e., receiving additional items monotonically increases (in case of goods) or decreases (in case of chores) an agent's utility. However, many realistic settings involve *non-monotonic* or even *non-additive* valuations. Moreover, whether the inclusion of a given item increases or decreases an agent's utility, could depend on the subset of items she already owns. For example, consider a firm looking to hire a new employee. While this individual might be highly valuable in isolation, the marginal utility of bringing them on could be negative if the team already has several people with the same expertise, or if the new hire's work style is not aligned with the team. The value the firm derives from the employee depends not only on their abilities, but also on the existing team composition and dynamics. Similarly, in parallel processing, adding more processors does not always improve performance. Initially, tasks complete faster as workload divides, but beyond a point, communication overhead and synchronization costs dominate — each new processor adds more waiting than work.

Motivated by these examples, we study the fairness notion of equitability under the most *general valuation* functions, extending beyond standard monotonic or additive assumptions. Our focus is on its prominent relaxation, *equitability up to one item* (EQ1), which requires that any inequality between two agents can be eliminated by removing at most one item from an agent's bundle.

When valuations are additive and monotone, an EQ1 allocation is known to exist when all items are goods for all agents [16], and likewise when all items are chores [17]. Even in *mixed-manna* settings, if agents agree on whether each item is a good or a chore, an EQ1 allocation is guaranteed to exist [23].[1] In contrast, if agents disagree on the sign of items' marginal values, an EQ1 allocation need not exist. For instance, consider two agents with additive valuations over more than two items: one assigns a value of 1 to every item, while the other assigns −1. In this case, no complete allocation can satisfy EQ1.

Motivated by this impossibility, we introduce a mild assumption: all agents agree on the sign of the *grand bundle*, that is, whether the collection of all items is overall desirable or undesirable. Under this assumption, we ask the following theoretical questions: *For which classes of valuation functions is an* EQ1 *allocation guaranteed to exist, and do the corresponding decision problems admit efficient algorithms?*

1.1 Our Contributions

We study the existence and computation of EQ1 allocations under general valuations and several subclasses, assuming that all agents agree on the sign of their value for the grand bundle. Figure 1 presents the resulting landscape of existence across major valuation classes. Specifically, we examine EQ1 allocations for (i) general valuations without additional assumptions, (ii) doubly monotone valuations where items can be partitioned into goods and chores, (iii) submodular and supermodular valuations, (iv) subadditive and superadditive valuations, and (v) non-negative and non-positive valuations. For clarity of exposition, we first present all our results under the assumption that all agents value the grand bundle *non-negatively*. We then establish a technical result (deferred to the full version [21]) that extends these findings to the complementary case in which the grand bundle is valued non-positively, thereby providing a complete characterization of EQ1 allocations.

General valuations. We show that, for three or more agents with supermodular valuations (and therefore arbitrary general valuations), an EQ1 allocation may not exist—even when all agents value the grand bundle non-negatively. Moreover, the corresponding decision problem is NP-complete (Theorem 3.2). In contrast, for two agents, an EQ1 allocation always exists and can be computed in polynomial time whenever both agents agree on the sign of the grand bundle (Theorem 3.1).

Doubly monotone and submodular valuations. We introduce a new class of valuations—those satisfying the *marginal witness property*.[2] For this class, when all agents value the grand bundle non-negatively, an EQ1 allocation always exists and can be computed in polynomial time. Since both *doubly monotone* (Theorem 4.5) and *submodular* (Theorem 4.4) valuations also satisfy this property and hence are contained within this class, our results immediately imply existence and efficient computation of EQ1 allocations for these two important valuation classes.

Non-negative valuations. A particular valuation class that has recently received attention in the literature [5, 10] is that of *non-negative valuations*, where every subset of items has non-negative value, implying that agents never view any bundle as a net loss. Within this class, recent works study weaker relaxations of equitability obtained via rounding equitable cake divisions and fixed-point arguments [5, 10]. Specifically, [10] achieve equitability after removing at most two items (at most one from each bundle), while [5] guarantee equitability after the removal or addition of at most three items. Both works leave the existence of EQ1 allocations as an intriguing open question. In this work, we resolve this question positively by proving, through a combinatorial approach, that EQ1 allocations always exist (Theorem 5.1).

Identical subadditive valuations. We show the existence of EQ1 allocations for identical subadditive valuations when agents value the grand bundle non-negatively (Theorem 6.1). This finding strengthens the result of [5].

Since EQ1 and EF1[3] coincide under identical valuations, this result immediately implies the existence of EF1 under identical subadditive valuations (Corollary 6.2).

1.2 Related Work

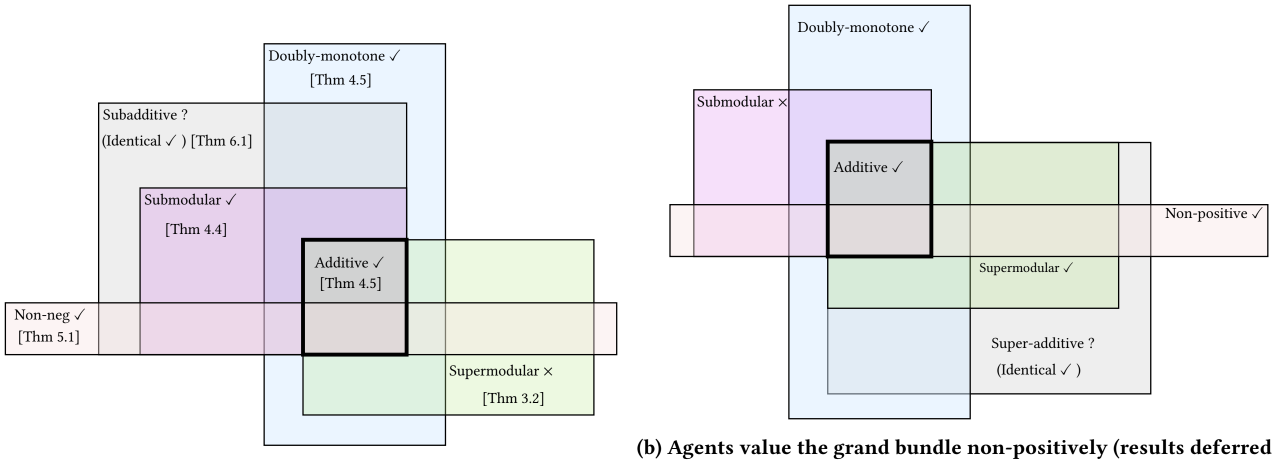
Divisible Items. In the divisible setting, Dubins and Spanier [15] showed that an equitable division always exists for additive valuations. CechláRová et al. [12] showed that connected EQ allocations exist for valuation functions that are non-negative, non-decreasing, and continuous, and in such settings, nearly equitable allocations can be computed efficiently [13]. Chèze [14] gave a simpler and shorter existence proof based on a fixed-point result (Borsuk-Ulam Theorem). Avvakumov and Karasev [2] showed the existence for general identical valuations. Aumann and Dombb [1] showed that there is a connected equitable division that also maximizes the egalitarian welfare. Recently, Bhaskar et al. [8] showed the existence of EQ allocations for non-negative valuations using Sperner's Lemma. Their proof also generalizes to subclasses of possibly negative valuations like identical valuations and single-peaked valuations. The above results are only existential, Procaccia and Wang [25] showed that there is no bounded runtime algorithm for finding an equitable division, even without the connectedness constraint.

Indivisible Items. When the items are indivisible, an exact EQ allocation may not exist, but EQ1 allocations are known to exist and are efficiently computable for monotone additive valuations [16, 17]. Under additive valuations, several works have also studied equitability in conjunction with efficiency guarantees [16, 18], and welfare trade-offs [7, 11, 26].

Equitability Beyond Additivity and/or Monotonicity. Recently, approximate equitability has been considered in the non-additive and/or non-monotone settings. Barman et al. [4] considered a stronger relaxation, EQX, where the removal of *any* good from the rich agent's bundle or the removal of *any* chore from the poor agent's bundle gets rid of the inequity. They showed that EQX exists for monotone valuations and can be efficiently computed for *weakly well-layered valuations.* For non-monotone valuations, they showed the existence of EQX allocations for the case of two agents with additive valuations where each item is either a good for both agents or a chore for both agents. Hosseini and Sethia [23] showed

[1]Such valuations are called *objective*, meaning that no item is valued positively by some agents and negatively by others.
[2]Formal definition in Section 4.

[3]An allocation is envy-free (EF) if no agent values another agent's bundle more than its own. Analogous to EQ1, EF1 is a relaxation where envy, if any, can be eliminated by removing at most one item.



(a) Agents value the grand bundle non-negatively.

(b) Agents value the grand bundle non-positively (results deferred to the full version [21]).

Figure 1: A pictorial representation of various valuation classes, their intersections, and an overview of our results. ✓ implies existence of EQ1 allocations, × implies non-existence, ? implies that the existence is an open question. For two agents, we show the existence and efficient computation of EQ1 allocations under *general valuations* when the grand bundle is valued non-negatively (resp. non-positively) (Theorem 3.1).

the existence and efficient computation of EQ1 allocations when every item is valued at $\{-\alpha, 0, \alpha\}$ and the valuations are additive. In [5], Barman and Verma study a weaker notion of equitability under non-negative valuations. They show the existence of allocations where the difference between the utilities of any two agents can be eliminated by the removal or addition of at most three items. Bilò et al. [10] prove a stronger result by showing the existence of EQ1_g^c allocations[4] for non-negative valuations.

Envy-Freeness Beyond Additivity and/or Monotonicity. EF1 allocations have been known to exist for general monotone valuations and can be computed efficiently with envy-cycle eliminations [24]. For non-monotone but additive valuations, Aziz et al. [3] showed the existence and efficient computation of EF1 by double round-robin algorithm. Bhaskar et al. [9] extended this to doubly monotone valuations. Recently, Bhaskar et al. [6] showed that EF1 always exists for (i) identical trilean valuations and (ii) a newly introduced class of valuations–*separable single-peaked (SSP) valuations.* Hosseini et al. [22] considered EF1 allocation of the vertices of a graph among the agents, where the value of a bundle is determined by its cut value, capturing settings where valuations are inherently non-monotonic.

2 PRELIMINARIES

The Setting. An instance of a fair division problem is given by a tuple $\mathcal{I} = \langle N, M, \{v_i\}_{i\in N}\rangle$, where N is a set of $n \in \mathbb{N}$ agents and M is a set of $m \in \mathbb{N}$ indivisible items. For the sake of brevity, we will often assume $N = \{1, 2, \dots n\}$. Every agent i has a valuation function $v_i : 2^M \to \mathbb{R}$, associating a real value to every set $S \subseteq M$ of items, denoted by $v_i(S)$. Throughout the paper, we assume a polynomial time oracle access to the agent valuation functions. An allocation $\mathcal{A} = (A_1, \dots A_n)$ is a partition of M into n bundles, where A_i denotes the bundle allocated to agent i. As is standard in the literature, we assume that $v_i(\emptyset) = 0$ for every agent $i \in N$.

[4]An allocation is EQ1_g^c if for every pair of agents equitability can be guaranteed by the removal of up to removal of one item *each* from their bundles [10].

Valuation Classes. The marginal value of an item o, for an agent i, with respect to a subset $S \subseteq M$ is defined as $v_i(S|e) = v_i(S \cup \{e\}) - v_i(S)$. We consider non-monotone valuations, meaning that, for an agent, an item may have either positive, negative, or zero marginal value with respect to a given subset of items. We additionally assume that the sign of the valuation of the *grand bundle* (the entire set M of items) is the same for all agents. Formally, either $v_i(M) \geq 0$ for all agents $i \in N$, or $v_i(M) \leq 0$ for all agents $i \in N$. This assumption is quite natural and reasonable; without it,EQ1 allocations may fail to exist even in simple cases with just two items and two agents with additive valuations, one valuing each item at 1, another at -1. Here, no (complete) allocation can satisfyEQ1.

We next define the classes of valuations that we consider in this work.

(1) *Additive*: A valuation function f is said to be *additive* if the value of a bundle S of items is equal to the sum of the values of the items in S. Formally, $f(S) = \sum_{e\in S} f(e)$ for all subsets $S \subseteq M$.
(2) *Submodular / Supermodular*: A valuation function f is said to be *submodular* if it satisfies the property of diminishing marginal value. Formally, for subsets $S, T \subseteq M$ such that $S \subseteq T$, and any item $e \notin T$, we have $f(S \cup e) - f(S) \geq f(T \cup e) - f(T)$. Likewise, a valuation function f is said to be *supermodular* if it satisfies the property of increasing marginal value. Formally, for subsets $S, T \subseteq M$ such that $S \subseteq T$, and any item $e \notin T$, we have $f(S \cup e) - f(S) \leq f(T \cup e) - f(T)$.
(3) *Subadditive*: We say that a valuation function f is said to be *subadditive* if for any two **disjoint** subsets $S, T \subseteq M$, we have $f(S \cup T) \leq f(S) + f(T)$. Note that, this is a weaker condition

than the standard definition of subadditivity, which requires the above condition to hold for any two (not necessarily disjoint) subsets $S, T \subseteq M$.[5]

(4) *Superadditive*: A valuation function f is said to be *superadditive* if for any two **disjoint** subsets $S, T \subseteq M$, we have $f(S \cup T) \geq f(S) + f(T)$.

(5) *Doubly Monotone*: A valuation function f is said to be *doubly monotone*, if the set of items M can be partitioned into *goods* G and *chores* C, such that $M = G \sqcup C$ and for all $S \subseteq M$, we have $f(S \cup g) \geq f(S)$ for each good $g \in G$ and $f(S \cup c) \leq f(S)$ for each chore $c \in C$. It is important to note that in an instance of fair division with doubly monotone valuations, different agents may have different sets of goods and chores.

(6) *Non-negative / Non-positive*: A valuation function f is said to be *non-negative* if the value of any subset of items is a non-negative real number. Formally, if $f(S) \geq 0$ for all subsets $S \subseteq M$. Likewise, a valuation function f is said to be *non-positive* if the value of any subset of items is a non-positive real number. Formally, if $f(S) \leq 0$ for all subsets $S \subseteq M$.

The general containment of valuation classes is as follows.

$$\text{Additive} \subset \text{Submodular} \subset \text{Subadditive}$$

$$\text{Additive} \subset \text{Doubly Monotone}$$

$$\text{Additive} \subset \text{Supermodular} \subset \text{Superadditive}$$

Fairness Notion. An allocation $\mathcal{A} = (A_1, \ldots A_n)$ is said to be *equitable* (EQ) if all the agents derive equal value from their respective bundles, that is, for every pair of agents i and j, we have $v_i(A_i) = v_j(A_j)$. An EQ allocation need not always exist. Thus, we consider a popular relaxation of EQ, called asEQ1. An allocation $\mathcal{A}$ is said to be *equitable up to one item* (EQ1) if for every pair of agents i and j such that $v_i(A_i) < v_j(A_j)$, either there exists some item g in A_j such that $v_i(A_i) \geq v_j(A_j \setminus \{g\})$ or there exists some item c in A_i that satisfies $v_i(A_i \setminus \{c\}) \geq v_j(A_j)$.

We now introduce a stronger notion of *lower EQ1 witness* that implies EQ1. This captures EQ1 through the existence of a single *witness value* that represents an equitable utility level. Intuitively, an allocation admits a *lower EQ1 witness* θ if all agents value their bundles at least as much as θ, and the value of each agent could be made to fall (weakly) below θ by removing at most one item from the agent's bundle.

Definition 2.1 (Lower EQ1 Witness). An allocation $\mathcal{A}$ is said to *admit a lower EQ1 witness* $\theta \in \mathbb{R}$ if the following conditions hold:

(1) For every agent i, $v_i(A_i) \geq \theta$, and
(2) For every agent i, either $v_i(A_i) = \theta$, or there exists an item $g \in A_i$ such that $v_i(A_i \setminus \{g\}) \leq \theta$.

The existence of a lower witness guarantees that all agents are "almost" at a common value level—within the change caused by one item. Hence, it immediately implies EQ1.

Proposition 2.2. *If an allocation $\mathcal{A}$ admits a lower EQ1 witness, then $\mathcal{A}$ is EQ1.*

[5]Putting $S = T$ in the standard definition, we get $f(S) \leq 2f(S)$, or equivalently, $f(S) \geq 0$ for all subsets $S \subseteq M$. Thus, the standard definition of subadditivity implies that the valuation function is non-negative. However, in this work, we consider more general subadditive valuation functions that may take negative values as well.

Definition 2.3 (Rich and Poor Agents). Given an allocation $\mathcal{A} = (A_1, \ldots A_n)$, an agent i is said to be a *rich* agent if $v_i(A_i) \geq v_j(A_j)$ for all $j \in N$. Similarly, an agent i is said to be a *poor* agent if $v_i(A_i) \leq v_j(A_j)$ for all $j \in N$.

2.1 Organization of the Paper.

We first consider the case of non-negatively valued grand bundle in Sections 3-6. We begin with the case of general valuations in Section 3, followed by doubly monotone valuations and submodular valuations in Section 4, non-negative valuations in Section 5, and identical subadditive valuations in Section 6. All the missing proofs, as well as the analogous case where the grand bundle is valued non-positively can be found in the full version([21]).

3 GENERAL VALUATIONS

In this section, we consider general valuations with non-negative grand bundles ($v_i(M) \geq 0 \; \forall \, i \in N$). We show that for two agents, an EQ1 allocation always exists and can be efficiently computed. On the other hand, for more than two agents, we exhibit non-existence and prove that its corresponding decision problem is NP-complete.

3.1 Two Agents: An Existence Result

We first present an efficient algorithm for the case of two agents.

Theorem 3.1. *Given a fair division instance $\mathcal{I} = \langle N, M, \{v_i\}_{i \in N} \rangle$ with two agents such that the grand bundle is valued non-negatively by both the agents, an EQ1 allocation always exists and can be computed efficiently.*

Algorithm Overview. We begin by checking a trivial case: if agent 1 assigns zero total value to all items, give everything to agent 1 and we are done. Otherwise, imagine the items laid out in some fixed order on a table. A "knife" sweeps from left to right, moving items from the right pile to the left pile one at a time. After each move, the left pile is what agent 1 would receive and the right pile is what agent 2 would receive. We keep moving the knife one item at a time until, for the first time, agent 1's value for the left pile becomes strictly larger than agent 2's value for the right pile. Call that first stopping point position i. At that moment we have two neighboring cut points to consider: just after item $i-1$ or just after item i. It turns out that at least one of these two allocations is EQ1, and we can check which one in polynomial time. The details are in Algorithm 1.

Proof of Theorem 3.1. If $v_1(M) = 0$, the algorithm returns the allocation $\mathcal{A}$ such that $A_1 = M$ and $A_2 = \emptyset$ which is clearly EQ1. So assume $v_1(M) > 0$. Let $f(t) = v_1(S_t) - v_2(T_t)$ for all $t \in \{0, 1, \ldots m\}$. Then, we have $f(0) = -v_2(M) \leq 0$ and $f(m) = v_1(M) > 0$. Our algorithm finds the smallest $i \in [m]$ such that $f(i) > 0$. By the minimality of i, and the fact that $f(0) \leq 0$, we have $f(i-1) \leq 0$. Thus, we have the following two inequalities:

(1) $v_1(S_i) > v_2(T_i)$
(2) $v_1(S_{i-1}) \leq v_2(T_{i-1})$

The algorithm sets $\mathcal{A} = (S_i, T_i)$ and $\mathcal{B} = (S_{i-1}, T_{i-1})$, returning $\mathcal{A}$ if it is EQ1, otherwise $\mathcal{B}$.

Algorithm 1: Two Agents.

Input: An instance $\mathcal{I} = \langle N, M, \{v_i\}_{i \in N} \rangle$ with two agents such that $v_i(M) \geq 0$ for $i \in \{1, 2\}$
Output: An EQ1 allocation.

if $v_1(M) = 0$ **then**
  return $(M, \emptyset)$
end
Order the items arbitrarily as $e_1, e_2, \ldots, e_m$.
for $t \in \{0, 1, \ldots m\}$ **do**
  $S_t \leftarrow \{e_1, \ldots, e_t\}$
  $T_t \leftarrow M \setminus S_t$
end
$i \leftarrow \min\{t \in \{1, \ldots, m\} \mid v_1(S_t) > v_2(T_t)\}$
$\mathcal{A} \leftarrow (S_i, T_i)$
$\mathcal{B} \leftarrow (S_{i-1}, T_{i-1})$
if $\mathcal{A}$ *is* EQ1 **then**
  return $\mathcal{A}$
end
return $\mathcal{B}$

If $\mathcal{A}$ is EQ1 we are done. Otherwise removing e_i from S_i does not eliminate the inequality. Thus $v_1(S_{i-1}) = v_1(S_i \setminus \{e_i\}) > v_2(T_i)$. Now, in the allocation $\mathcal{B}$, $v_1(S_{i-1}) \leq v_2(T_{i-1})$ by (2) and $v_2(T_{i-1} \setminus e_i) = v_2(T_i) < v_1(S_{i-1})$. Hence $\mathcal{B}$ is EQ1.

The runtime is clearly polynomial since checking the inequalities can be done in constant time.

□

3.2 Beyond Two Agents: Non-existence and Hardness Result

For more than two agents, we show that even with the assumption of the grand bundle being non-negatively valued by all the agents, an EQ1 allocation may not exist, and the corresponding decision problem is NP-complete. In fact, our result holds even for the restricted class of supermodular valuations.

Theorem 3.2. *For valuations where each agent values the grand bundle non-negatively,* EQ1 *allocations may not exist. Furthermore, it is NP-Complete to decide whether an* EQ1 *allocation exists, even when the valuations are supermodular.*

4 MARGINAL WITNESS VALUATIONS

This section establishes the existence and polynomial-time computability of EQ1 allocations for doubly monotone and submodular valuations, under the assumption that every agent values the grand bundle non-negatively. While our results are stated for these familiar valuation classes, they in fact hold more generally for any valuation class that satisfies the *marginal-witness* property. Informally, a valuation function is said to satisfy the marginal-witness property, if, any non-empty subset having a non-negative marginal value with respect to a given bundle contains a singleton "witness" that also has a non-negative marginal.

Definition 4.1 (Marginal Witness Property). A valuation function $v_i : 2^M \to \mathbb{R}$ is said to satisfy the *marginal witness property* if, for any two disjoint bundles $A, B \subseteq M$ with $B \neq \emptyset$ and $v_i(A \cup B) \geq v_i(A)$, there exists an item $b \in B$ such that $v_i(A \cup \{b\}) \geq v_i(A)$. Any valuation function that satisfies the marginal witness property is referred to as a *marginal witness valuation.*

The main result of this section is stated as the following theorem:

Theorem 4.2. *Given a fair division instance $\mathcal{I} = \langle N, M, \{v_i\}_{i \in N} \rangle$ with marginal-witness valuations such that every agent values the grand bundle non-negatively, an EQ1 allocation always exists and can be computed in polynomial time.*

We first present an overview of our algorithm.

Algorithm Overview. Our algorithm is presented in pseudo-code form as Algorithm 2. It maintains a partial EQ1 allocation A and attempts to allocate either all the remaining items or a single item to some agent. If the set of remaining items R can be allocated to some agent while preserving the EQ1 property, the algorithm performs the allocation and terminates. For this purpose, the algorithm performs a couple of early termination checks (lines 6 and 10) respectively. If both the checks fail, leveraging the marginal witness property, it identifies an item with a non-negative marginal value for the poorest agent and assigns it to that agent, ensuring that the EQ1 property is maintained. This process continues until all items are allocated.

Algorithm 2: Marginal-witness valuations.

Input: An instance $\mathcal{I} = \langle N, M, \{v_i\}_{i \in N} \rangle$ with marginal witness valuations, with $v_i(M) \geq 0 \; \forall \; i \in N$.
Output: An EQ1 allocation.

1 $A_i \leftarrow \emptyset$ for all $i \in N$.
2 $R \leftarrow M$ `// Set of remaining items`
3 **while** $R \neq \emptyset$ **do**
4   Let $p \in \arg\min_{i \in N} v_i(A_i)$ `// A poor agent.`
5   $\mu \leftarrow v_p(A_p)$
6   **if** $\exists i \in N$, *such that* $v_i(A_i \cup R) \leq \mu$ **then**
7     $A_i \leftarrow A_i \cup R$
8   **return** A
9   **end**
10   **if** $\exists i \in N, h \in R$, *such that* $v_i(A_i \cup R \setminus \{h\}) \leq \mu$ **then**
11     $A_i \leftarrow A_i \cup R$
12   **return** A
13   **end**
14   Let $g \in R$ be such that $v_p(A_p \cup g) \geq \mu$
15   $A_p \leftarrow A_p \cup g$
16 **end**
17 **return** A

Before we begin the analysis, let us first argue that the algorithm is well defined, that is, when the termination tests fail, there indeed exists an item $g \in R$, such that $v_p(A_p \cup \{g\}) \geq \mu$. This is true because, the first early termination test (line 6) fails, and hence $v_p(A_p \cup R) > \mu = v_p(A_p)$. Therefore, the existence of an item with a non-negative marginal follows from the marginal witness property.

4.1 Analysis of Algorithm 2

Let A^t and R^t denote the partial allocation and the set of remaining items, respectively, at the beginning of the t^{th} iteration of the while loop (line 3). Also, let p^t, μ^t and g^t denote the variables p (poor agent), μ (value derived by the poor agent), and g (item corresponding to the marginal witness property) respectively in the t^{th} iteration of the while loop. For the sake of completeness, we define $A_j^0 = \phi$ for all $j \in N$, $R^0 = M$, $\mu^0 = 0$. The correctness of our algorithm hinges on the following two invariants:

(1) **Invariant 1:** For every iteration t of the while loop, μ^{t-1} is a lower EQ1 witness for the partial allocation A^t. That is, for each $j \in N$:
   (a) $v_j(A_j^t) \geq \mu^{t-1}$, and
   (b) $\min_{S \subseteq A_j^t : |S| \leq 1} v_j(A_j^t \setminus S) \leq \mu^{t-1}$.
(2) **Invariant 2:** For every iteration t, we have $v_j(A_j^t \cup R^t) \geq \mu^{t-1}$ for all $j \in N$.

Lemma 4.3. *Algorithm 2 maintains* **Invariant 1** *and* **Invariant 2**.

Proof. We proceed via induction on t.

Base case. ($t = 1$): Invariant 1 is satisfied because $\mu^0 = 0$ and $A_j^1 = \emptyset$ for all agents $j \in N$. Invariant 2 is satisfied because the grand bundle is valued non-negatively, that is, $v_j(M) \geq 0$ for all $j \in N$. Hence, $v_i(A_i^0 \cup R) = v_i(M) \geq 0 = \mu^0$.

Inductive step. Consider the $(t+1)^{\text{th}}$ iteration of the while loop. Since this iteration is reached, the two early termination checks (line 6 and line 10) must have failed in the previous (t^{th}) iteration. Note that, $A_j^{t+1} = A_j^t$ for all $j \neq p^t$, and $A_{p^t}^{t+1} = A_{p^t}^t \cup \{g^t\}$. Also, $v_{p^t}(A_p^{t+1}) = v_{p^t}(A_{p^t}^t \cup \{g^t\}) \geq \mu^t = v_{p^t}(A_{p^t}^t)$. Hence, agent utilities are non-decreasing, and so is the sequence $(\mu^0, \mu^1, \mu^2, \ldots)$.

Invariant 1. Clearly, $v_j(A_j^{t+1}) \geq \mu^{t+1} \geq \mu^t$ for all $j \in N$. Hence, the first requirement of invariant 1 holds. For each agent $j \neq p^t$, the second requirement of invariant 1 holds because $A_j^{t+1} = A_j^t$ and $\mu^t \geq \mu^{t-1}$. For agent p^t, clearly, $v_{p^t}(A_{p^t}^{t+1} \setminus \{g^t\}) = \mu^t$. Hence the invariant is satisfied in the $(t+1)^{\text{th}}$ iteration.

Invariant 2. Since the second early termination check (line 10) failed in the t^{th} iteration, we have $v_j(A_j^t \cup R^t \setminus \{g^t\}) > \mu^t$ for all $j \in N$. Since $R^{t+1} = R^t \setminus \{g^t\}$, this means that $v_j(A_j^{t+1} \cup R^{t+1}) = v_j(A_j^t \cup R^t \setminus \{g^t\}) \geq \mu^t$ for all $j \neq p^t$. Also, $v_{p^t}(A_{p^t}^{t+1} \cup R^{t+1}) = v_{p^t}((A_{p^t}^t \cup \{g^t\}) \cup (R^t \setminus \{g^t\})) = v_{p^t}(A_{p^t}^t \cup R^t) > \mu^t$, where the last inequality due to the failure of the first early termination check (line 6). Hence, the invariant is satisfied in the $(t+1)^{\text{th}}$ iteration. □

We will now prove Theorem 4.2.

Proof of Theorem 4.2. Clearly, Algorithm 2 terminates in polynomial time, since each iteration takes polynomial time, and the number of iterations is bounded by $|M|$. There are three possibilities for the termination state:

(1) The first early termination test (line 6) passes in some iteration (say the t^{th} iteration), that is $\exists i \in N$, such that $v_i(A_i^t \cup R^t) \leq \mu^t$. In this case, we allocate R^t to i and return an allocation $\mathcal{B} = (B_1, \ldots B_n)$ where $B_i = A_i^t \cup R^t$ and $B_j = A_j^t$ for all $j \neq i$. We claim that $\theta = v_i(A_i^t \cup R^t)$ is a lower EQ1 witness for the allocation $\mathcal{B}$. Indeed, note that every agent's value under $\mathcal{B}$ is atleast θ (since $v_i(B_i) = v_i(A_i^t \cup R^t) = \theta$ and for all $j \neq i$, we have $v_j(A_j^t) \geq \mu^t \geq \theta$.) Also, from invariant 2, we have $\mu^{t-1} \leq \theta$. Now, from invariant 1, it follows that if $v_j(A_j^t) > \theta \geq \mu^{t-1}$, then there exists $h \in A_j^t$, such that $v_j(A_j^t \setminus \{h\}) \leq \mu^{t-1} \leq \theta$. Therefore, $\mathcal{B}$ is an EQ1 allocation.
(2) The first early termination test fails but the second early termination test (line 10) passes in some iteration (say the t^{th} iteration). Hence, there exists $i \in N$, and an item $h \in R^t$, such that $v_i(A_i^t \cup R^t \setminus \{h\}) \leq \mu^t$. In this case, we allocate R^t to agent i and return an allocation $\mathcal{B} = (B_1, \ldots B_n)$ where $B_i = A_i^t \cup R^t$ and $B_j = A_j^t$ for all $j \neq i$. Note that $v_i(B_i) = v_i(A_i^t \cup R^t) \geq \mu^t$, since the first early termination test (line 6) fails. Also, for all agents $j \neq i$, $v_j(B_j) = v_j(A_j^t) \geq \mu^t$ by the definition of μ^t. Hence, μ^t is a lower bound on the agent utilities. Additionally, from invariant 1, for each $j \neq i$, $\min_{S \subseteq B_j : |S| \leq 1} v_j(B_j \setminus S) = \min_{S \subseteq A_j^t : |S| \leq 1} v_j(A_j^t \setminus S) \leq \mu^{t-1} \leq \mu^t$. For the agent i, clearly $v_i((A_i^t \cup R^t) \setminus \{h\}) \leq \mu^t$. Thus, μ^t is a lower EQ1 witness for $\mathcal{B}$.
(3) The algorithm terminates in line 17. Here, the algorithm runs for exactly $|M|$ iterations, allocating a single item in each iteration. It follows from invariant 1 that $\mu^{|M|}$ is a lower EQ1 witness for the final allocation.

This completes the proof. □

As an immediate corollary, Theorems 4.4 and 4.5 follow, since submodular and doubly monotone valuations satisfy the marginal witness property.

Theorem 4.4. *Given a fair division instance $\mathcal{I} = \langle N, M, \{v_i\}_{i \in N} \rangle$ where each agent has a submodular valuation and values the grand bundle non-negatively, an* EQ1 *allocation always exists and can be computed in polynomial time.*

Theorem 4.5. *Given a fair division instance $\mathcal{I} = \langle N, M, \{v_i\}_{i \in N} \rangle$ where each agent has a doubly monotone valuation and values the grand bundle non-negatively, an* EQ1 *allocation always exists and can be computed in polynomial time.*

5 NON-NEGATIVE VALUATIONS

In this section, we show the existence of EQ1 allocations for non-negative valuations. Recall that a valuation function is *non-negative* if every subset of items has a non-negative value. Such valuations need not be monotone: the marginal value of adding an item to a bundle can be negative. That is, for some agent i, item e, and bundle S, it may hold that $v_i(S \cup \{e\}) < v_i(S)$. Nevertheless, the total value of any bundle is always non-negative, i.e., $v_i(S) \geq 0$ for all $S \subseteq M$. Our result settles the open question from [10].

Theorem 5.1. *Given a fair division instance $\mathcal{I} = \langle N, M, \{v_i\}_{i \in N} \rangle$ with non-negative valuations, an* EQ1 *allocation always exists. Furthermore, if $|M| \geq |N|$, then there exists an* EQ1 *allocation where each agent receives a non-empty bundle.*

The proof of Theorem 5.1 is constructive and is based on Algorithm 3, which we describe next.

Algorithm Overview. We start with an empty allocation and a pool R containing all the items. In each step of our algorithm, we would like to select a subset $S \subseteq R$ and an agent i that minimize $v_i(A_i \cup S)$ over all agents and subsets, then give S to i and remove S from R. Throughout this process, two invariants are maintained: (i) no agent is ever made poorer upon receiving a set—if $v_i(A_i \cup S) < v_i(A_i)$, then at an earlier step when A_i was formed, the union $A_i \cup S$ would have been a strictly better minimizer; and (ii) after a set S is given to an agent i, she becomes one of the rich agents—if some other agent j were richer than i after i received S, it would contradict the formation of A_j. Furthermore, due to the minimality of $v_i(A_i \cup S)$, every other agent $j \neq i$ can be made (weakly) richer than i by adding any non-empty subset, and in particular, any single item from the pool. Consequently, if at some point, by sheer luck, there are exactly $n-1$ items remaining in the pool R, the process can be stopped, and each of the agents, except the last one to receive a set, can be given one item from the pool to obtain an EQ1 allocation, with a lower EQ1 witness being the value of the last agent who received a set.

Now, to ensure that the pool indeed ends with $n-1$ items, Algorithm 3 restricts attention to *valid* bundles S whose removal leaves at least $n-1$ items in the pool, i.e., $|R \setminus S| \geq n-1$. Lemma 5.2 shows that the two invariants continue to hold under this validity constraint. Hence, when the loop terminates, $|R| = n-1$, and the remaining items can be assigned one per agent (except the last agent to receive some subset) to obtain an EQ1 allocation.

Algorithm 3: Non-negative valuations.

Input: An instance $\mathcal{I} = \langle N, M, \{v_i\}_{i \in N}\rangle$ with non-negative valuations.
Output: An EQ1 allocation.

1 $A_i \leftarrow \emptyset$ for all $i \in N$.
2 $R \leftarrow M$ `// Set of remaining items`
3 $\ell \leftarrow$ an arbitrary agent `// this variable will maintain the agent who received the last valid bundle`
4 **while** $|R| \geq n$ **do**
5 $\quad \mathbb{V} \leftarrow \{S \subset R : |R \setminus S| \geq n-1, S \neq \emptyset\}$ `// Set of all valid bundles`
6 $\quad (\ell, S) \leftarrow \arg\min_{j \in N, T \in \mathbb{V}} v_j(A_j \cup T)$ `// Break ties arbitrarily`
7 $\quad A_\ell \leftarrow A_\ell \cup S$
8 $\quad R \leftarrow R \setminus S$
9 **end**
10 **for** $j \in N \setminus \{\ell\}$ **do**
11 $\quad$ Pick any item $g \in R$ arbitrarily
12 $\quad A_j \leftarrow A_j \cup \{g\}$
13 $\quad R \leftarrow R \setminus \{g\}$
14 **end**
15 **return** $\mathcal{A} = (A_1, \dots A_n)$

We first establish a loop invariant stating that, the agent who receives a valid bundle in any iteration is a rich agent at the end of that iteration.

Lemma 5.2 (Loop Invariant)**.** *Let $\mathcal{A}^t$ be the allocation at the end of the t^{th} iteration of the while loop in Line 4 of Algorithm 3. Suppose agent i receives a bundle S_t in this iteration. Then i is a rich agent in $\mathcal{A}^t$, i.e.,*

$$v_i(A_i^t) \;\geq\; v_j(A_j^t) \quad \forall j \in N.$$

Suppose agent i receives a valid bundle S in an iteration. By the above loop invariant, agent i becomes a rich agent at the end of that iteration. Since i, S is the minimizer over all $i \in N$ and valid subsets S of R, any other agent who could have received a valid bundle in the same iteration would have obtained a value at least as large as that of agent i. We note this consequence as the following corollary.

Corollary 5.3. *Let $\mathbb{V}$ be the family of all valid subsets of the pool R at the beginning of some iteration t. Let $\mathcal{A}^t$ be the allocation at the end of iteration t, where agent j was allocated a valid subset S. Then, for any agent $i \in N \setminus \{j\}$ and any valid subset $T \in \mathbb{V}$,*

$$v_i(A_i^t \cup T) \geq v_j(A_j^t)$$

We now prove Theorem 5.1.

Proof of Theorem 5.1. Firstly, in each iteration of the while loop, we allocate a non-empty valid bundle to some agent. Since the pool starts with m items and each valid bundle contains at most $m-(n-1)$ items, the while loop runs at most $m-(n-1)$ times. Thus, the algorithm terminates in a finite number of steps.

Let $\mathcal{A}$ be the final allocation returned by the algorithm. If the instance had less than n items, then the while loop in Line 4 never runs. In this case, every agent gets at most one item, hence $\mathcal{A}$ is EQ1 with a lower EQ1 witness of 0. So, consider the case when there are at least n items.

Suppose the while loop in Line 4 runs for t iterations and let $\mathcal{A}^t$ be the allocation at the end of the t^{th} iteration. Let ℓ be the agent who received the last valid bundle S_t in iteration t. By Lemma 5.2, agent ℓ is a rich agent in $\mathcal{A}^t$. As agent ℓ does not receive any more items the end of the while loop, we have $A_\ell = A_\ell^t$ in the returned allocation $\mathcal{A}$. Let θ denote the final value of agent ℓ, i.e., $\theta = v_\ell(A_\ell)$.

Since t^{th} iteration was the last iteration of the while loop, and S_t was a valid bundle, the pool R contains exactly $n-1$ items at the end of iteration t. Now, to construct the final allocation A, we give one item, say g_i from the pool to each agent i in $N \setminus \{\ell\}$. Note that any singleton subset of R was a valid bundle at the beginning of iteration t. So, by Corollary 5.3, for any agent $i \in N \setminus \{\ell\}$ and any item $g \in R$, we have $v_i(A_i^t \cup \{g\}) \geq v_\ell(A_\ell^t) = \theta$. Since agent i receives exactly one item g_i from the pool, we have $v_i(A_i) = v_i(A_i^t \cup \{g_i\}) \geq \theta$. Also, $v_i(A_i \setminus \{g_i\}) = v_i(A_i^t) \leq \theta$. Hence, θ is a lower EQ1 witness for the final allocation $\mathcal{A}$. Thus, the algorithm always returns an EQ1 allocation. □

5.1 Applications Beyond Fair Division

Many important functions in statistics and graph theory are non-negative but not necessarily monotone. For example, statistical measures such as average, variance, and standard deviation, as well as graph-theoretic functions like the cut function and graph density, are all non-negative and often non-monotone. Consequently, Theorem 6.1 has broad applicability across diverse domains. In this section, we highlight two notable applications, both of which have been previously explored in the context of fair division.

Equitable Graph Partitioning

An interesting application of Theorem 6.1 is in equitable graph partitioning. Given an undirected graph $G = (V, E)$ and an integer $k \leq n$, the objective is to partition the vertex set V into k non-empty parts $V_1, V_2, \ldots, V_k$ such that for any pair of parts V_i and V_j, the difference in their cut values is at most α, i.e., $|\delta(V_j) - \delta(V_i)| \leq \alpha$ for some $\alpha \geq 0$. Here, $\delta(V_i)$ denotes the cut value of V_i, defined as the number of edges in E with exactly one endpoint in V_i. This problem captures the problem of distributing the "boundary" edges as evenly as possible among the parts, and arises naturally in load balancing, network design, and parallel computing. The cut function is a classic example of a non-negative, submodular, but non-monotone set function, making it a natural candidate for the application of our general existence results.

In [5], it was shown that for any undirected graph G and integer k, there exists a partition of V into k non-empty parts such that the cut values differ by at most $5\Delta + 1$, where Δ is the maximum degree in G. We improve this bound to Δ by applying Theorem 5.1. This improvement was also shown by [22] by exhibiting the existence of EF1 and *(weak) Transfer Stable* allocations for *cut-valuations*. Likewise, we model the problem as a fair division instance with k agents, where the items are the vertices and each agent has the identical non-negative cut function as their valuation. By applying Theorem 5.1, we obtain an EQ1 partition with a lower witness threshold. This means that for any two parts V_i and V_j with $\delta(V_i) < \delta(V_j)$, there exists a vertex $u \in V_j$ such that $\delta(V_j \setminus \{u\}) \leq \delta(V_i)$. Since adding or removing any vertex changes the cut value by at most Δ, it follows that $\delta(V_j) \leq \delta(V_i) + \Delta$. Therefore, we achieve a partition into k non-empty parts such that the cut values differ by at most Δ, improving upon the previous bound.

While the existence result above depends on an exponential time algorithm(algorithm 3), we can leverage additional structure in the cut function. Specifically, the cut function is both non-negative and submodular. By modifying Algorithm 2, we show that for any non-negative submodular function, an EQ1 allocation with non-empty bundles can be found in polynomial time, if the number of items is greater than or equal to the number of agents.

Theorem 5.4. *Given a fair division instance $\mathcal{I} = \langle N, M, \{v_i\}_{i \in N} \rangle$ with $|M| \geq |N|$, where each valuation v_i is both non-negative and submodular, there exists a polynomial-time algorithm that finds an* EQ1 *allocation in which every agent receives a non-empty bundle. Moreover, this allocation admits a lower* EQ1 *witness.*

The modification to the algorithm and the proof of this theorem are provided in the full version. Applying this result to the graph partitioning problem, we obtain the following corollary.

Corollary 5.5. *Given an undirected graph $G = (V, E)$ with maximum degree Δ and an integer $k \leq |V|$, there exists a polynomial-time algorithm to partition V into k non-empty parts such that the cut values of the parts differ by at most Δ.*

Uniformly Dense Graphs

Given an undirected graph $G = (V, E)$, the density $\rho(S)$ of a subset $S \subseteq V$ is defined as the ratio of the number of edges with both endpoints in S, to $|S|$, i.e., $\rho(S) = \frac{|E(S)|}{|S|}$, where $E(S)$ denotes the set of such edges. The density function is non-negative, and the marginal contribution of any vertex to the density is at most 1 [5].

Barman and Verma [5] showed that for any undirected graph and integer k, the vertex set can be partitioned into k non-empty parts such that the densities of any two parts differ by at most 4. As a direct consequence of Theorem 5.1, we improve this factor to 1.

Corollary 5.6. *Given an undirected graph $G = (V, E)$ and an integer $k \leq |V|$, there exists a partition of V into k non-empty parts such that the densities of any two parts differ by at most* 1.

6 IDENTICAL SUBADDITIVE VALUATIONS

We shall now consider a special case of subadditive valuations, where all agents have identical valuation functions. Given such an instance, we show that there exists an algorithm that always returns an EQ1 allocation, provided that the common valuation function $v(\cdot)$ is non-negative for the grand bundle M, i.e., $v(M) \geq 0$.

Theorem 6.1. *Given a fair division instance $\mathcal{I} = \langle N, M, \{v\}_{i \in N} \rangle$, where the agents share an identical subadditive valuation function and the grand bundle is valued non-negatively, there exists an* EQ1 *allocation.*

Note that when agents have identical valuations, an allocation that is EQ1 is also EF1. Therefore, we have the following corollary.

Corollary 6.2. *Given a fair division instance $\mathcal{I} = \langle N, M, \{v\}_{i \in N} \rangle$, where the agents share an identical subadditive valuation function and the grand bundle is valued non-negatively, there exists an EF1 allocation.*

The proofs and algorithms of this section are deferred to the full version.

7 CONCLUSION

In this paper, we studied the problem of finding an equitable up to one item (EQ1) allocation for indivisible items under general valuations. We explored the existence and computability of such allocations under the assumption that agents agree on the sign of their utility for the grand bundle.

We worked on the setting where every agent values the grand bundle non-negatively (since the case where all agents value the grand bundle non-positively is analogous). In this setting, our results establish non-existence and hardness for finding EQ1 allocations in the most general scenario, but also provide constructive existence guarantees for specific cases, such as submodular valuations, doubly monotone valuations, non-negative valuations and identical subadditive valuations. While our algorithm for submodular valuations and doubly monotone valuations runs in polynomial time, our algorithm for non-negative valuations runs in exponential time. It would be interesting to see whether EQ1 allocations can be computed for non-negative allocations in polynomial time. Another interesting question is whether EQ1 exists for non-identical subadditive valuations in this setting.

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